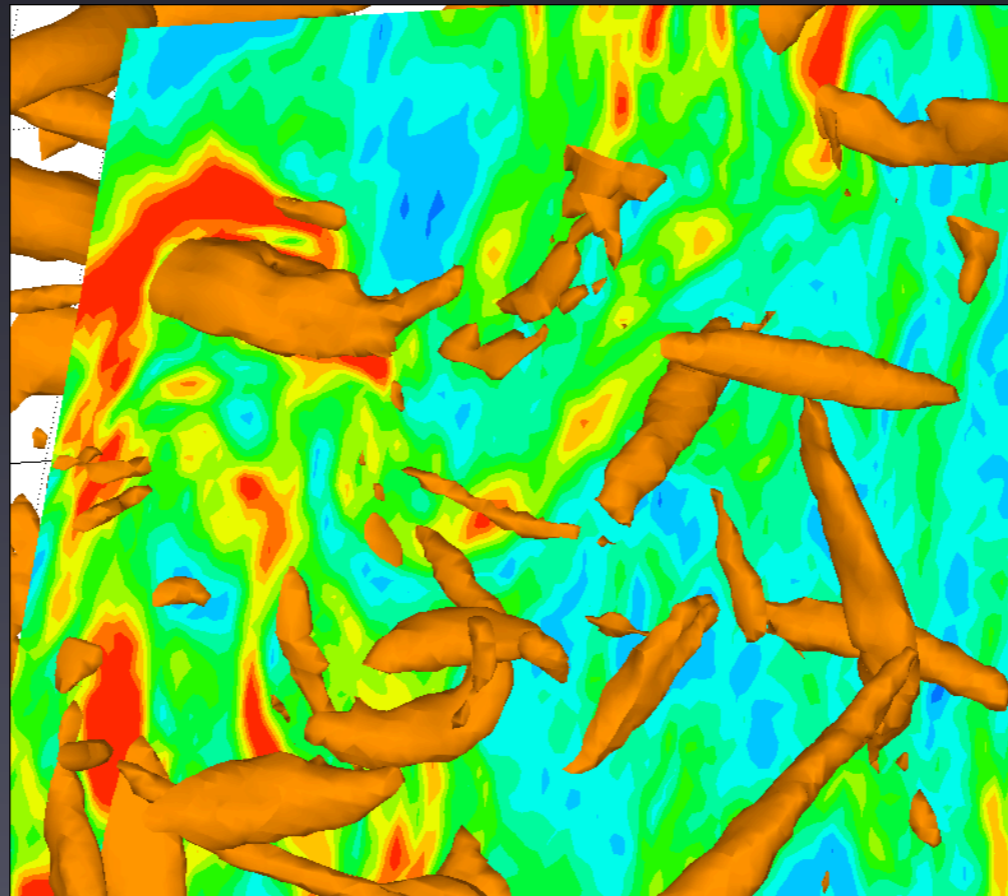


# The fine scale features of turbulent shear flows

O.R.H. Buxton

*Department of Aeronautics, Imperial College London*



Acknowledgements : EPSRC grant number EP/FO56206/1; Doctoral Training Account;  
HECToR RAP; RAeS Centenary Grant

## Introduction

“The most important unsolved problem of classical physics” [RICHARD FEYNMAN]

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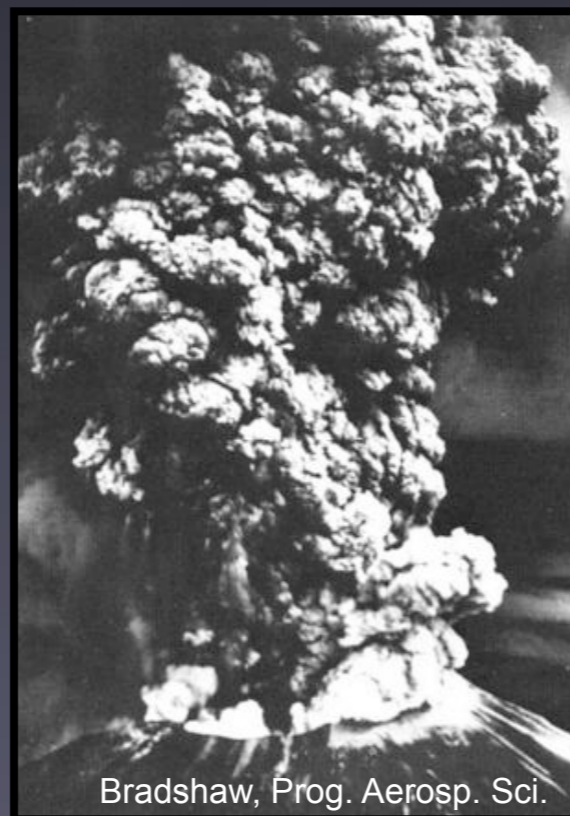
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  - Temporally and spatially



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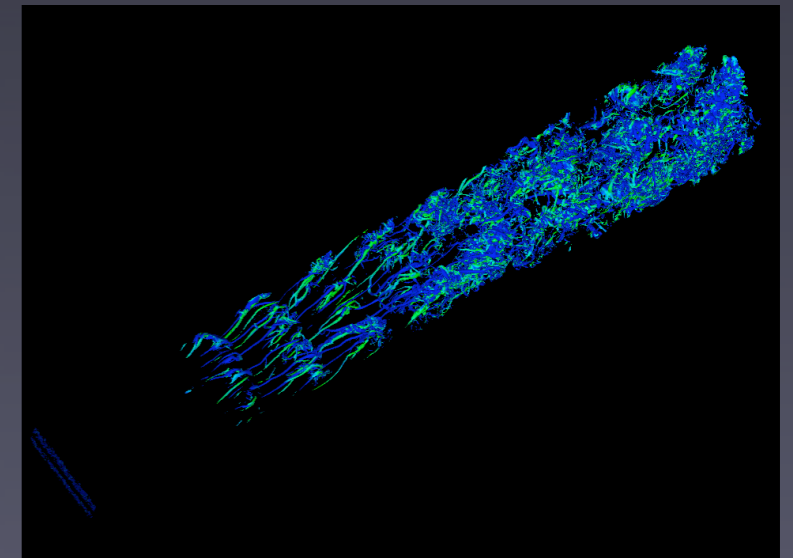
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  - High spatial resolution required to compute / measure them  $\eta = (\nu^3 / \langle \epsilon \rangle)^{1/4}$
- Models of fine scales required for LES





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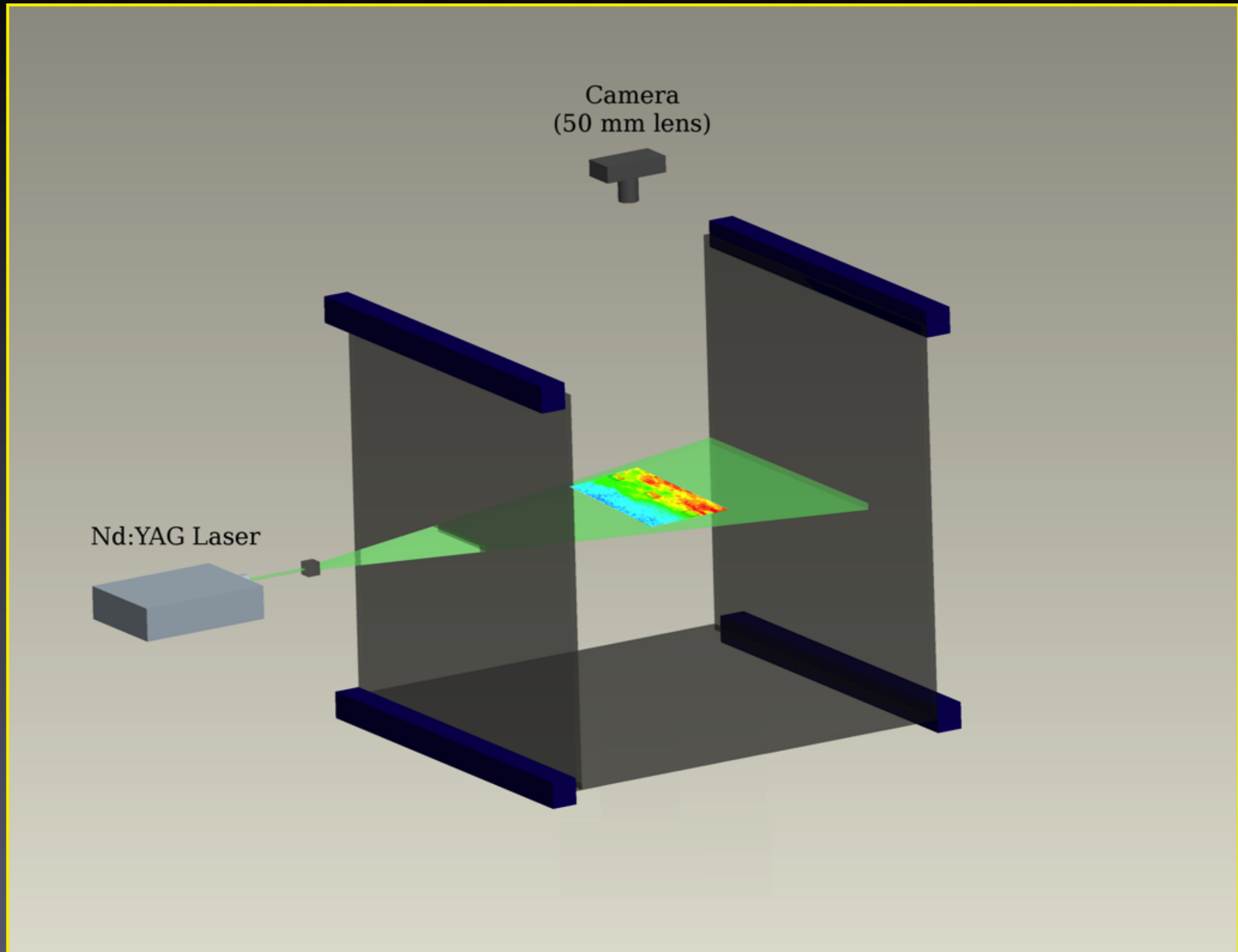
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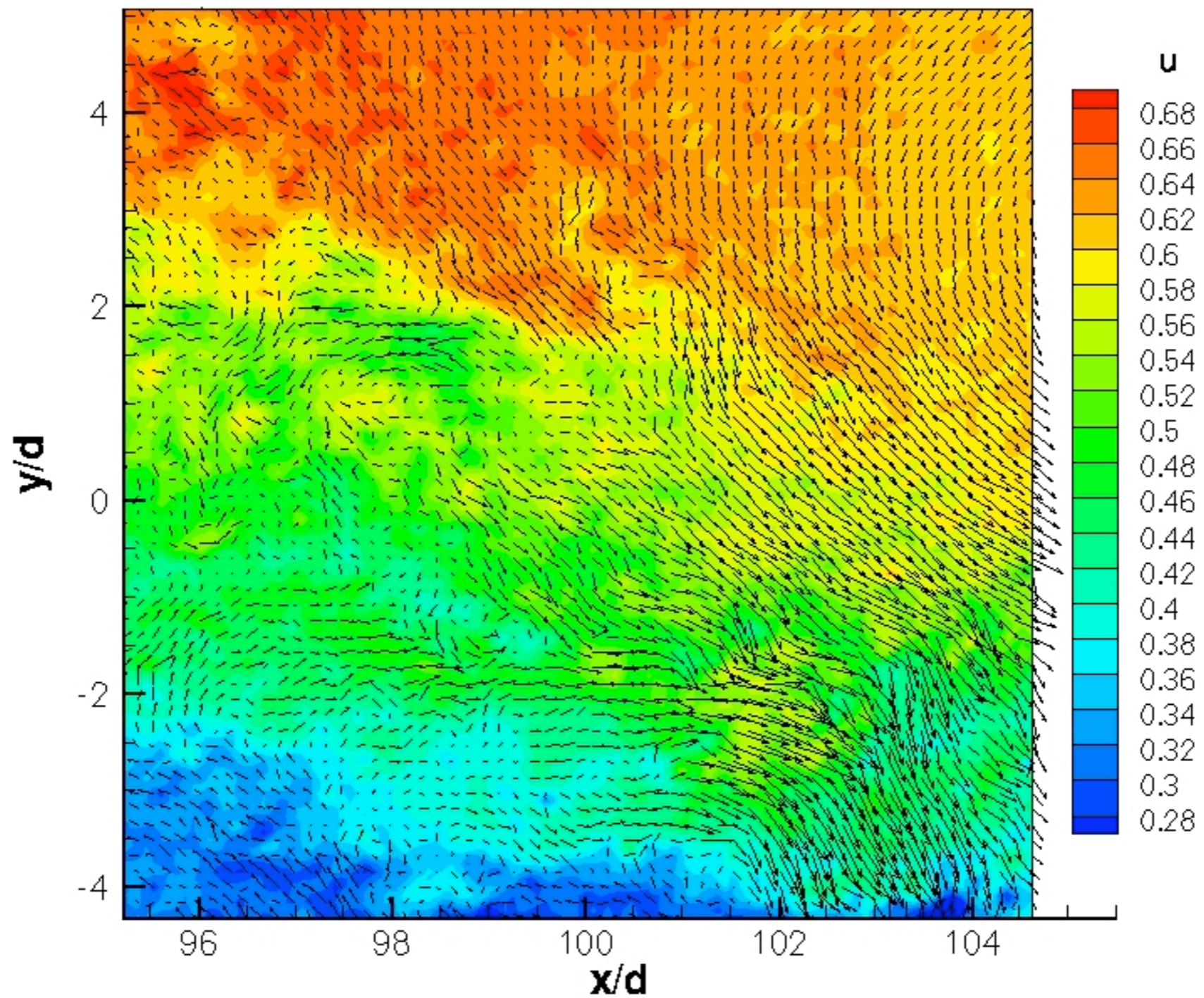
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  - Far field of planar 2D mixing layer at different Reynolds numbers
- Examine multi-scale interaction of fine-scales conditioned on large scales
  - Convection velocities
  - Probability density functions *pdfs*

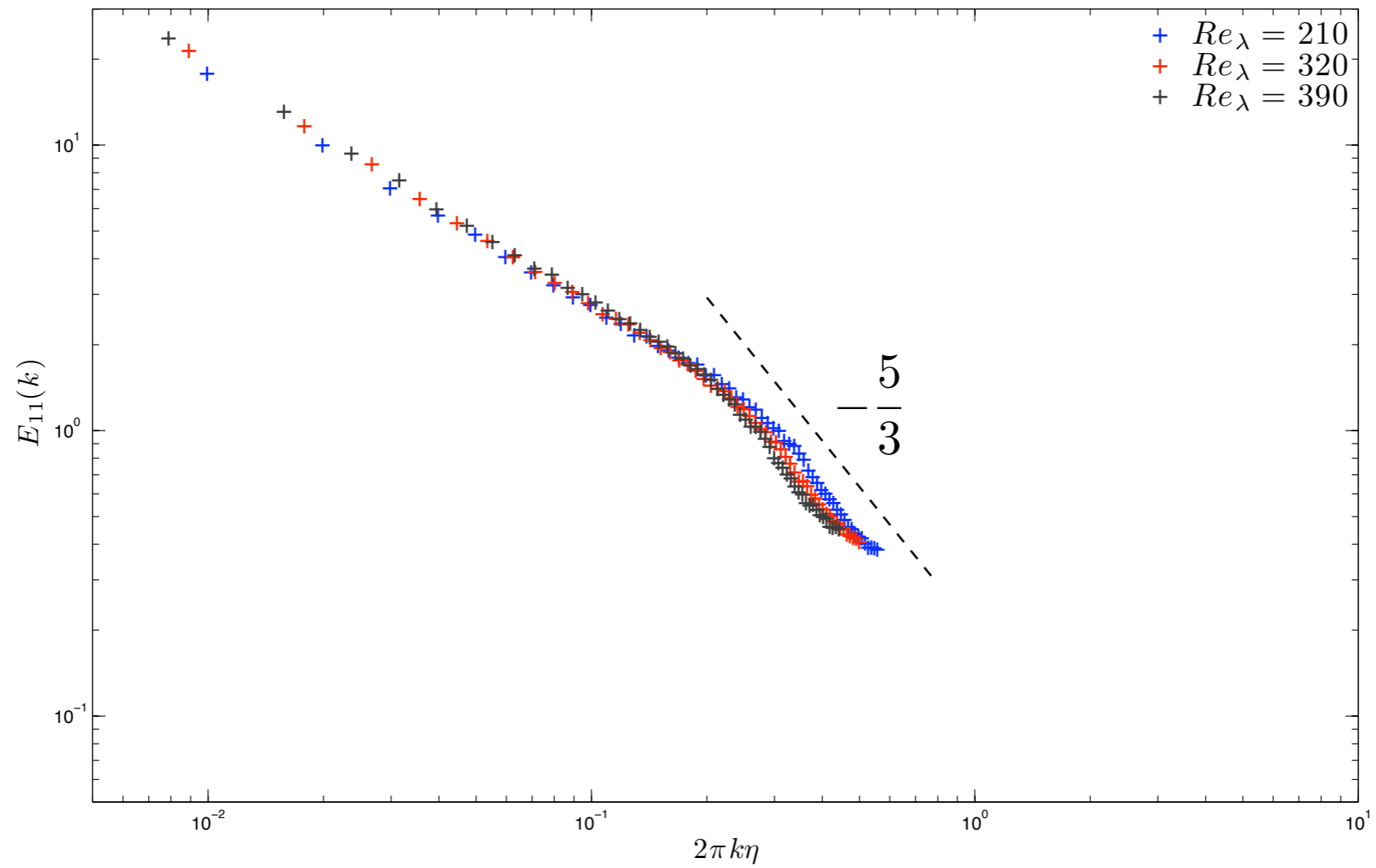
# PIV



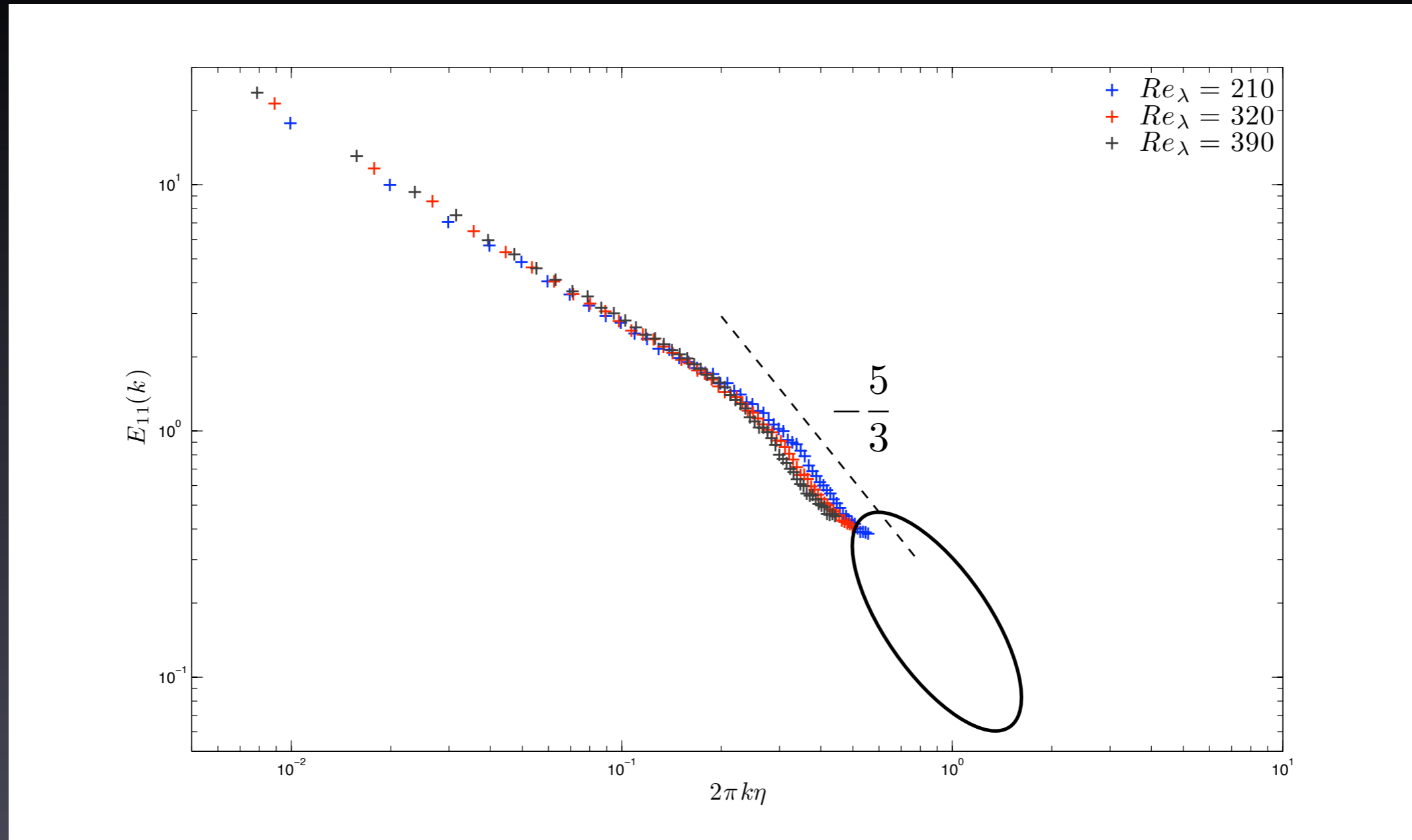
# Planar mixing layer



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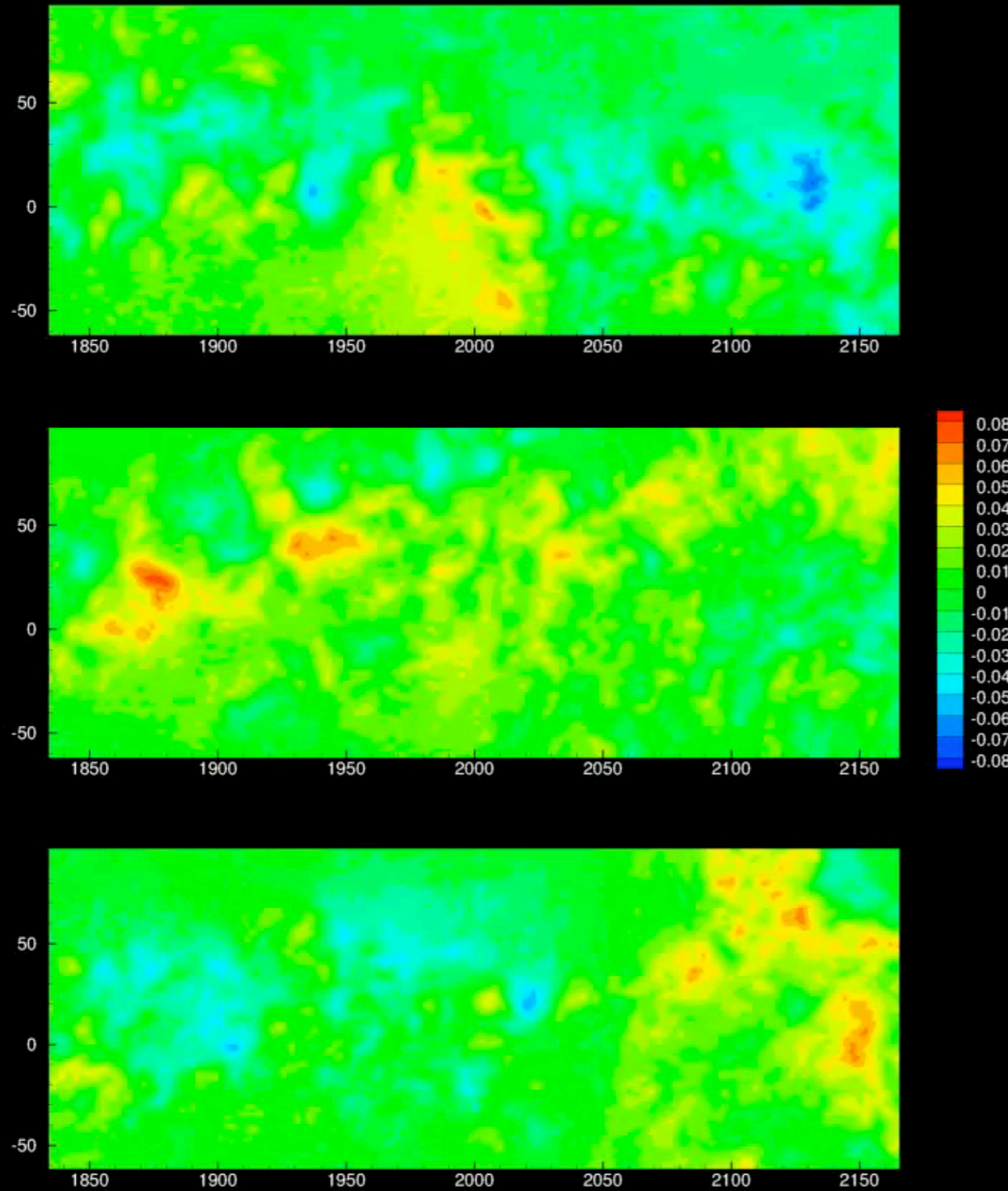


- What about the energy content of the fine-scales?



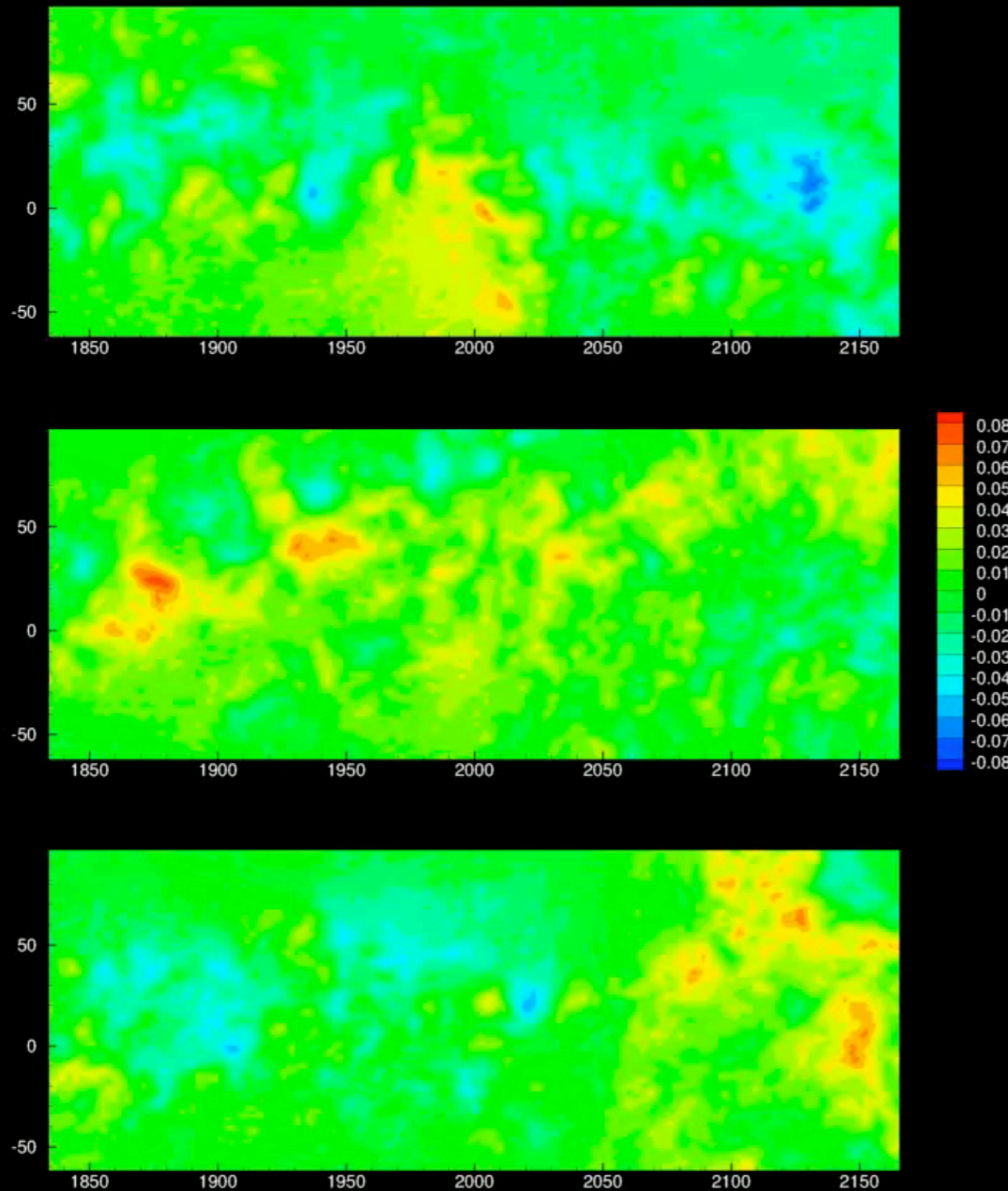
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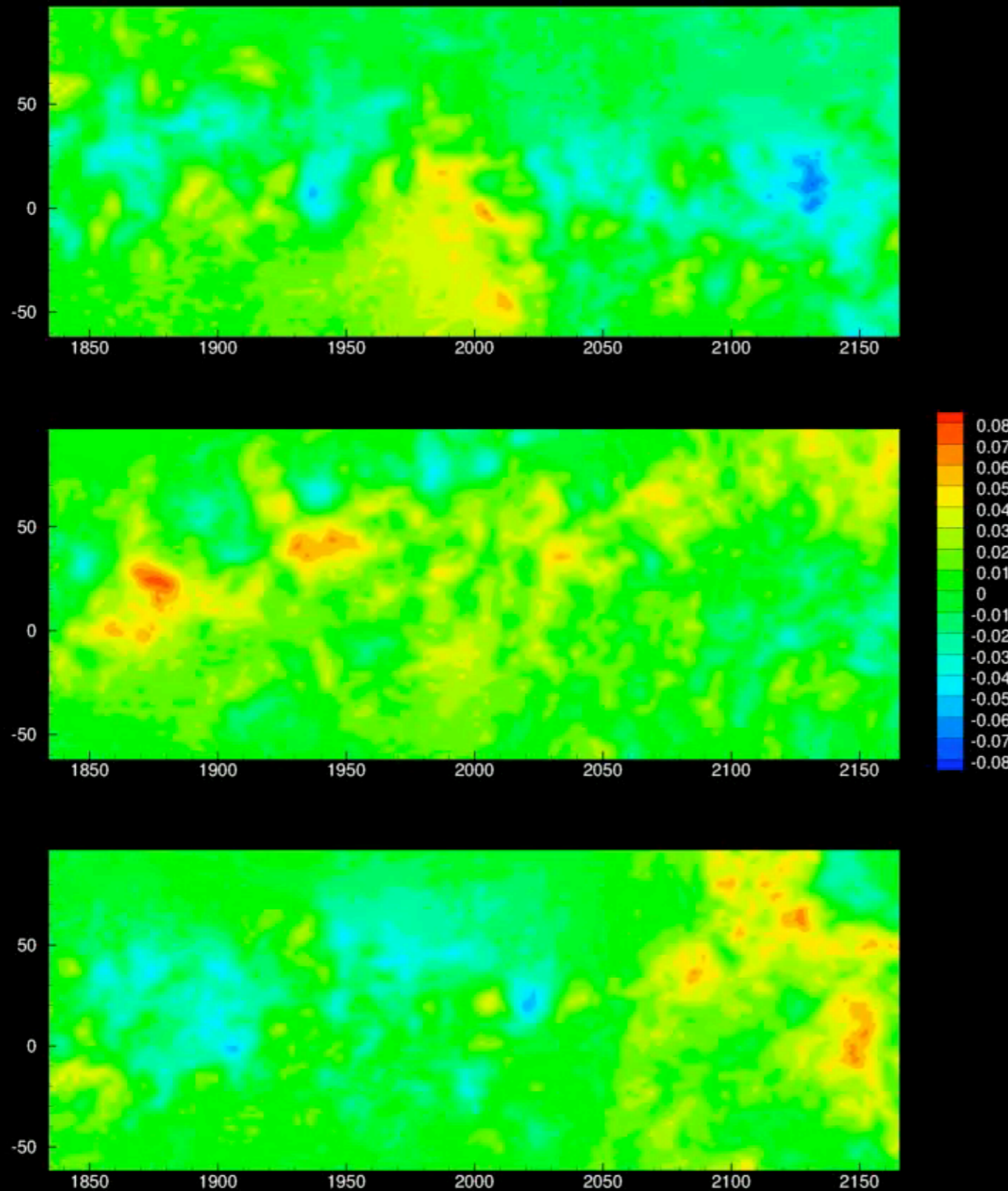


- Evidently multi-scale

# Planar mixing layer

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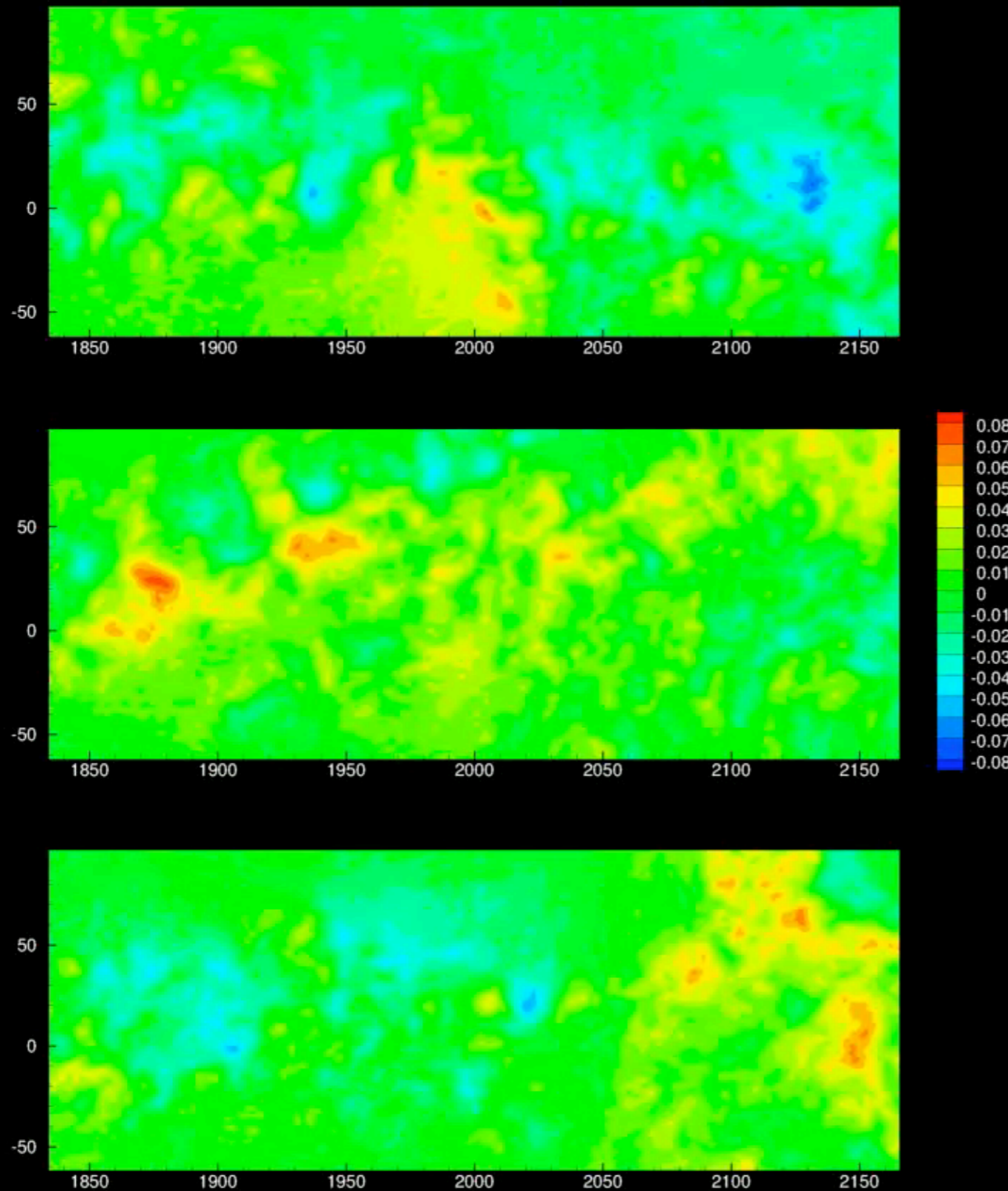


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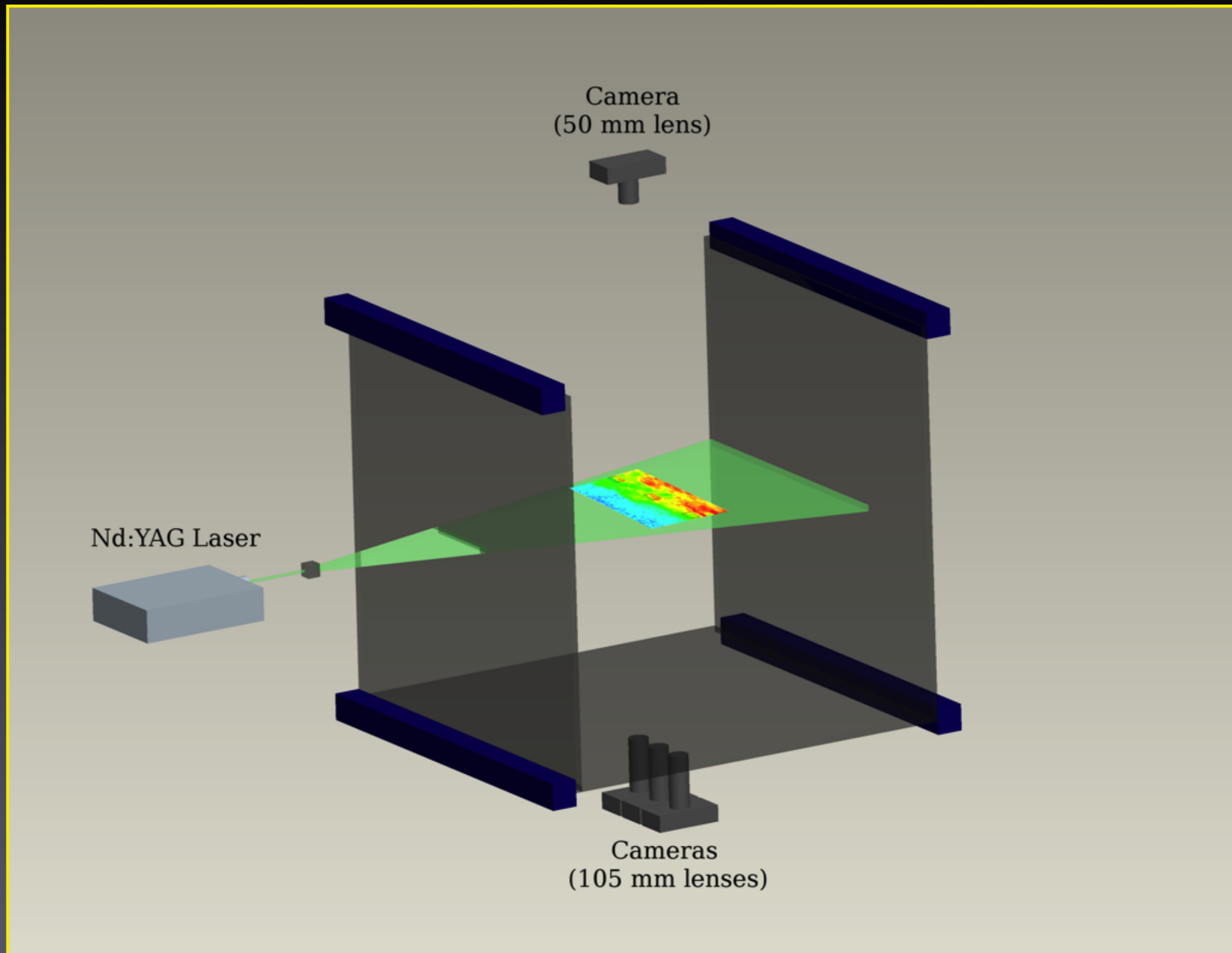
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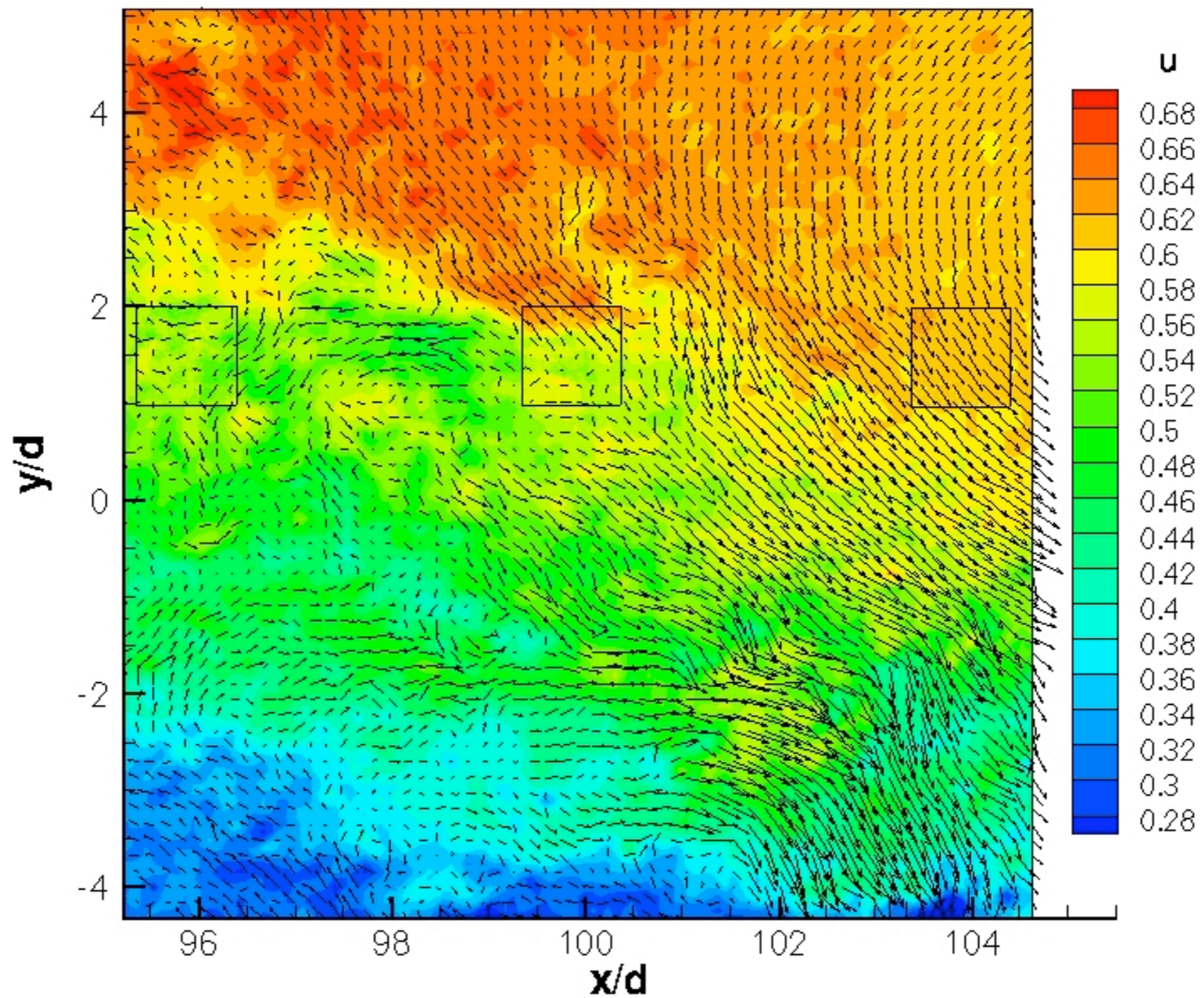


- Evidently multi-scale
- Convection velocities obtained by cross correlation
- Is the convection velocity scale dependent?

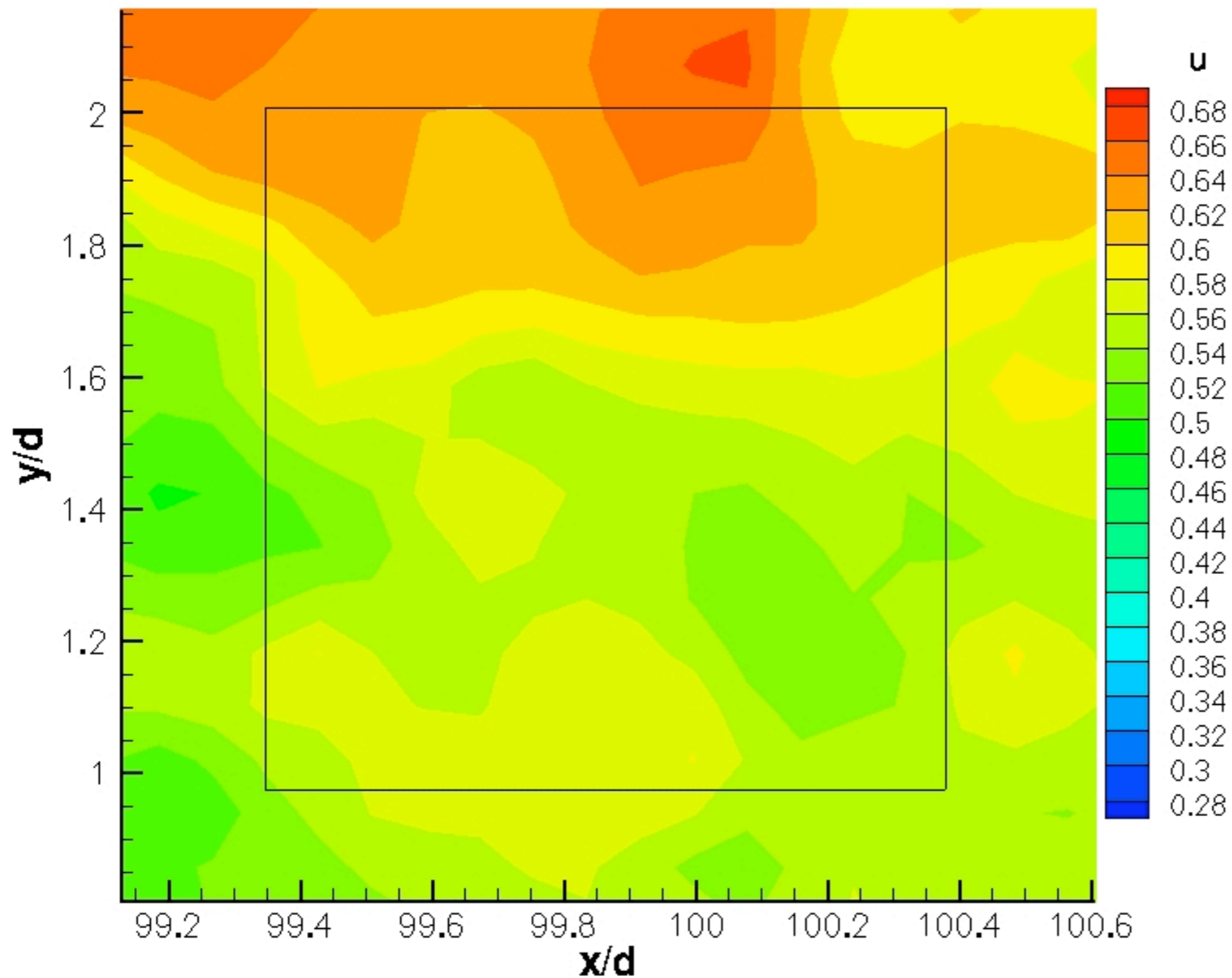
# Synchronised multi-scale PIV



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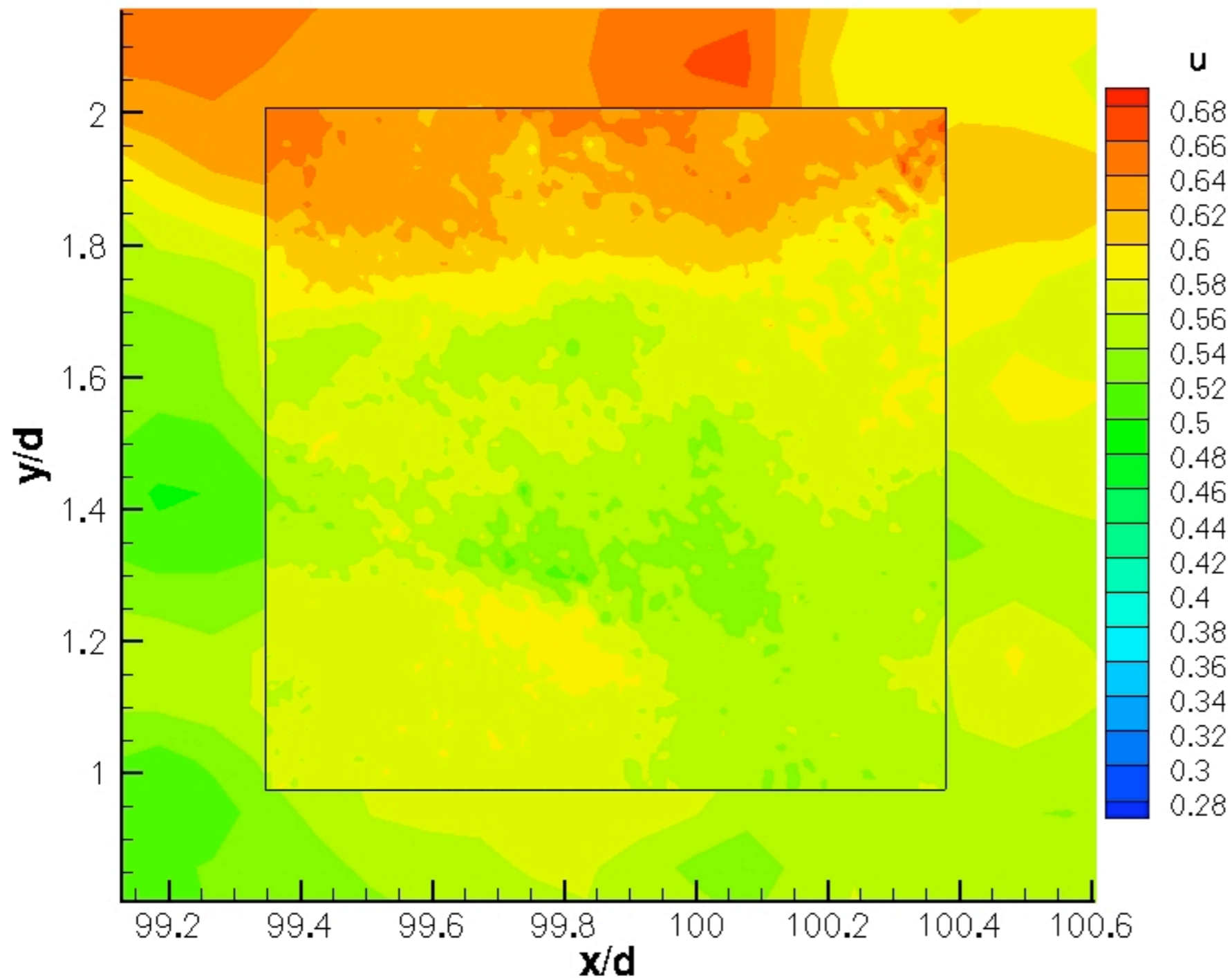


# Planar mixing layer



- Adjacent vectors separated by 1.62 mm (resolution = 3.24 mm, 50% overlap)
  - $5.6\eta$
  - $0.2\lambda$

# Planar mixing layer



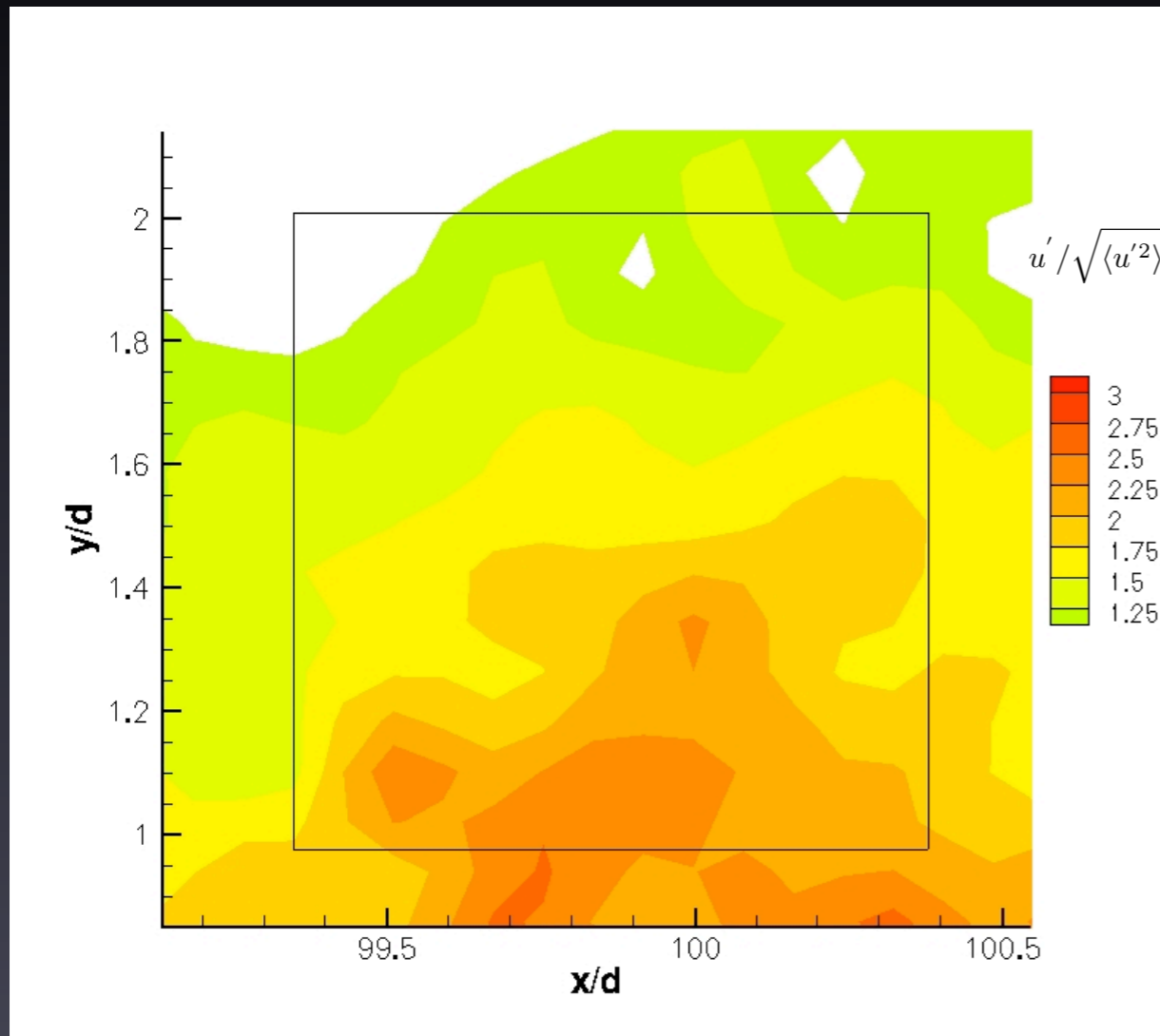
- Adjacent vectors separated by 0.18 mm (resolution = 0.36 mm, 50% overlap)
  - $0.62\eta$
  - $0.02\lambda$



## Planar mixing layer

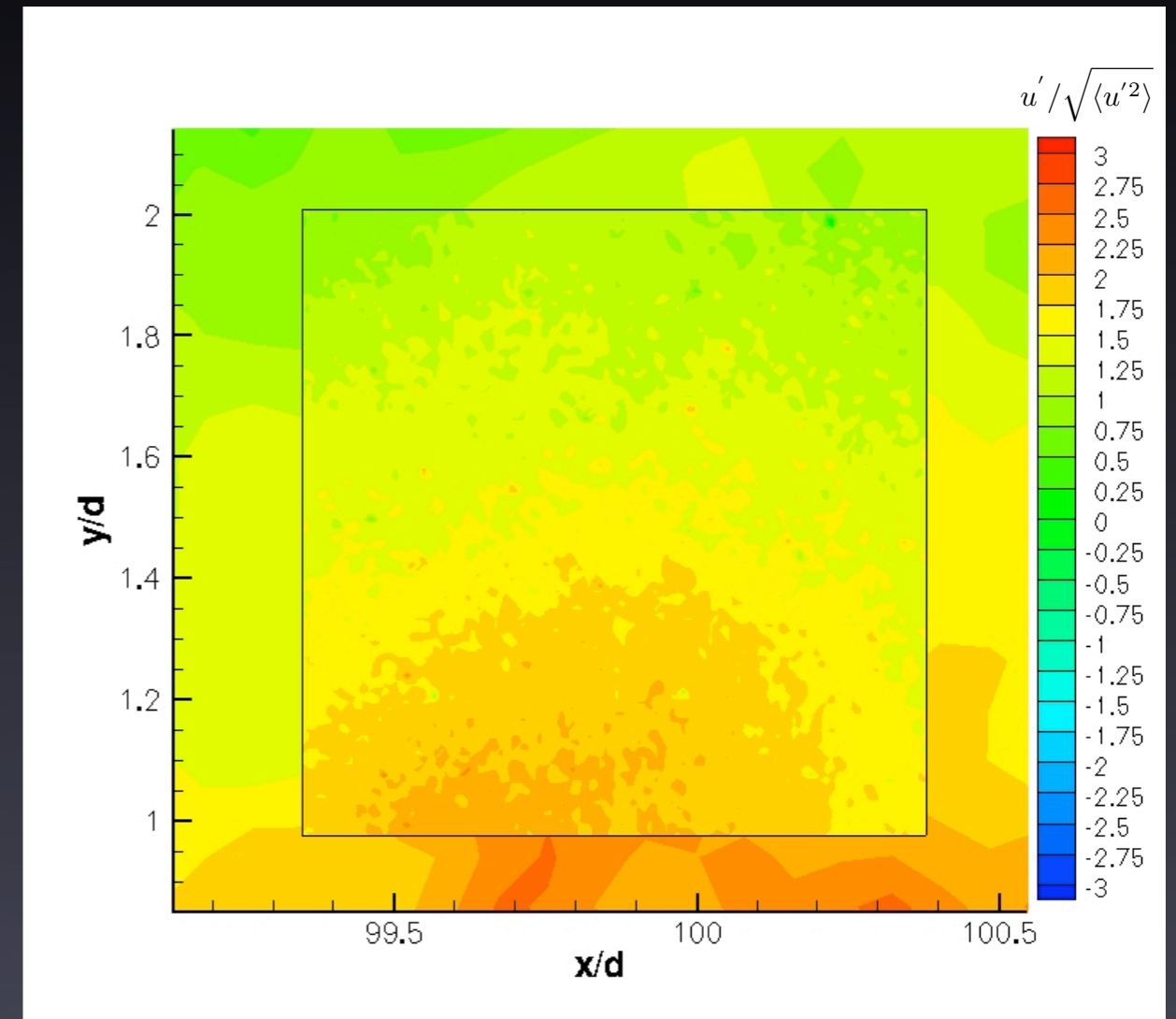
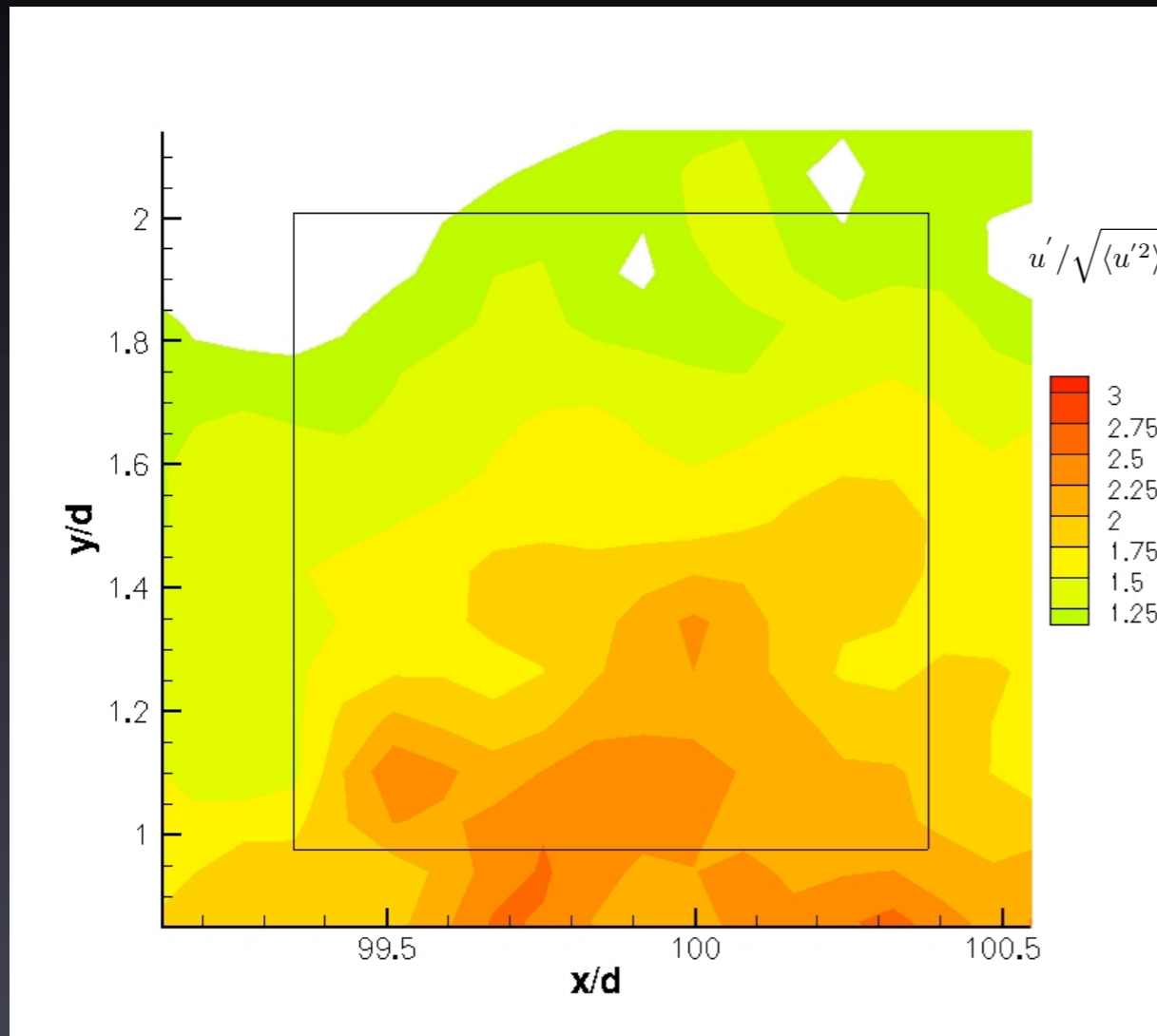
- Probability density function for  $|u'| > \sqrt{\langle u'^2 \rangle}$

## Planar mixing layer



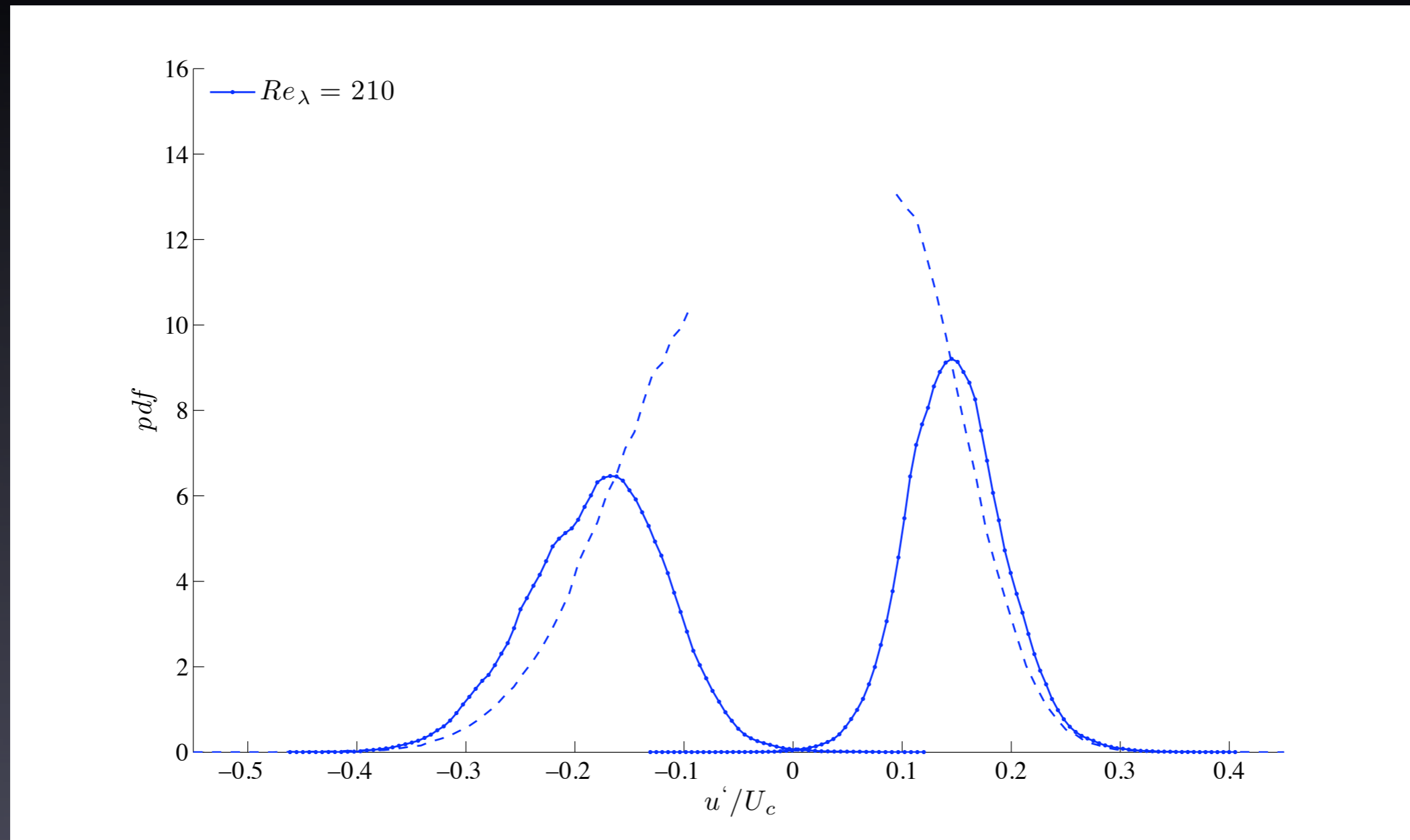
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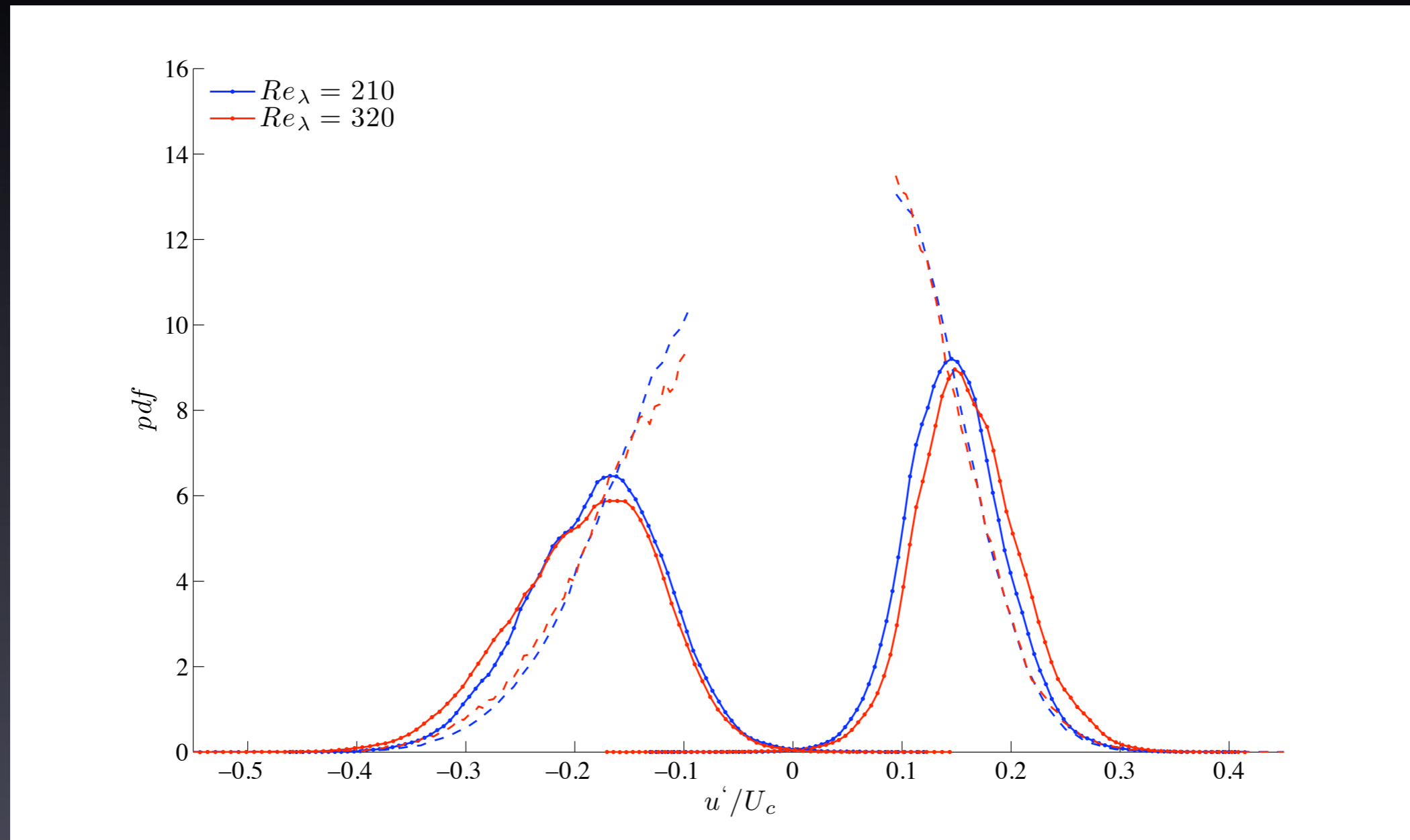
- Probability density function for  $|u'| > \sqrt{\langle u'^2 \rangle}$
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- Now use small scale data to produce conditional *pdf*

## Planar mixing layer



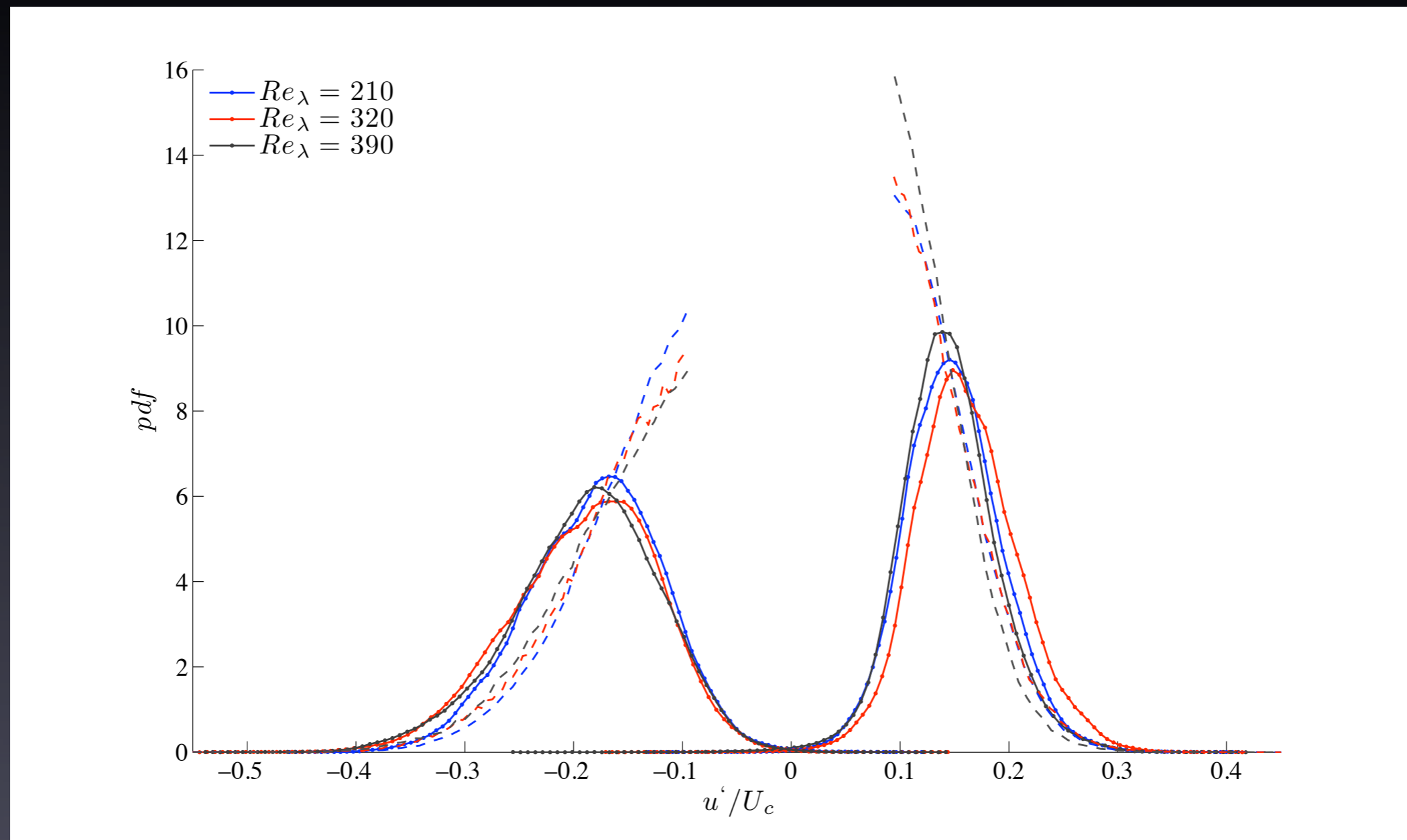
- Probability density function for  $|u'| > \sqrt{\langle u'^2 \rangle}$ 
  - Dashed line : large scale information
  - Solid line : proportion of  $1 - e^{-2}$  of large scale information exceeds threshold

## Planar mixing layer



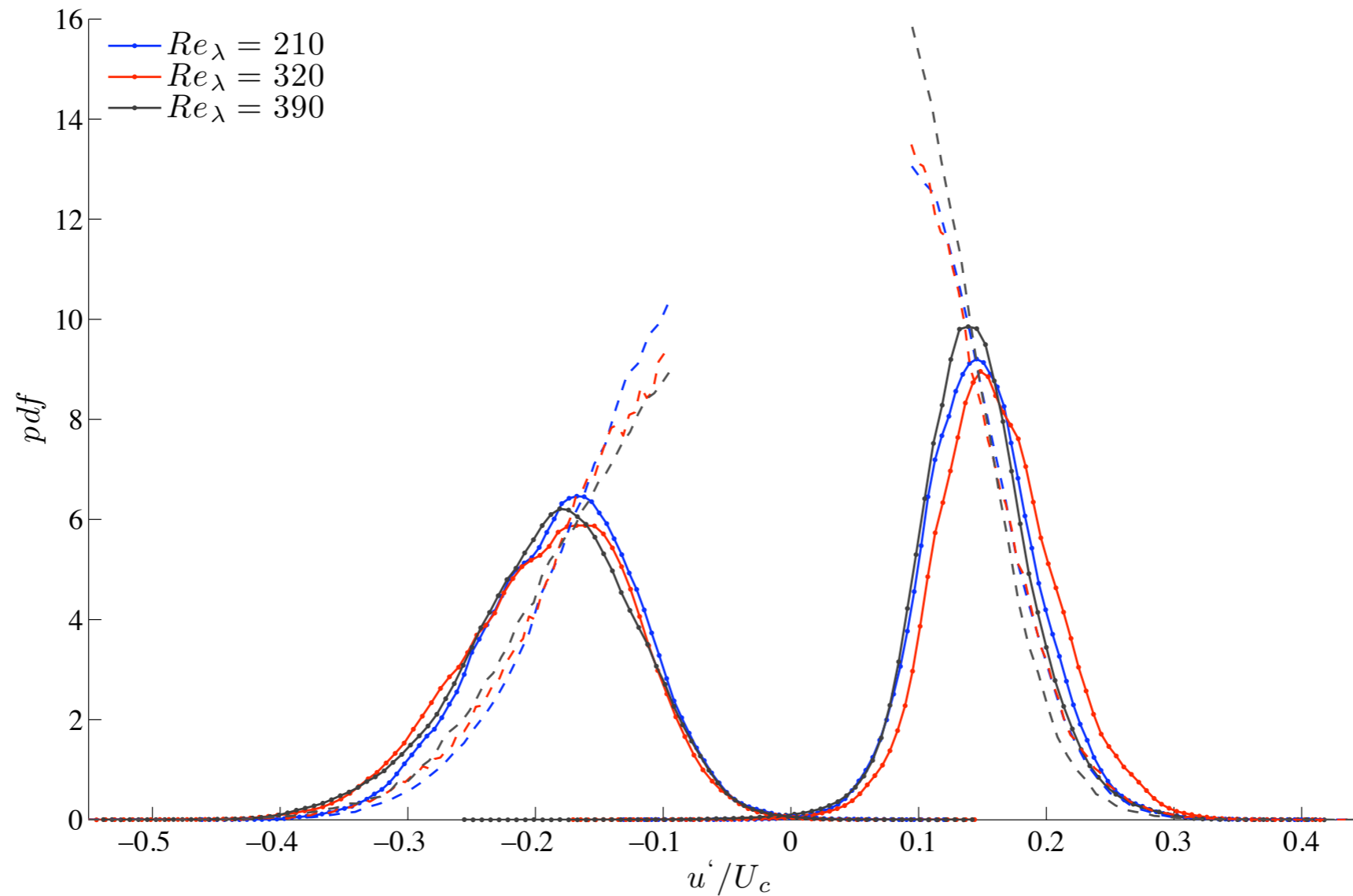
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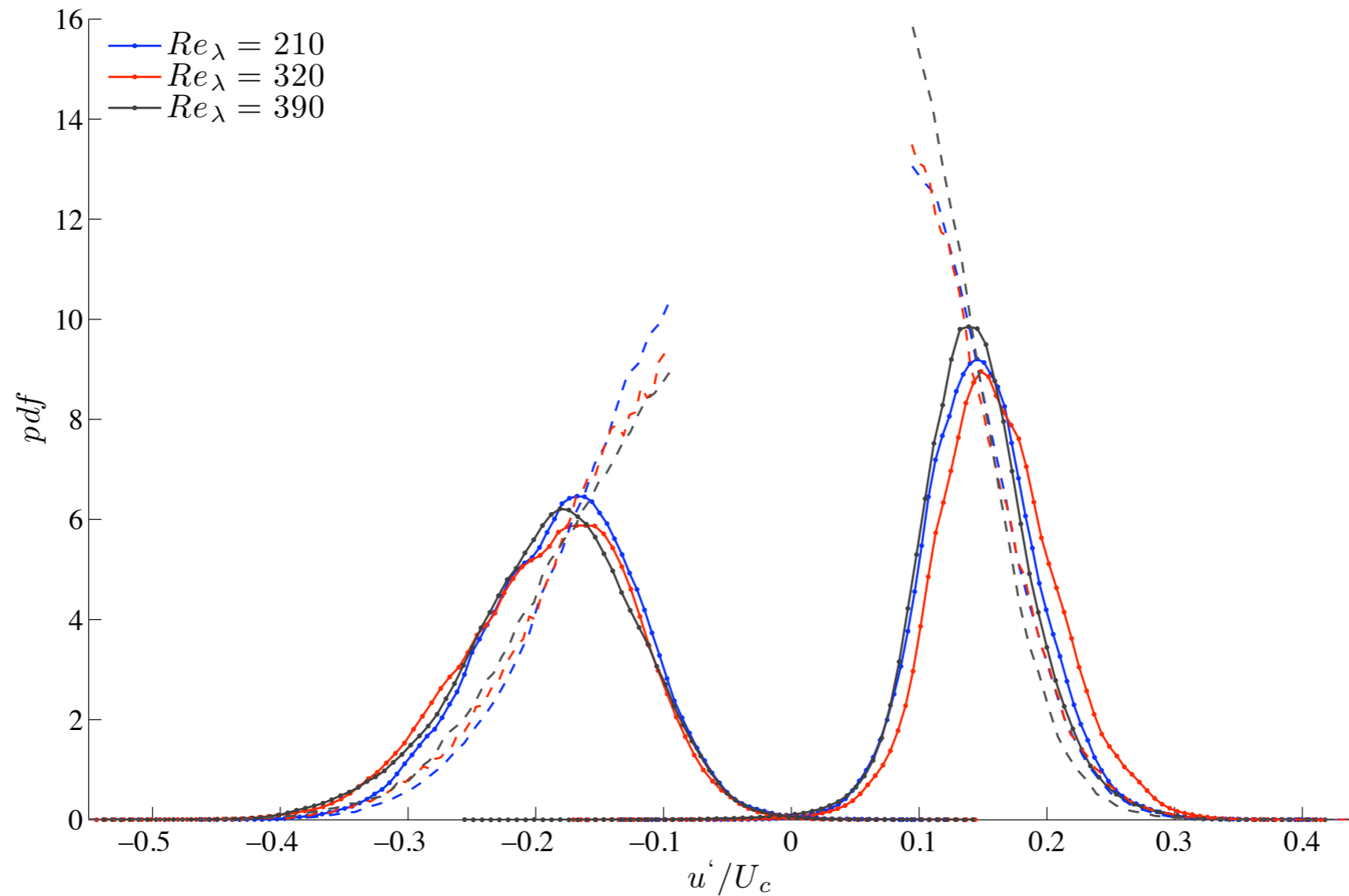
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## Planar mixing layer



- Different behaviour for positive and negative fluctuations
  - Longer tails for negative fluctuations

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  - Longer tails for negative fluctuations
- Reynolds number effect?



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3D data allows examination of velocity gradient quantities

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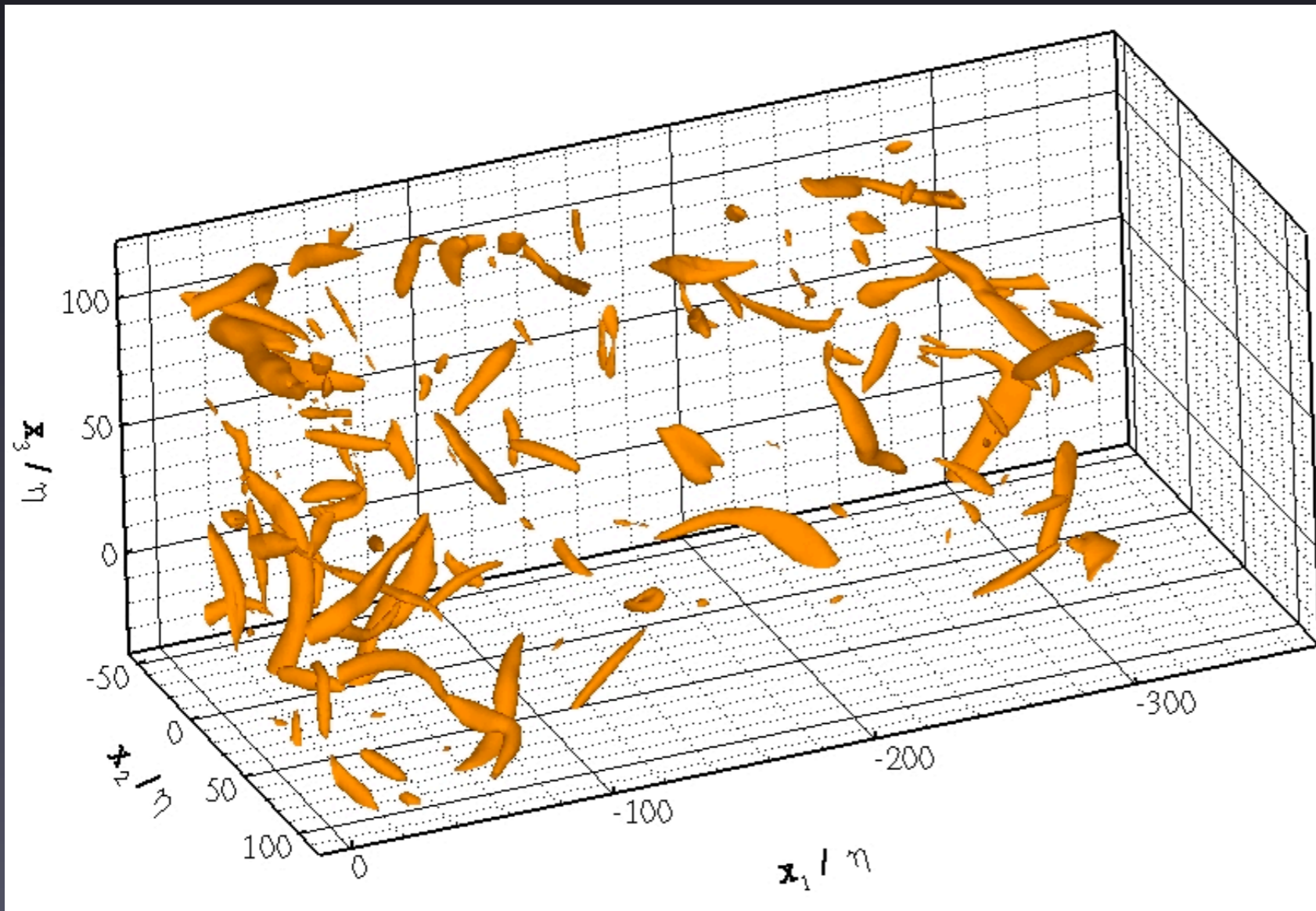
Dissipation ( $\epsilon$ )

$$\Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$$

Enstrophy ( $\omega^2$ )

## Turbulent axisymmetric jet

$$\Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$$



$\omega^2 = 75.0 \text{s}^{-1} = 3.55 \langle \omega^2 \rangle$

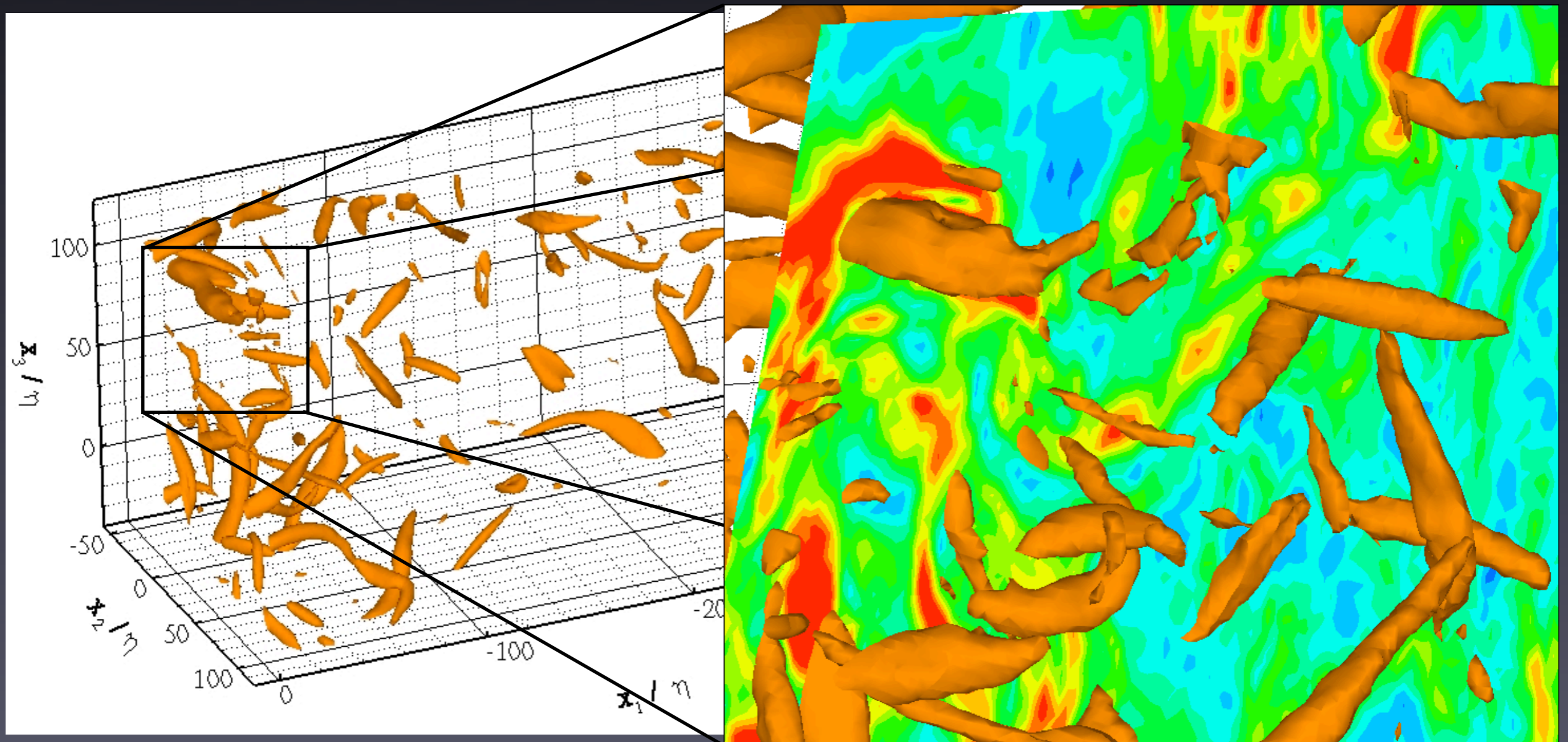
- Isosurfaces of enstrophy (rotation dominated)

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- Contours of dissipation (strain dominated)



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## Three dimensional data

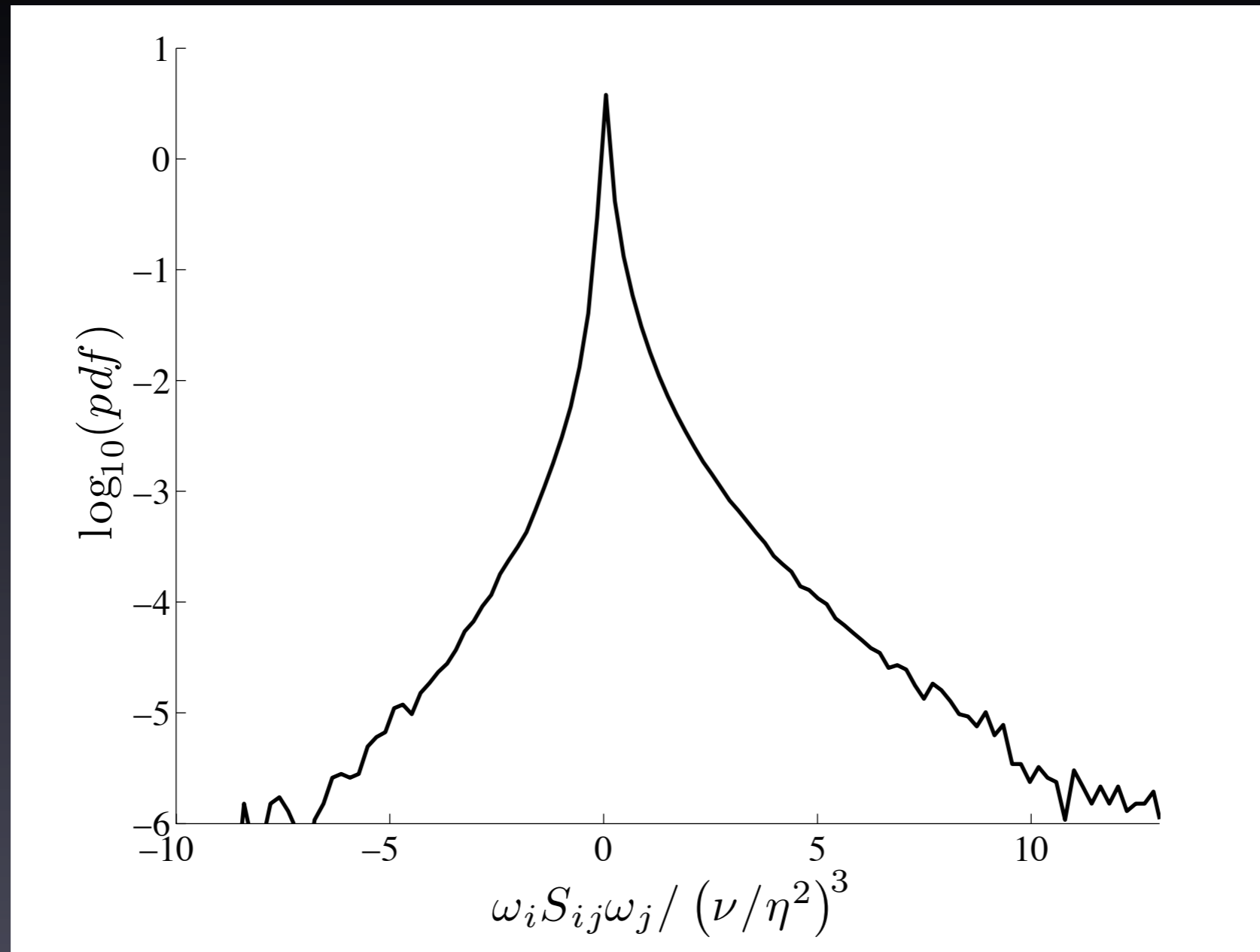
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- Quantity  $\omega_i S_{ij} \omega_j$  is the rate of enstrophy amplification

## Turbulent axisymmetric jet



- Enstrophy amplification favoured over attenuation
  - $\langle \omega_i S_{ij} \omega_j \rangle = 0.07 (\nu/\eta^2)^3$
  - 71% of data points enstrophy amplifying

## Three dimensional data

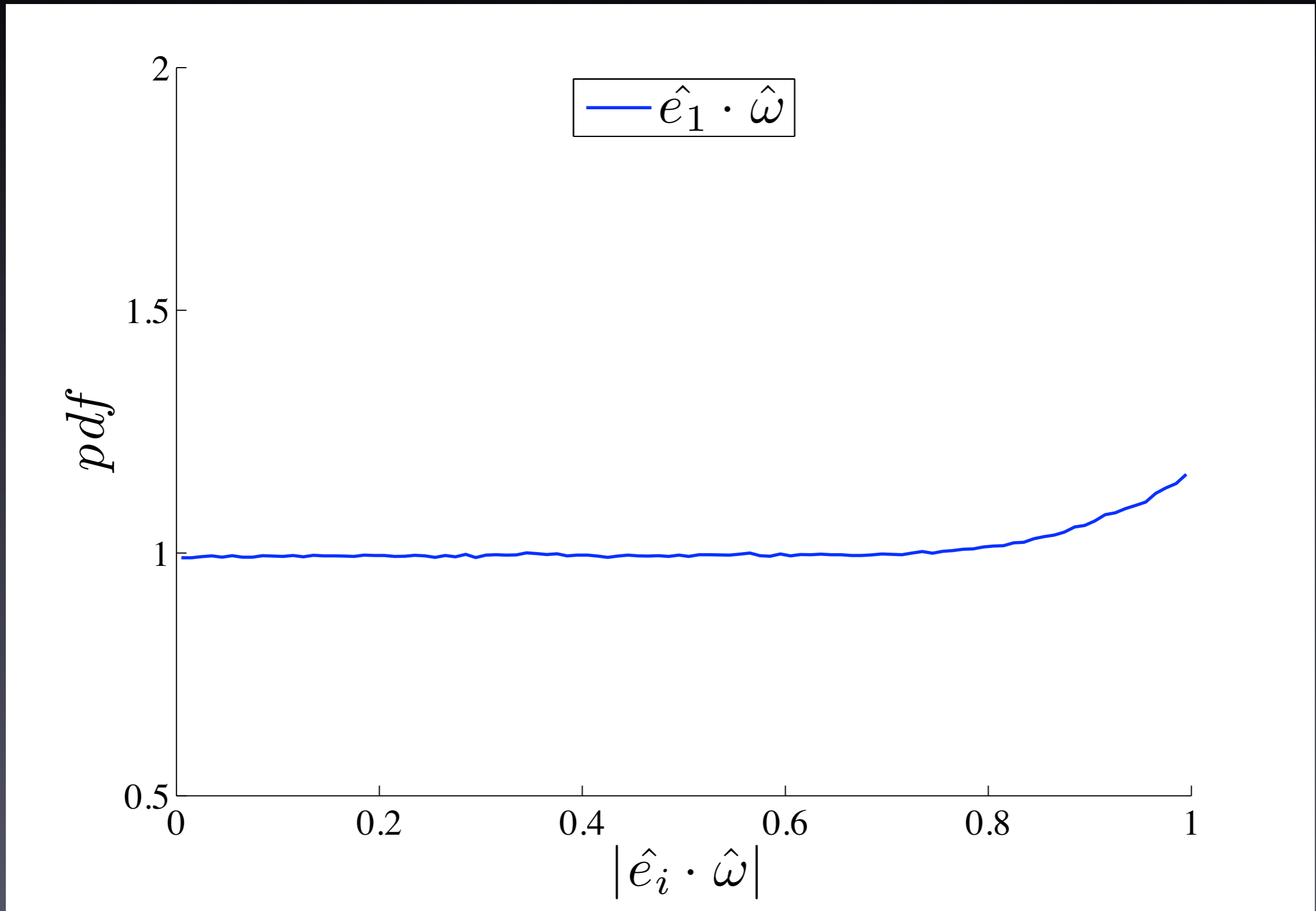
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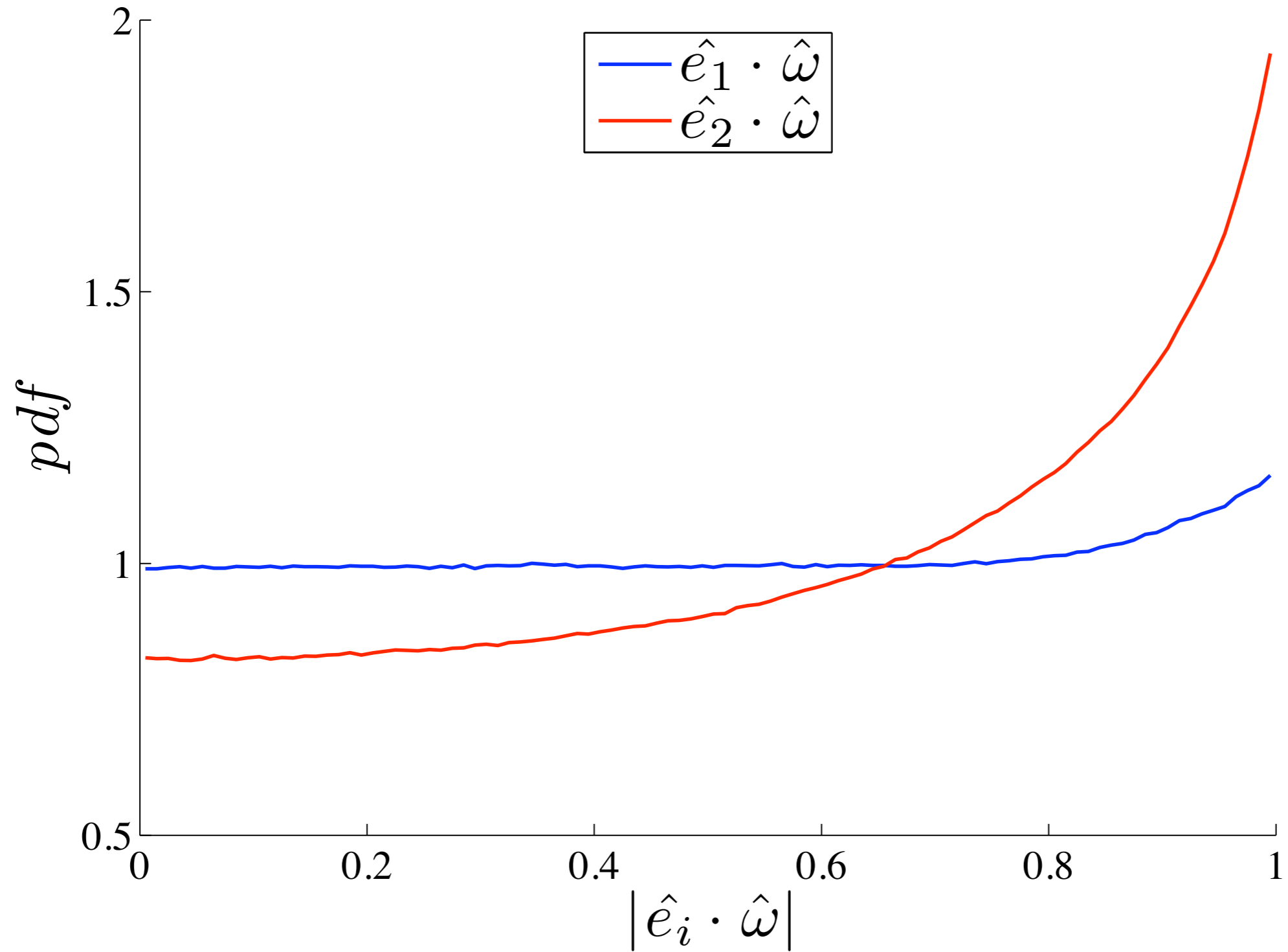
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- Quantity  $\omega_i S_{ij} \omega_j$  is the rate of enstrophy amplification
- Excellent metric for examining the interaction between strain-rate and rotation :  $\omega_i S_{ij} \omega_j = \omega^2 s_i (\hat{e}_i \cdot \hat{\omega})^2$

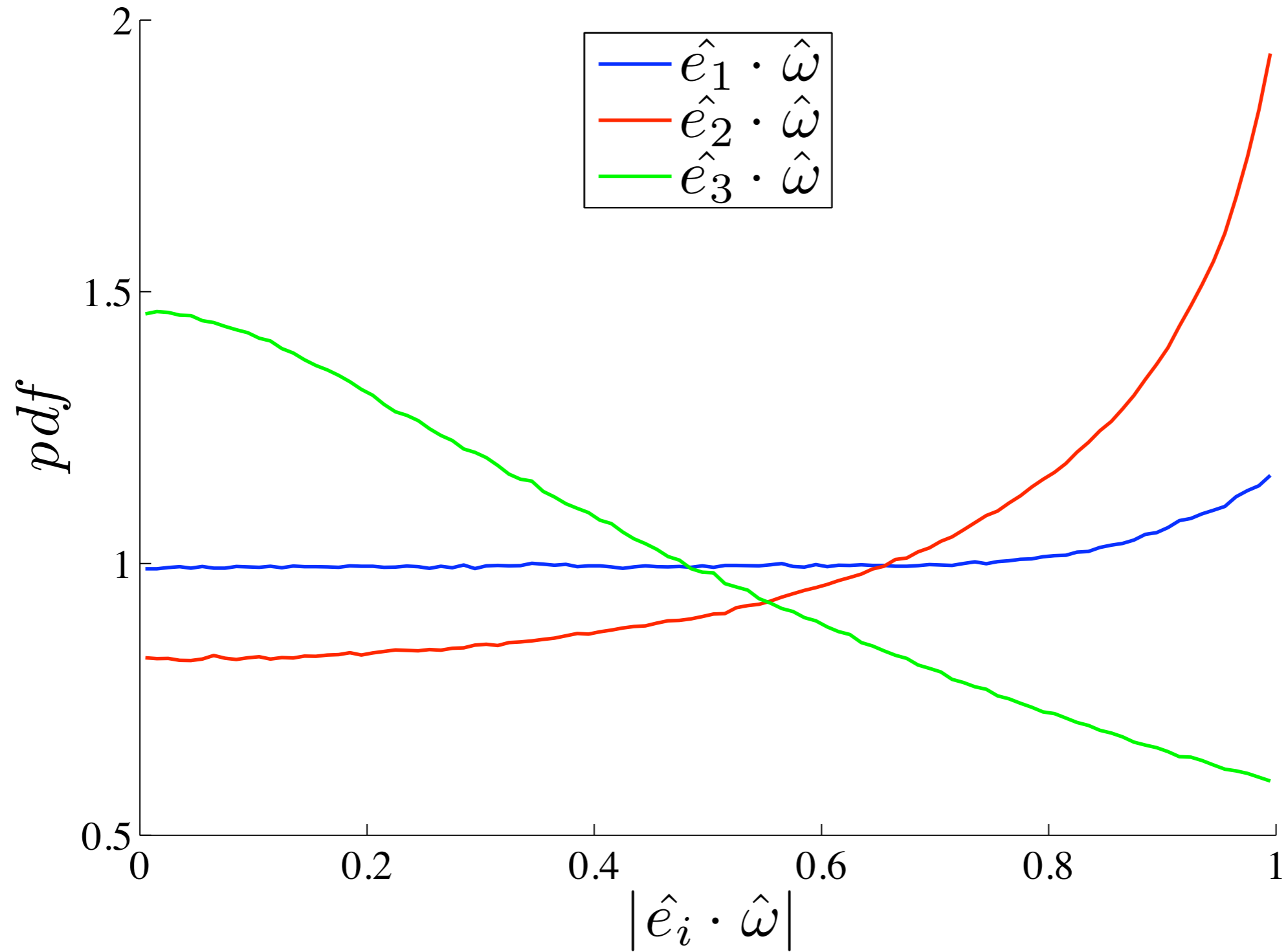
# Planar mixing layer



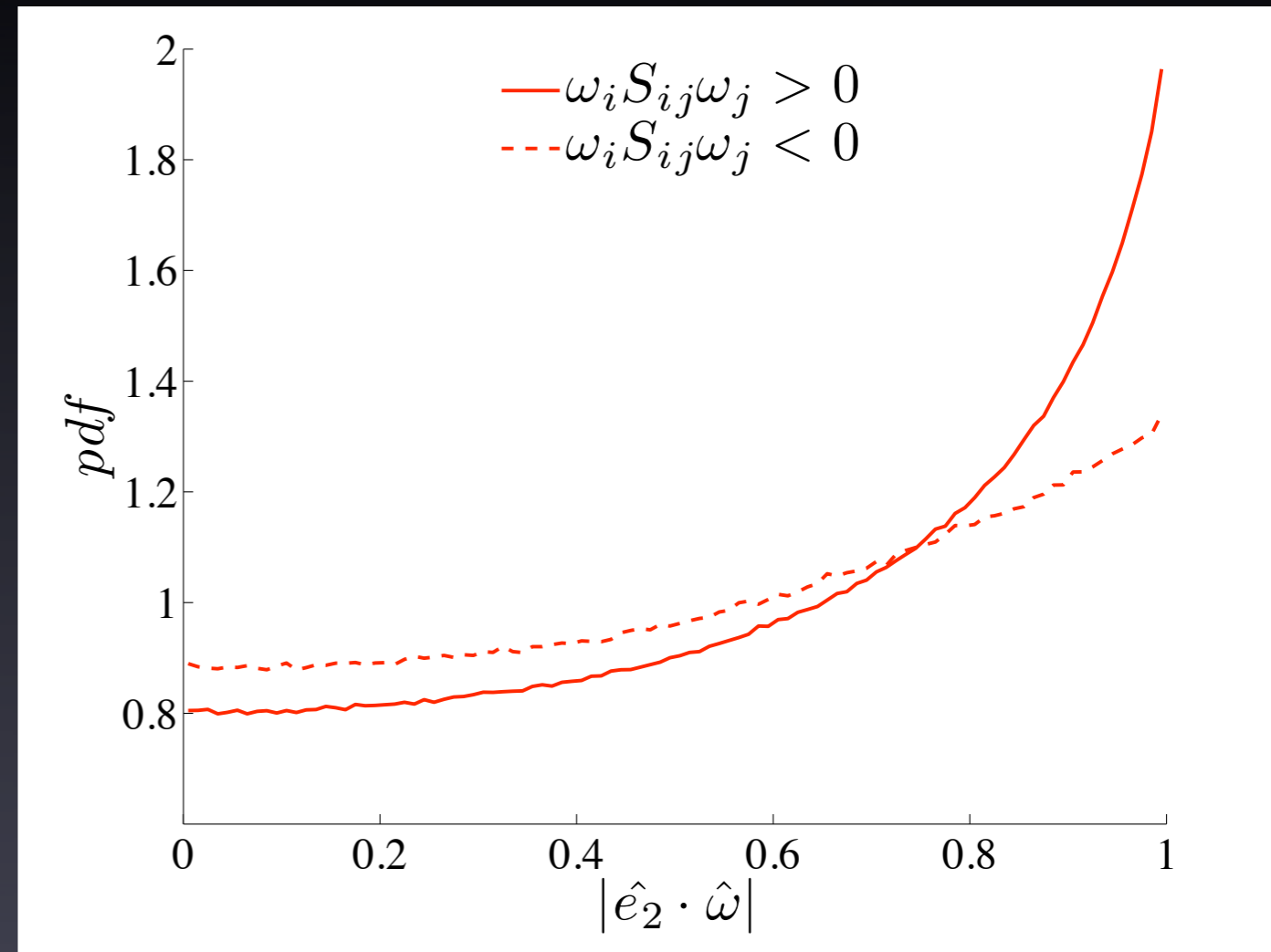
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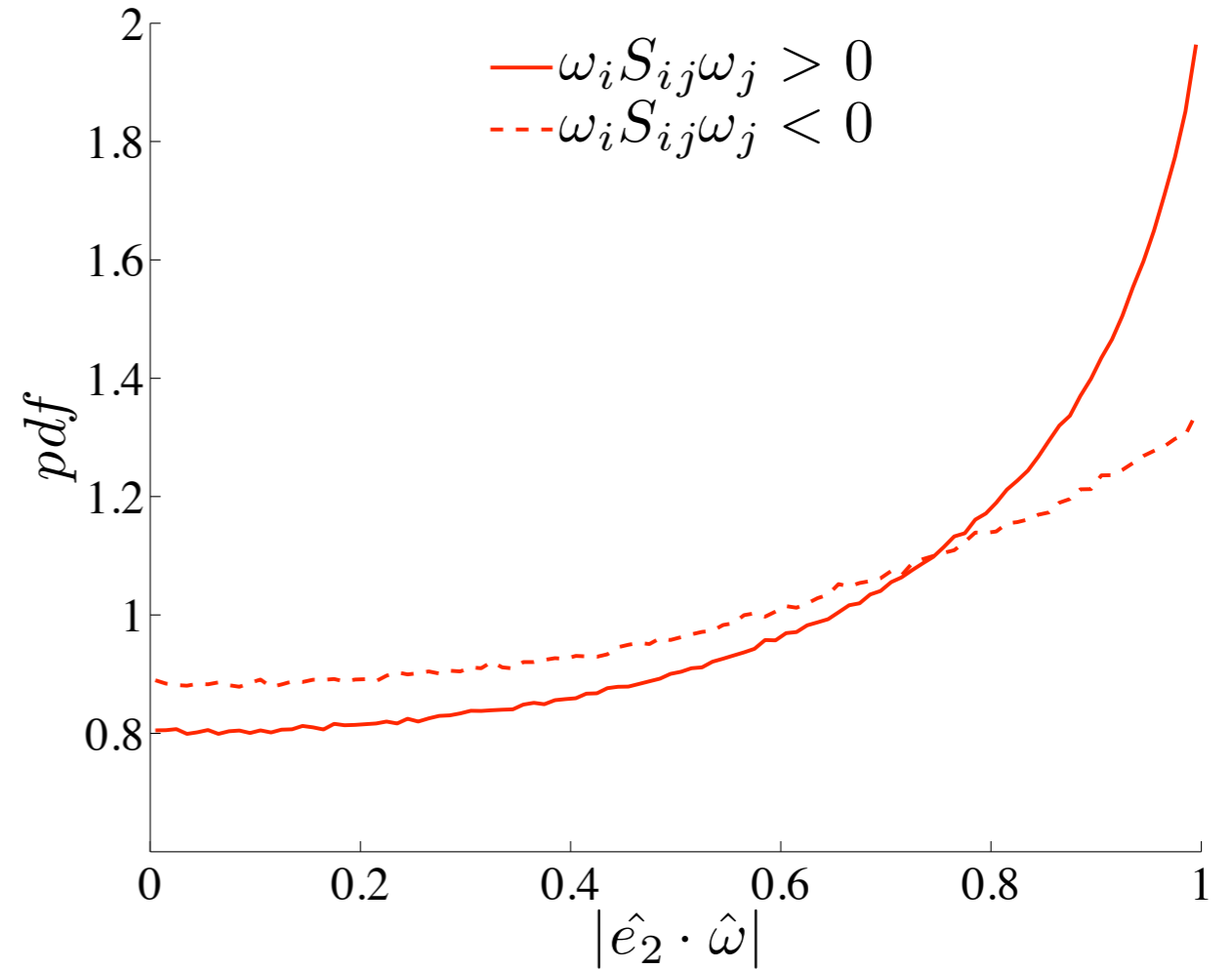
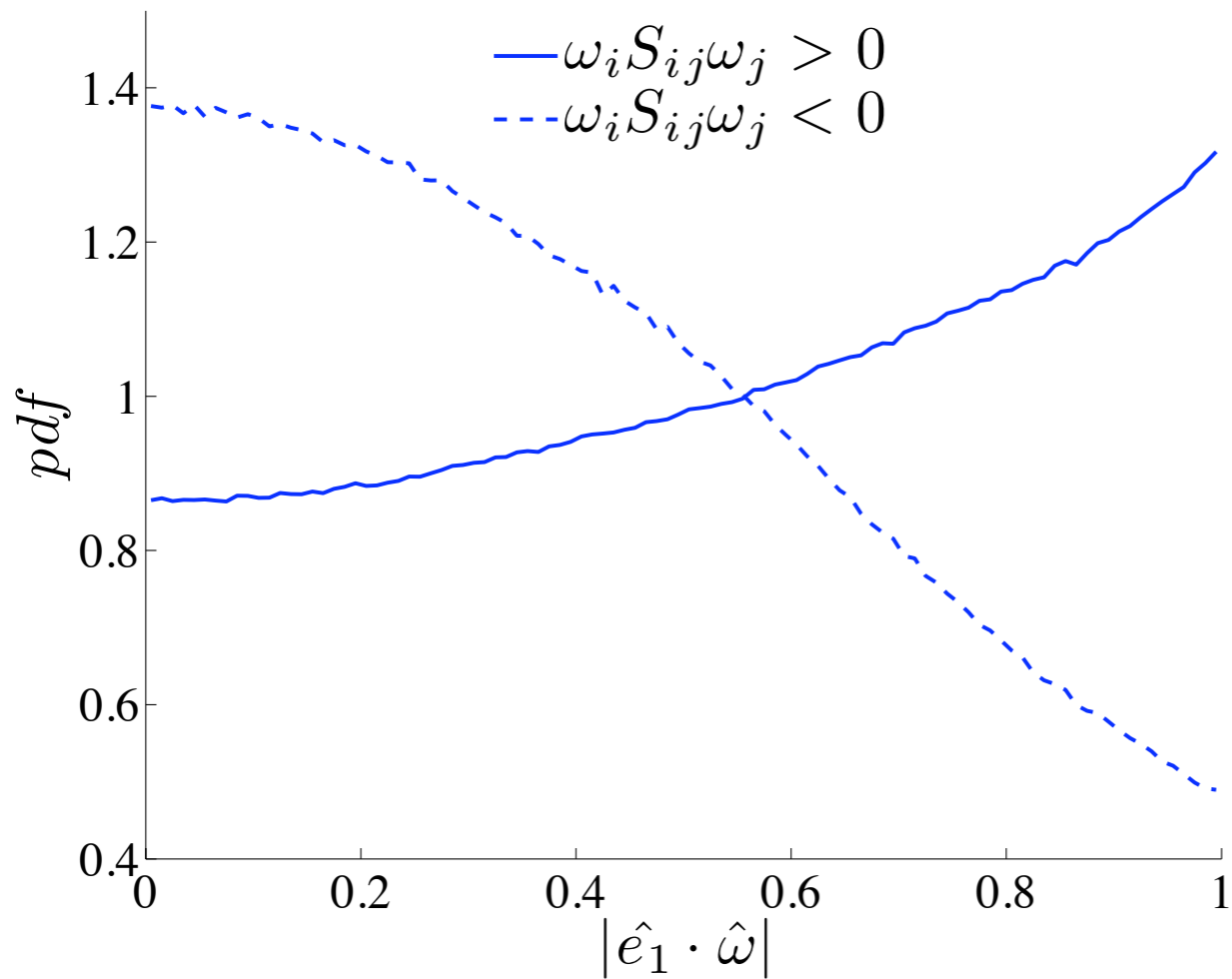
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# Turbulent axisymmetric jet



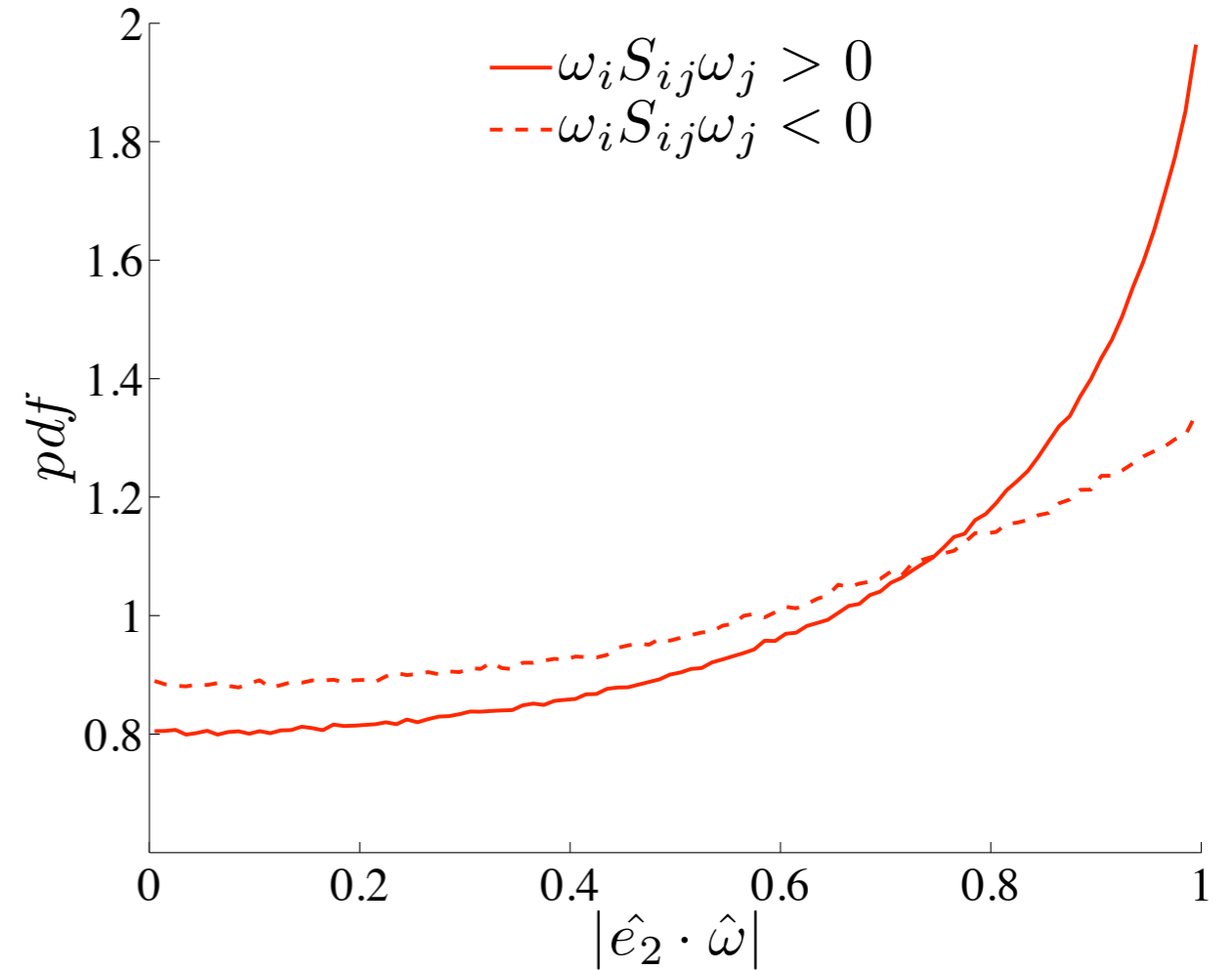
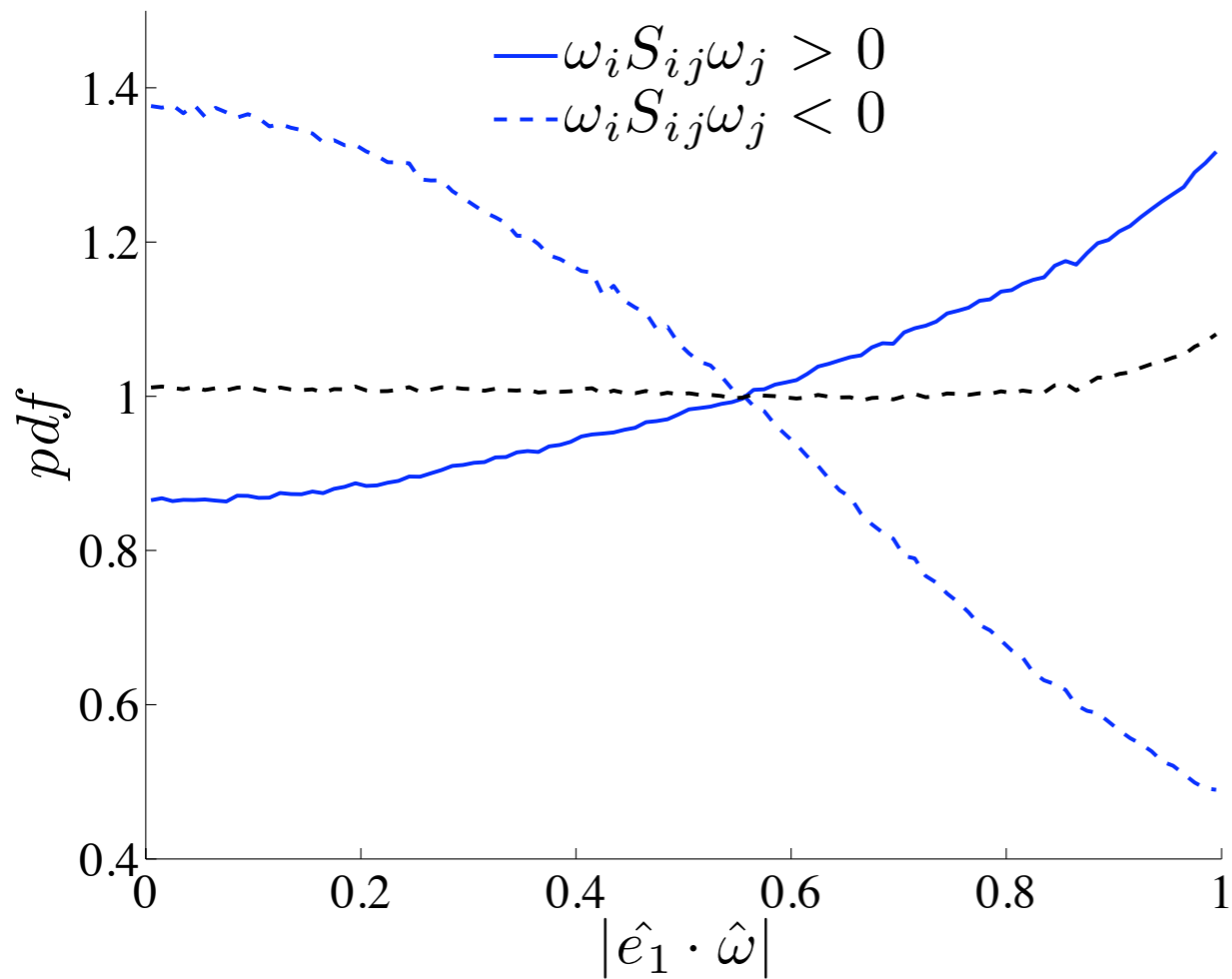
## Turbulent axisymmetric jet



- Alignment between  $\hat{e}_1$  and  $\hat{\omega}$  crucial to enstrophy production rate
  - parallel = enstrophy production
  - perpendicular = enstrophy destruction



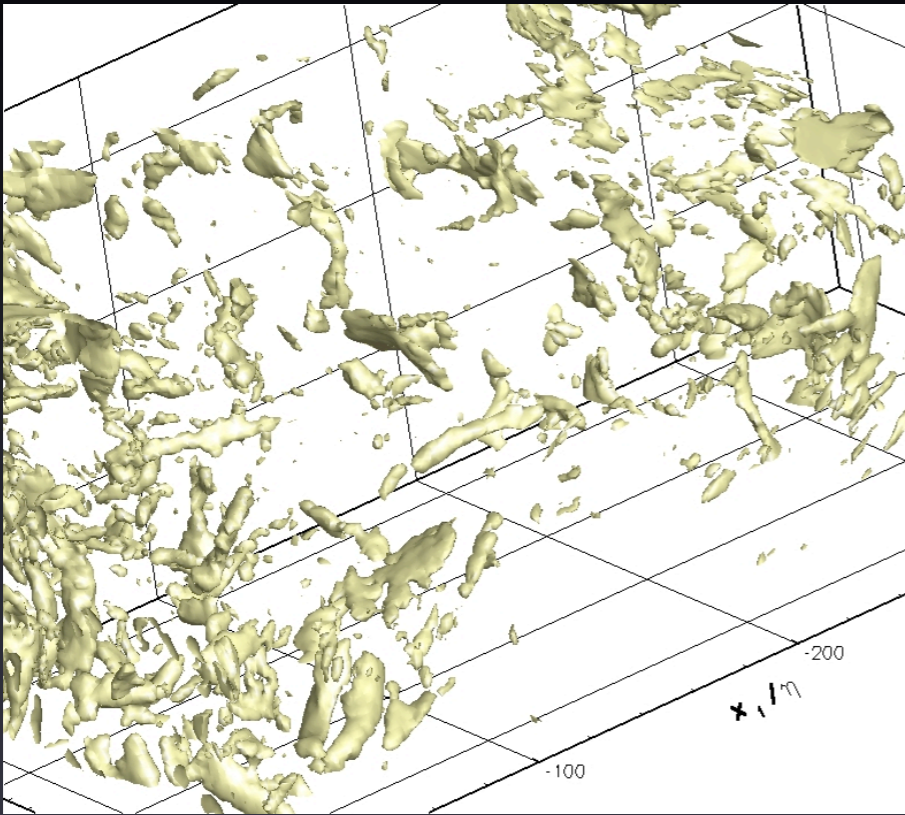
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“Traditional”  $|\hat{e}_1 \cdot \hat{\omega}|$  pdf the summation of enstrophy producing and enstrophy destroying data points

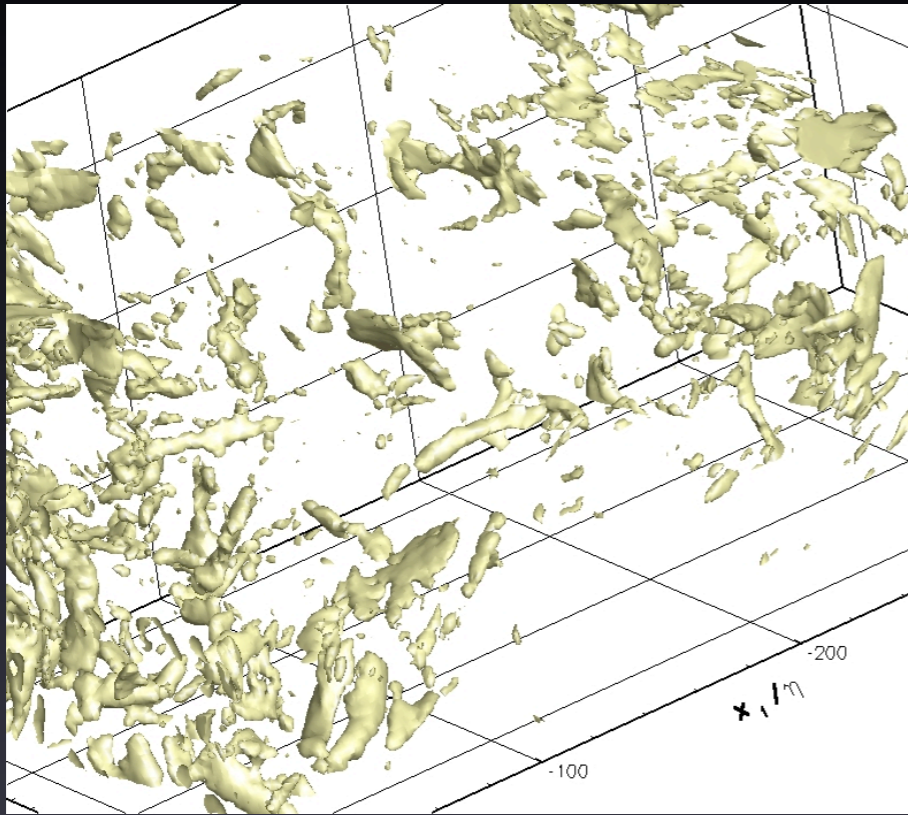
## Turbulent axisymmetric jet



$$\omega_i S_{ij} \omega_j > 0$$

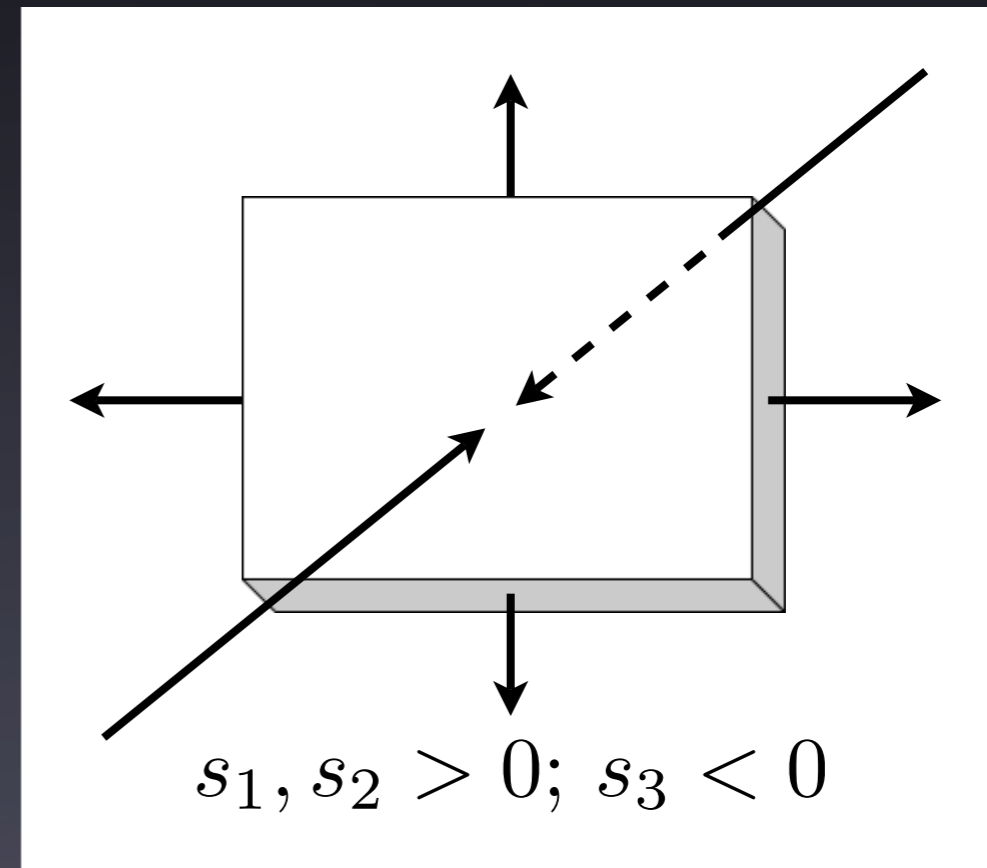
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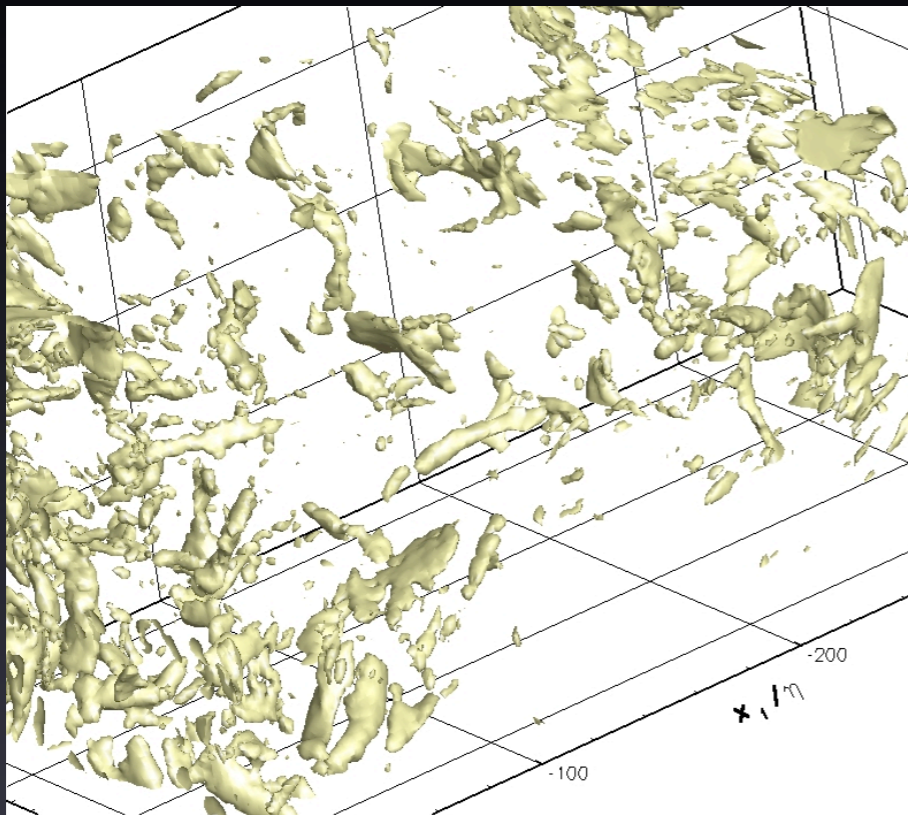


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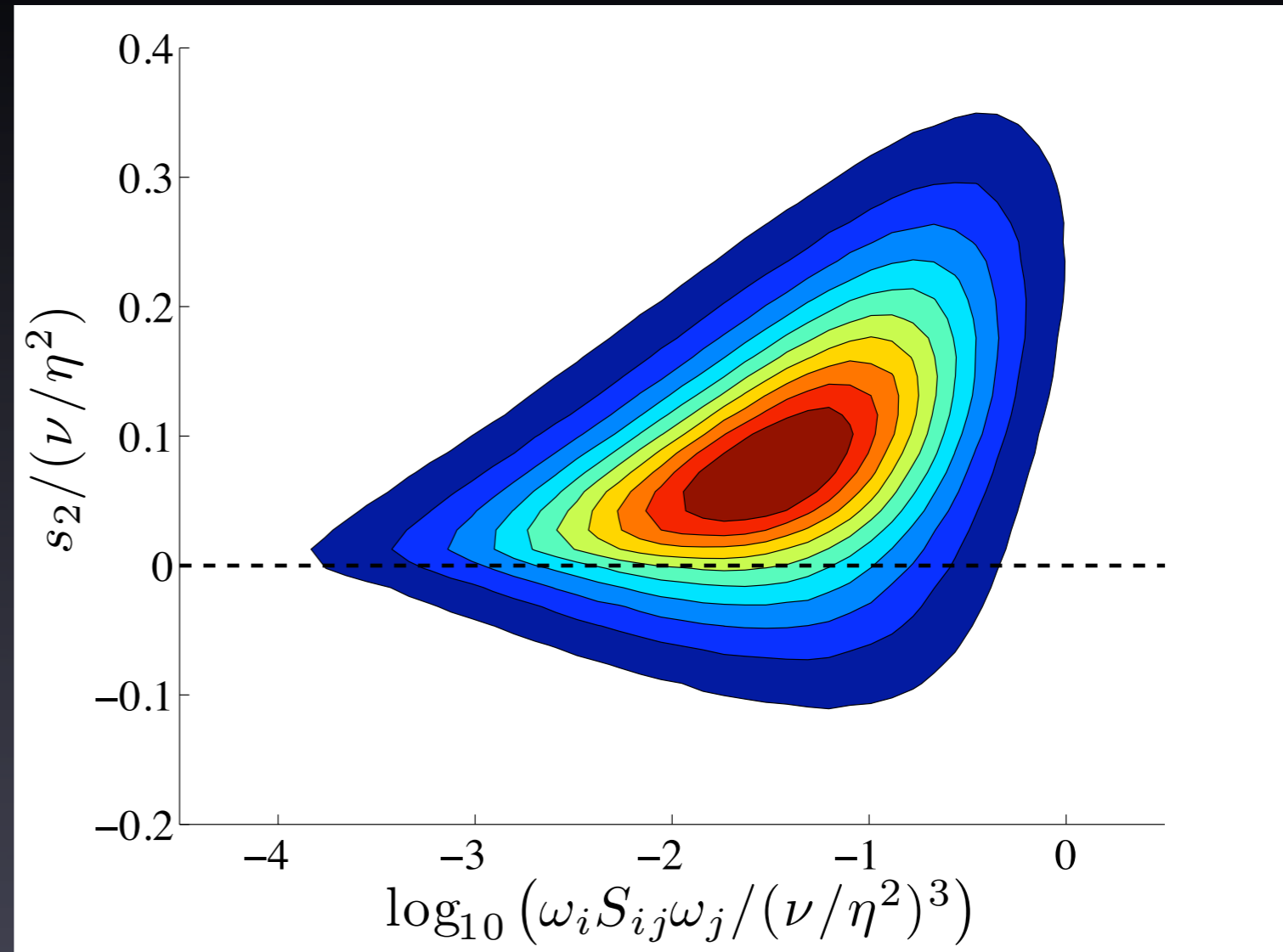


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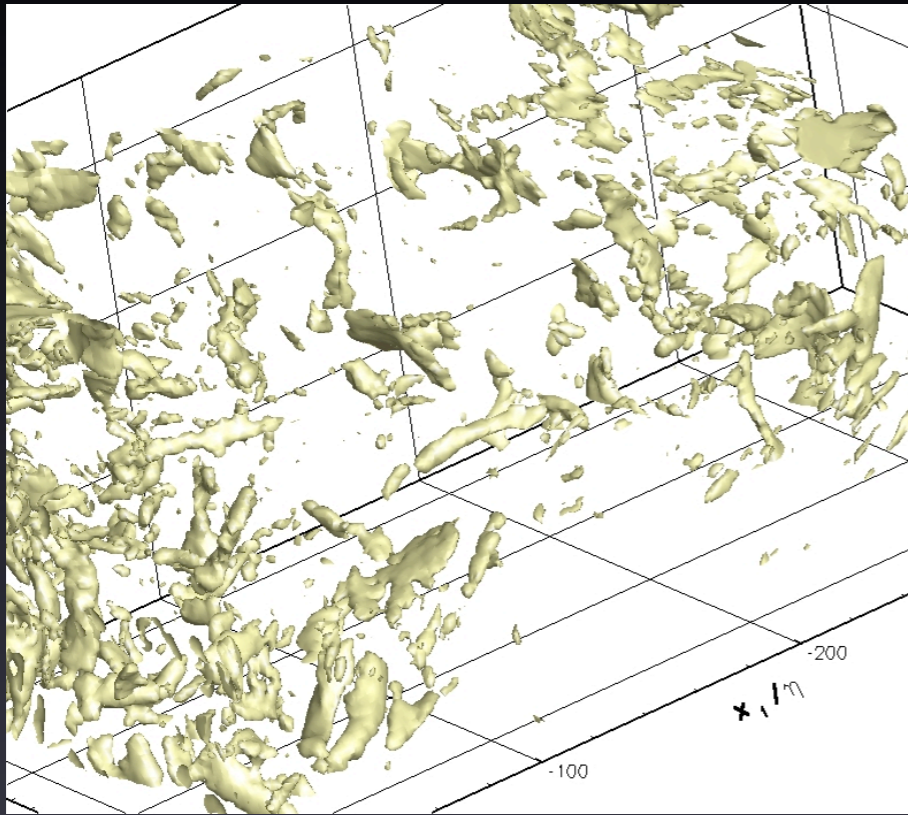
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- Positive  $s_2$  favoured over negative
- Particularly for high  $\omega_i S_{ij} \omega_j$

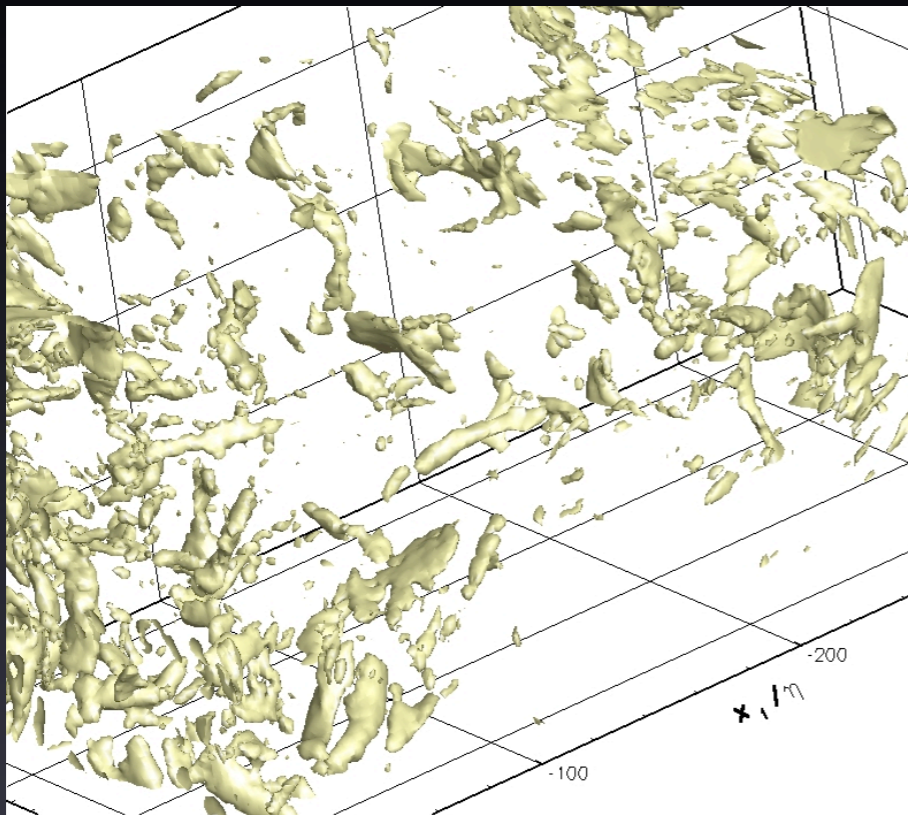
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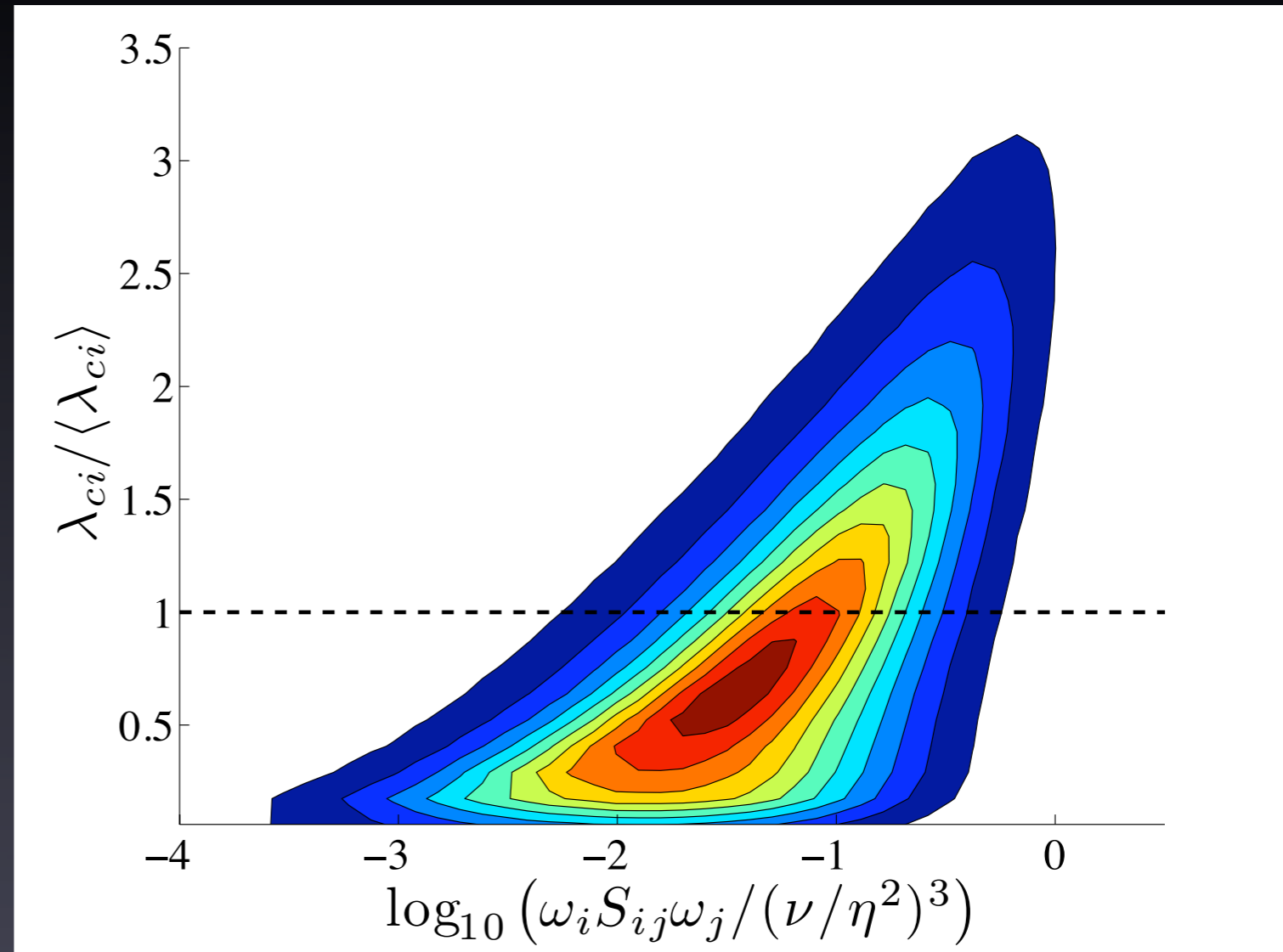
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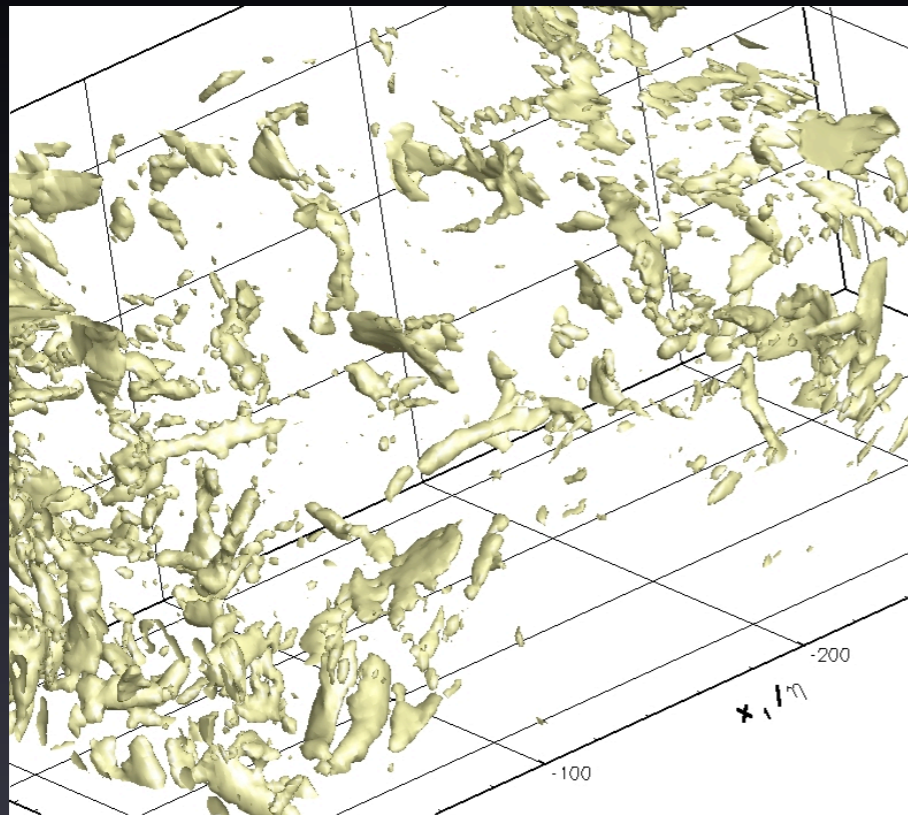


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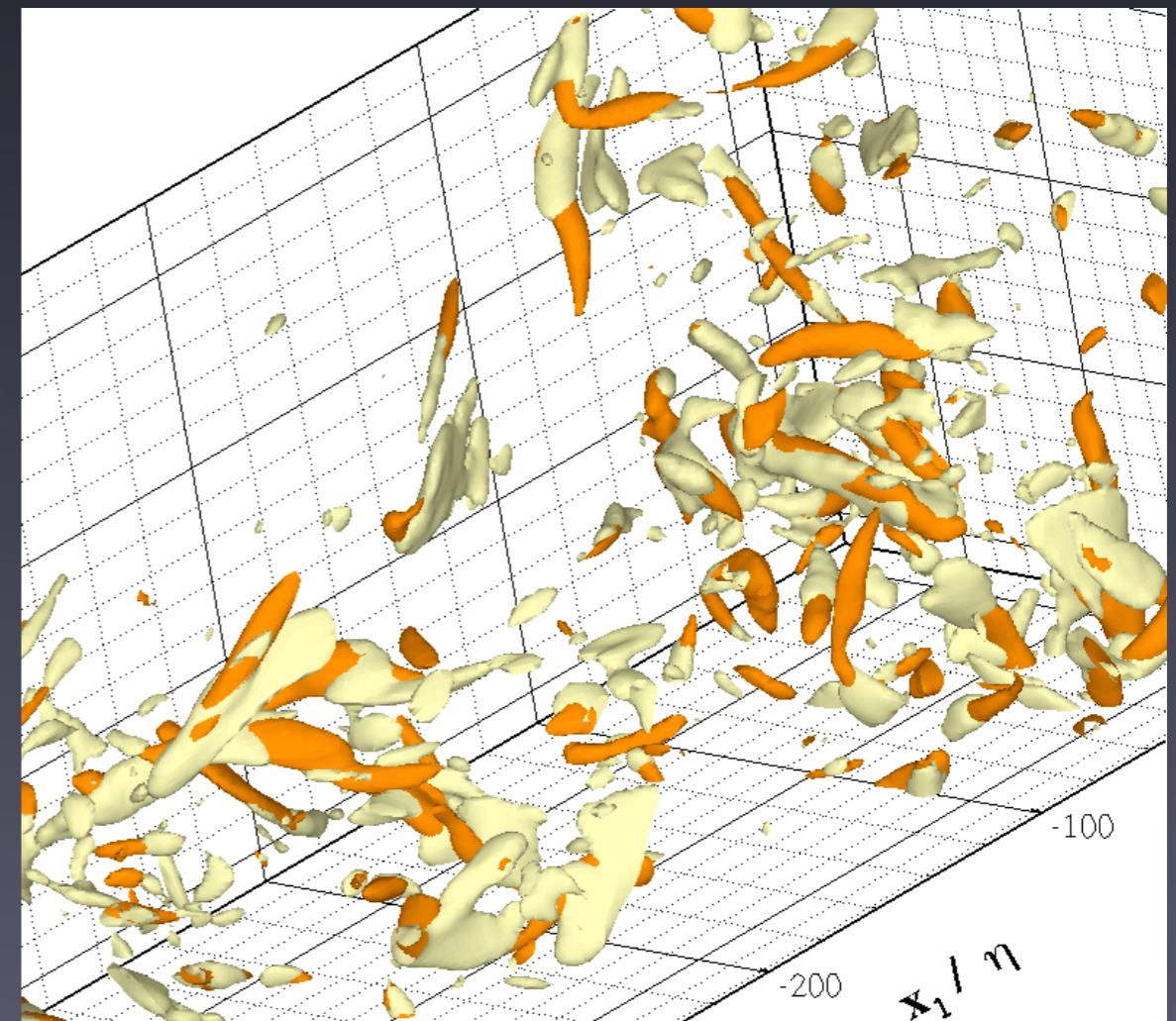


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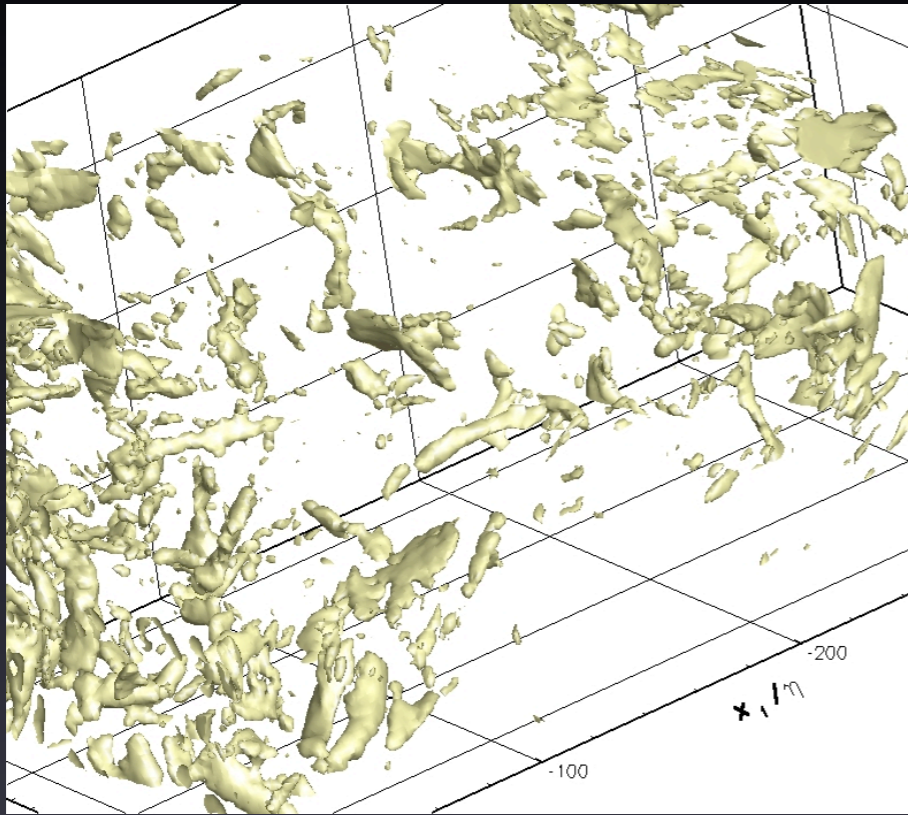
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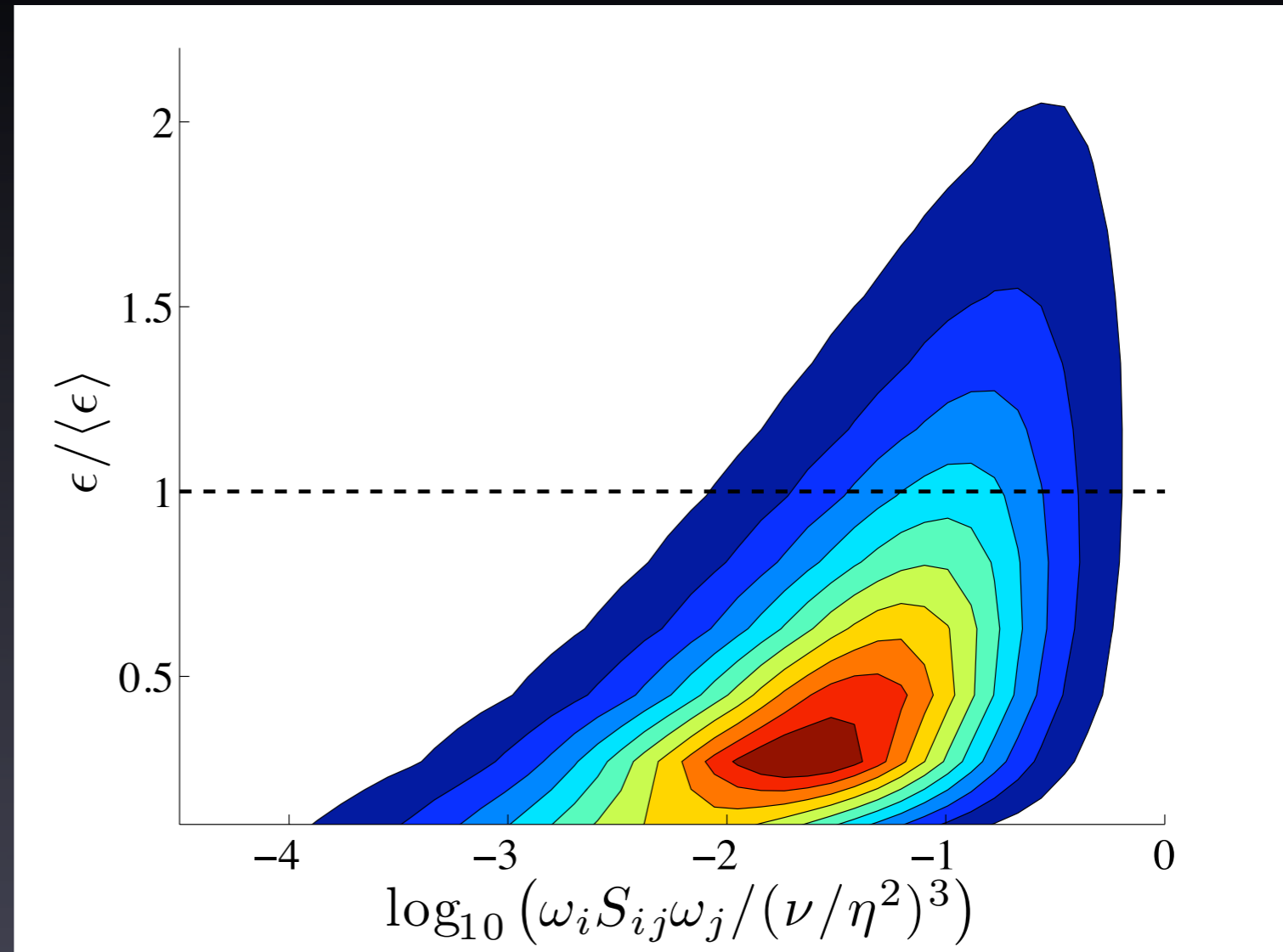


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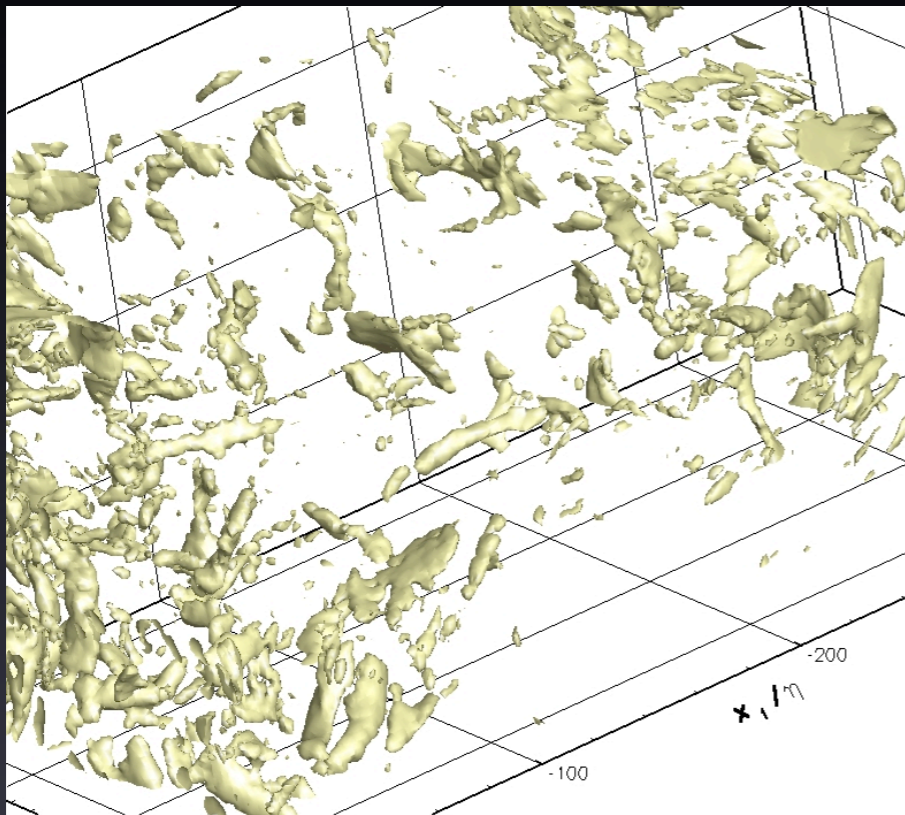
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- Strongly dissipative regions tend to coincide with enstrophy producing regions





# Turbulent axisymmetric jet

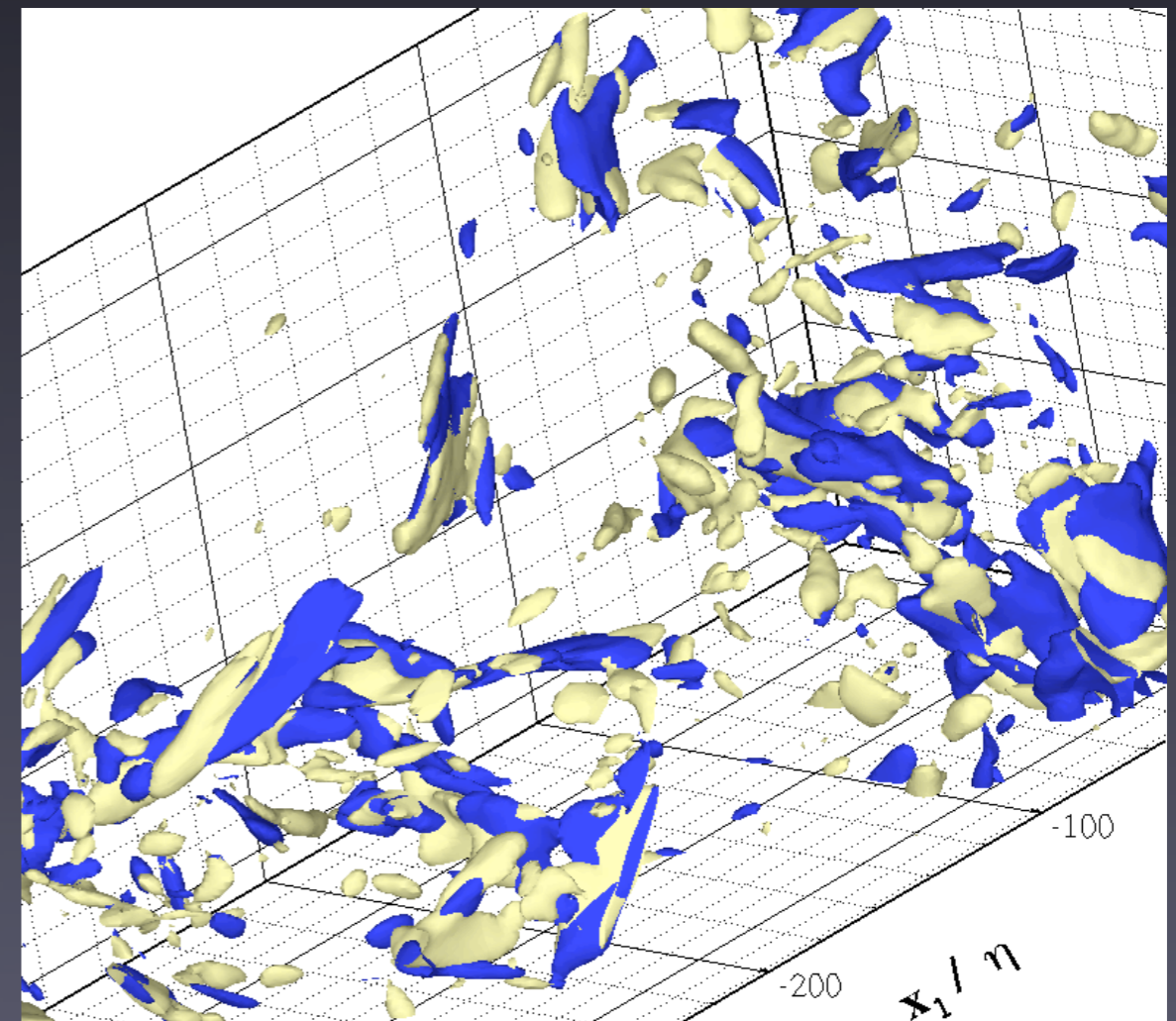


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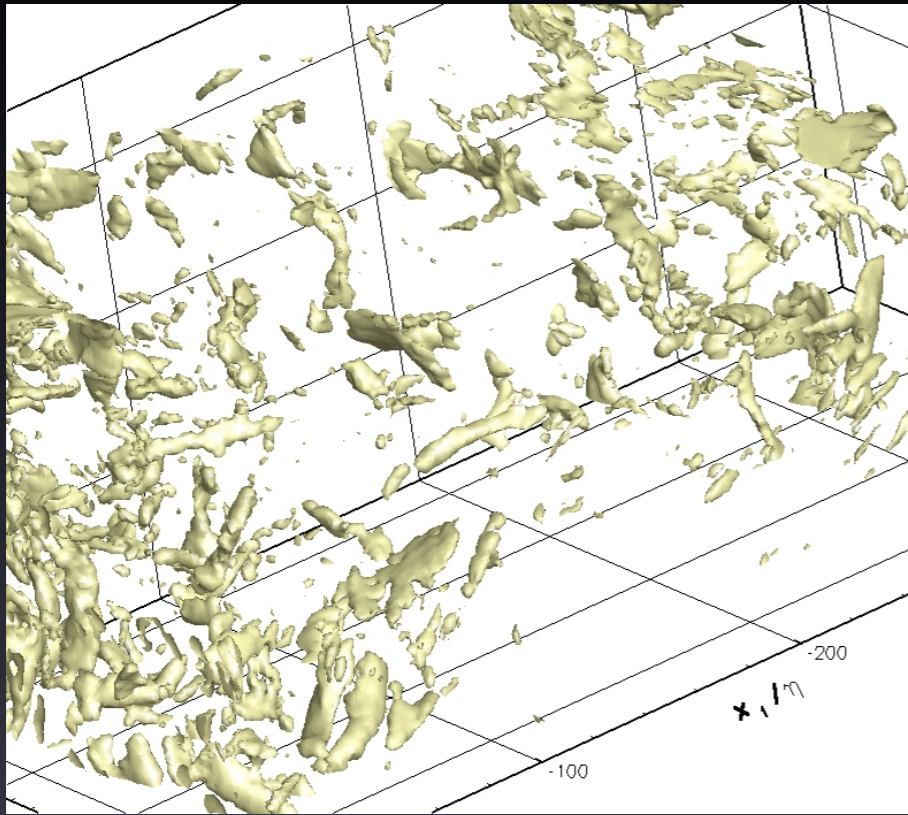
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$$\epsilon = 0.24 \text{m}^2 \text{s}^{-3} = 2.86 \langle \epsilon \rangle$$

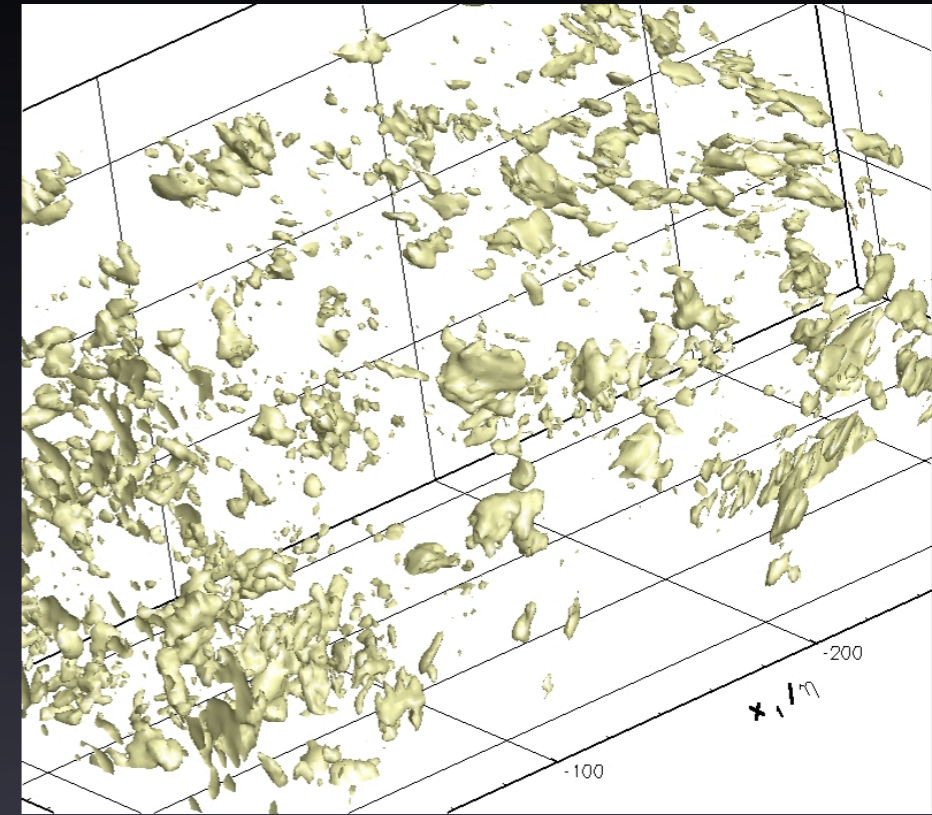


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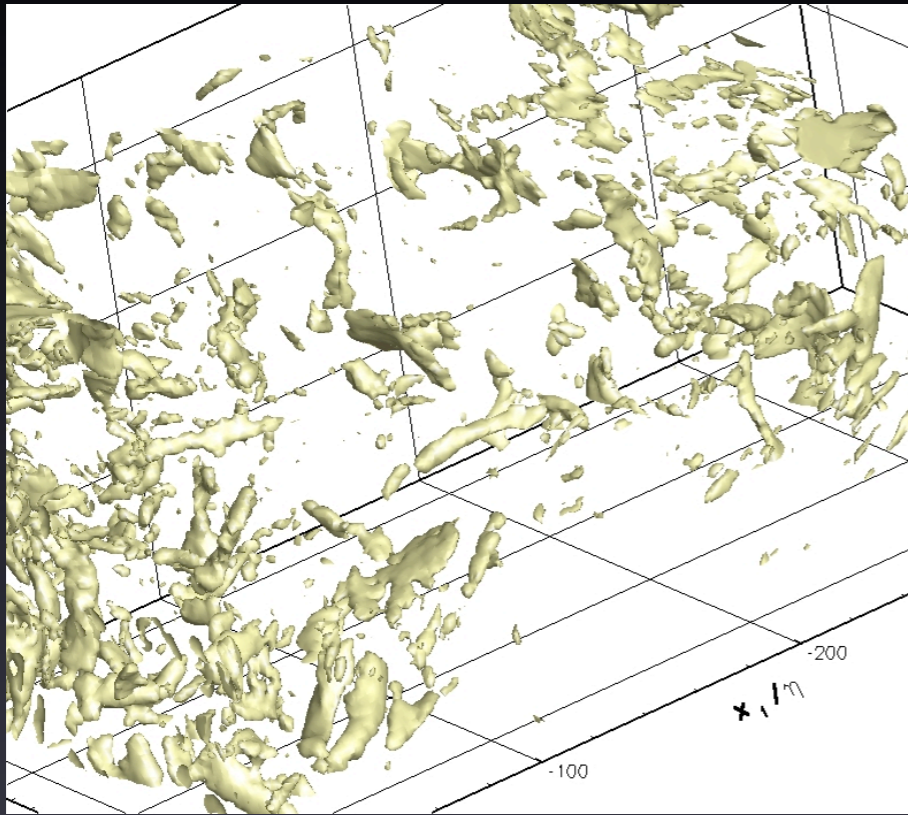
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$$\omega_i S_{ij} \omega_j < 0$$

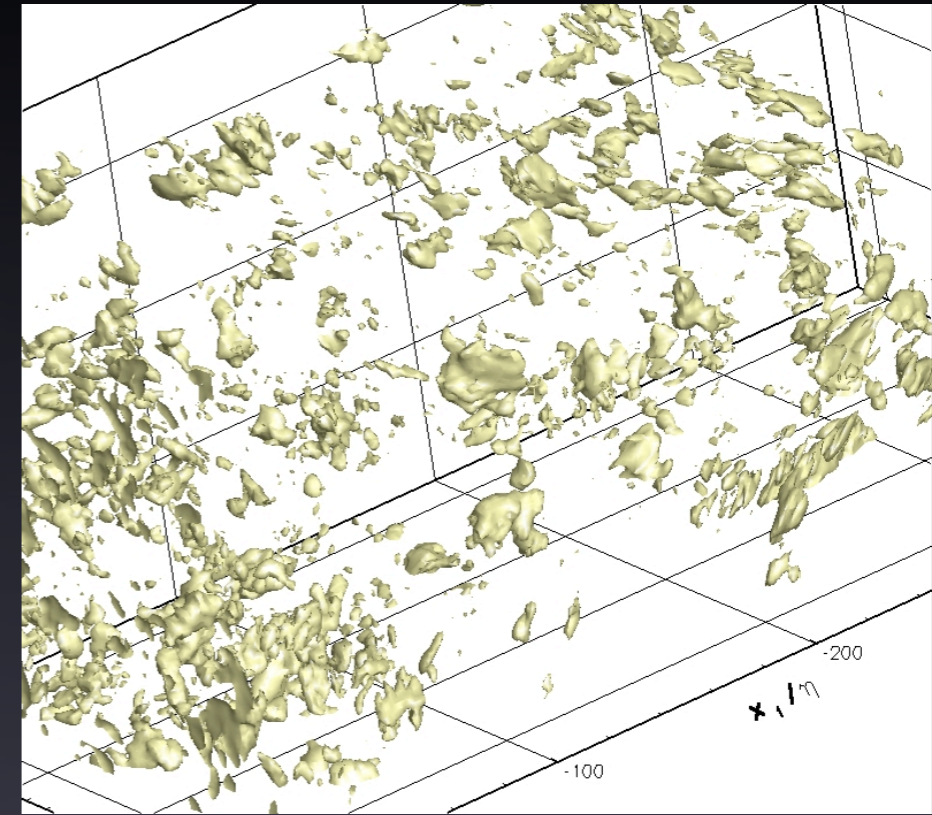
- Unstructured
- Strongly rotational regions do not favour enstrophy attenuation
- Strongly dissipative regions do not tend to coincide with enstrophy attenuating regions

## Turbulent axisymmetric jet



$$\omega_i S_{ij} \omega_j > 0$$

- Predominantly “sheet-like”
- Strongly rotational regions favour enstrophy production
- Strongly dissipative regions tend to coincide with enstrophy producing regions



$$\omega_i S_{ij} \omega_j < 0$$

- Unstructured
- Strongly rotational regions do not favour enstrophy attenuation
- Strongly dissipative regions do not tend to coincide with enstrophy attenuating regions

## Conclusions

- Multi-scale PIV experiment performed in planar mixing layer
  - Asymmetry in fine-scale *pdfs* for positive and negative fluctuations
  - Different behaviour for fine-scale *pdfs* of positive and negative fluctuations
  - Reynolds number effect?
  - Need to consider power spectral densities for the fine-scales conditioned on large scale fluctuations
  - Are convection velocities scale dependent?

## Conclusions

- Multi-scale PIV experiment performed in planar mixing layer
  - Asymmetry in fine-scale *pdfs* for positive and negative fluctuations
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  - Reynolds number effect?
  - Need to consider power spectral densities for the fine-scales conditioned on large scale fluctuations
  - Are convection velocities scale dependent?
- 3D velocity gradient data reveals strain - rotation interaction
  - Interaction directly leads to enstrophy amplification
    - Sustains turbulence in shear flows
  - Look for “universality” of interaction by examining different shear flows

## Ongoing / future work

- Volumetric three dimensional velocimetry
  - Fully three dimensional PIV data in a volume
- Dual plane stereoscopic particle image velocimetry
  - Three dimensional velocity and velocity gradient data in a plane
- Direct numerical simulations
  - incompact3d code run on HECToR
- Large eddy simulations
  - streamLES run on HECToR