

New avenues in computational fluid dynamics

Esteban Ferrer

esteban.ferrer@upm.es

<http://sites.google.com/site/eferrerdg>

<https://numath.dmae.upm.es/>



@FerrerCfd



Esteban Ferrer



UPM Collaborators: E Valero, G Rubio, S Le Clainche, L Gonzalez, J Garicano...

Ext. Collaborators: DA Kopriva (San Diego), C Hirsch (Numeca), Paniagua (Purdue), P García (Zaragoza)
R Vinuesa (KTH), S Sherwin (IC), R Willden (Oxford), H Blackburn (Monash)

Industrial collaborators: Numeca-Cadence, Airbus, Dassault Syst., Siemens-Gamesa...



Funding



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de Madrid

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Summary

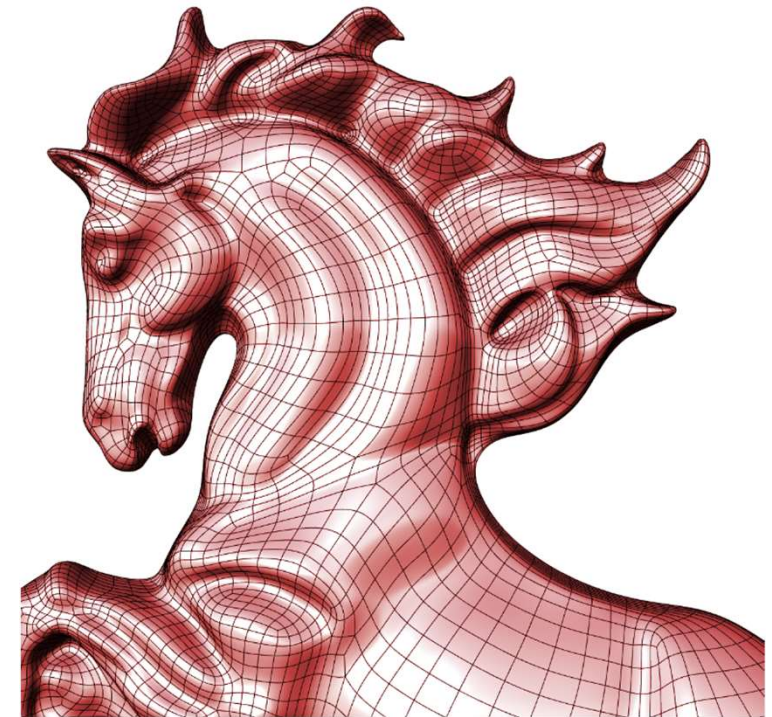
1- Introduction to DG & Horses3d

2- Multiphysics

- Wind turbines
- Turbulence

3. Machine Learning + CFD

- Mesh adaption
- NN acceleration
- RL for automation



DGSEM: nodal Discontinuous Galerkin Spectral Element Methods

- **Compressible & Incompressible**
- **Entropy / Energy conserving schemes for stability**
- **Local p-adaption / h-adaption (hanging nodes)**
- **Explicit / implicit time stepping**
- **Turbulence models: LES:** SVV-Smag., Wale, Vreman & **RANS:** Spallart-Almaras
- **Multi-physics:** Multiphase, Immersed Boundaries, Shock etc..

HORSES3D <https://github.com/loganoz/horses3d>

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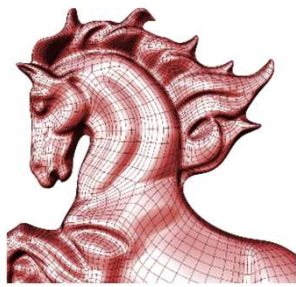
journal homepage: www.elsevier.com/locate/cpc



HORSES3D: A high-order discontinuous Galerkin solver for flow simulations and multi-physics applications ☆☆☆

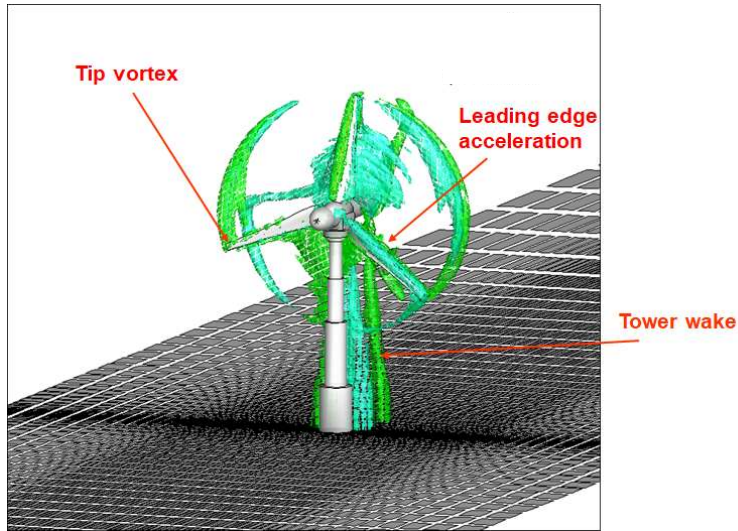
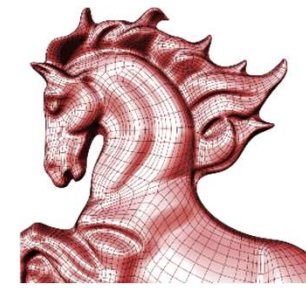
E. Ferrer^{a,b}, G. Rubio^{a,b,*}, G. Ntoukas^a, W. Laskowski^a, O.A. Mariño^a, S. Colombo^a, A. Mateo-Gabín^a, H. Marbona^a, F. Manrique de Lara^a, D. Huergo^a, J. Manzanero^e, A.M. Rueda-Ramírez^c, D.A. Kopriva^d, E. Valero^{a,b}



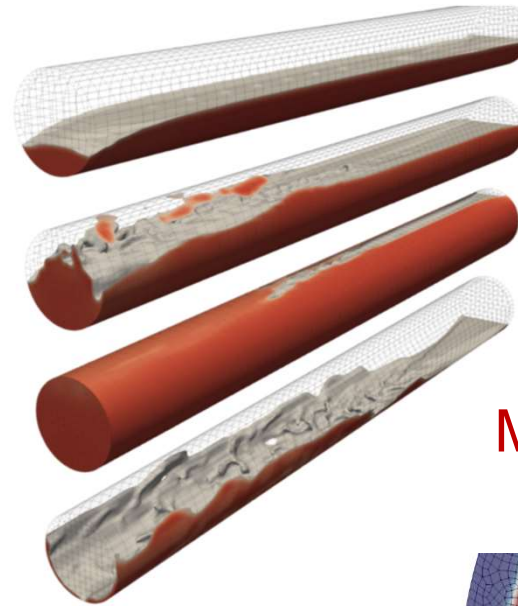


HORSES3D

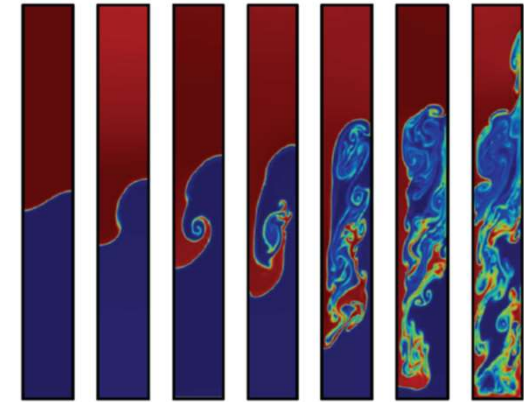
<http://github.com/loganoz/horses3d>



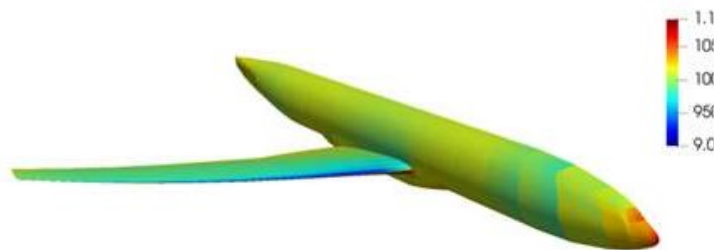
Mesh free



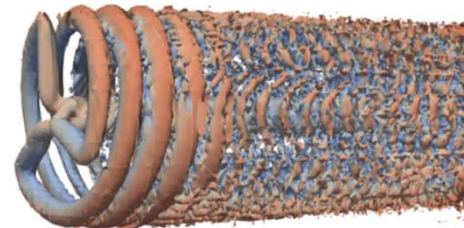
Shocks



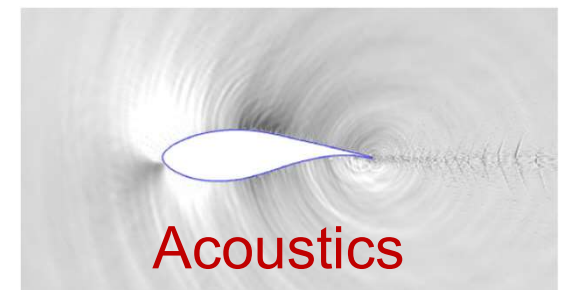
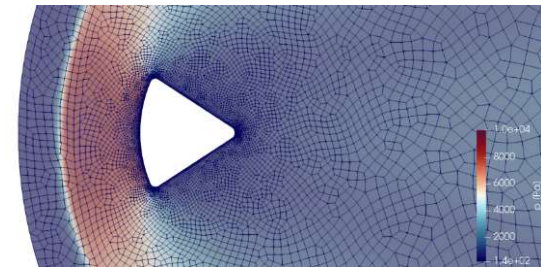
Multiphase



RANS



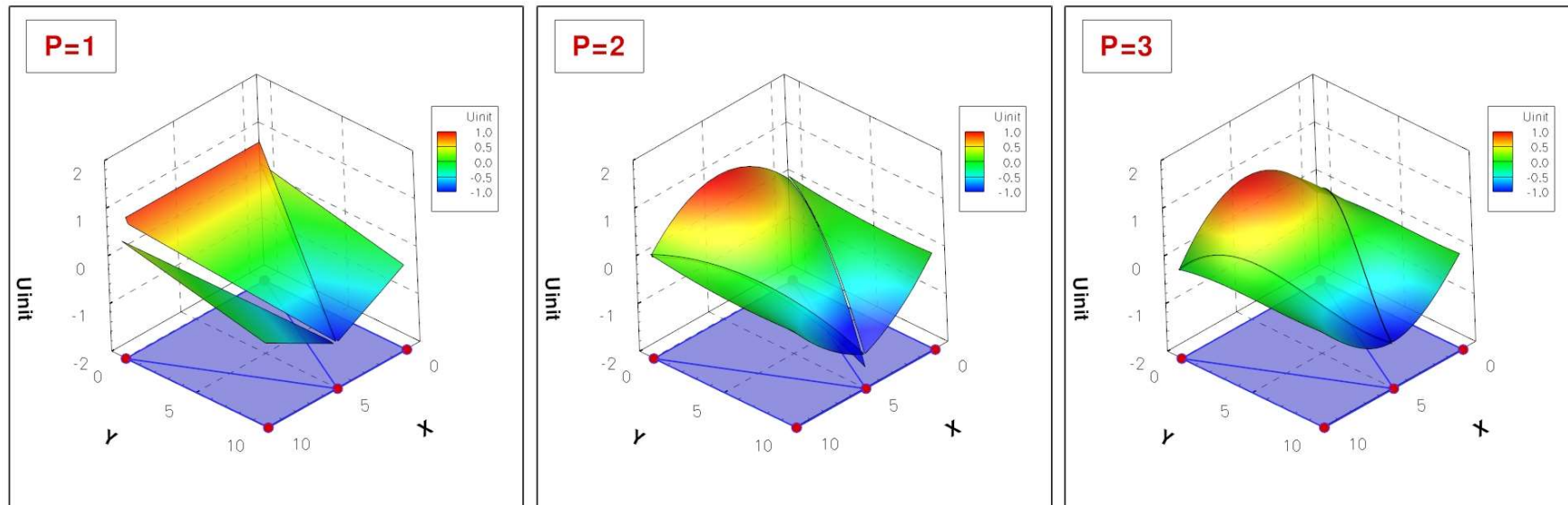
LES



Acoustics

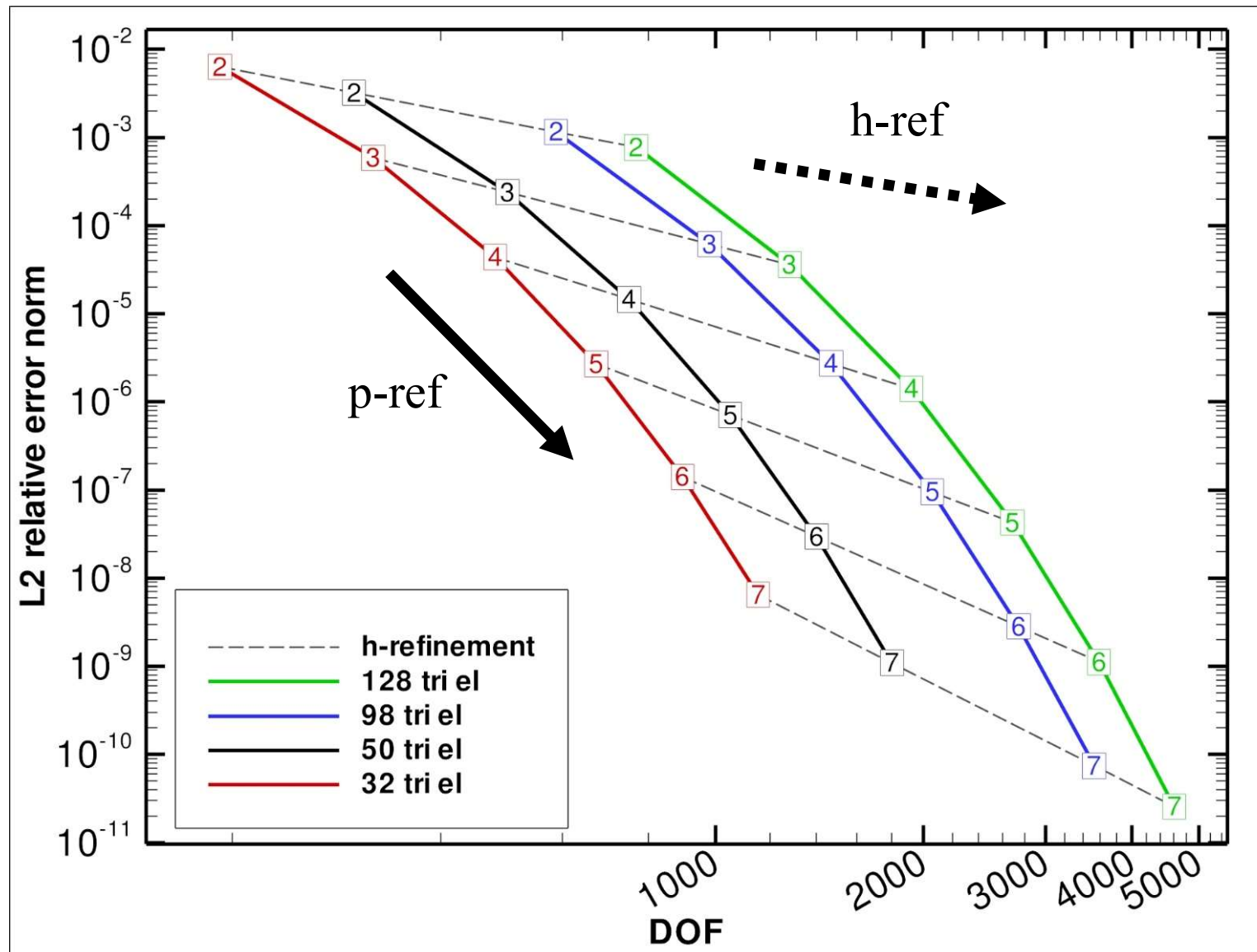
High order methods

2D Discontinuous Galerkin Projection on triangular elements for various polynomial order

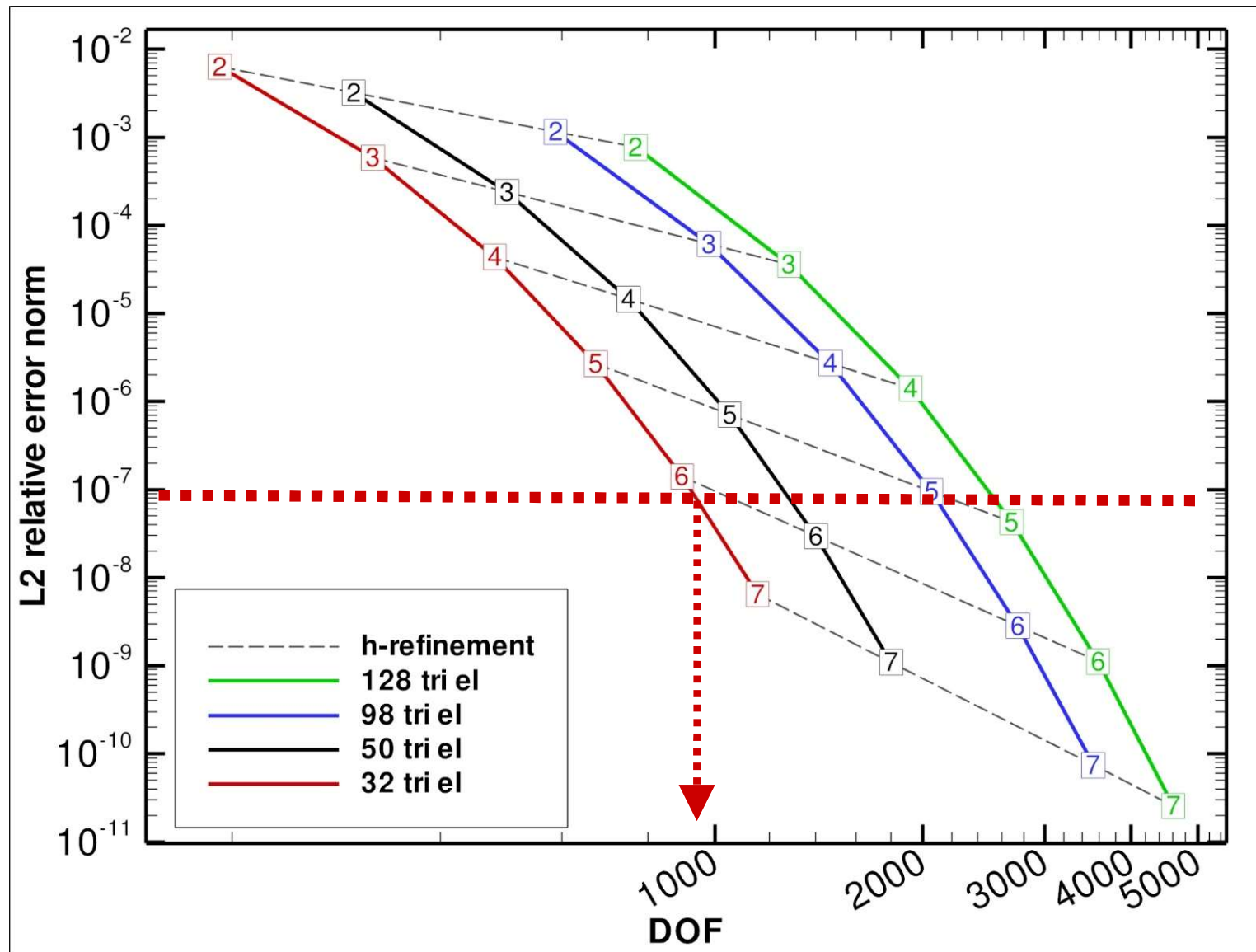


- **High order is generally defined for $P \geq 2$**
- **High order allows h/p refinement**
 - h -refinement offers constant decay of the error
 - p -refinement offers exponential decay of the error

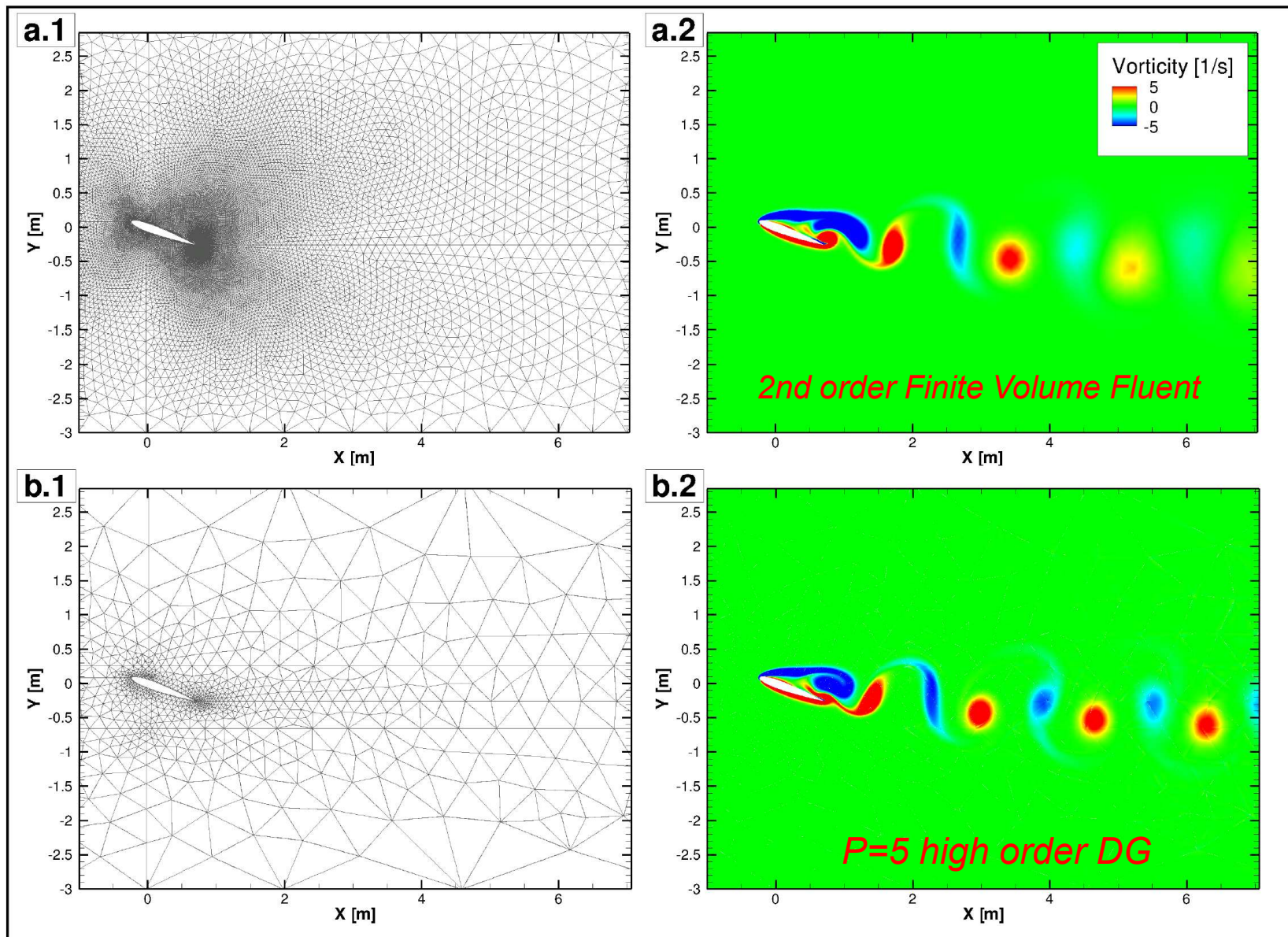
High order methods (Poisson eq.)



High order methods (Poisson eq.)



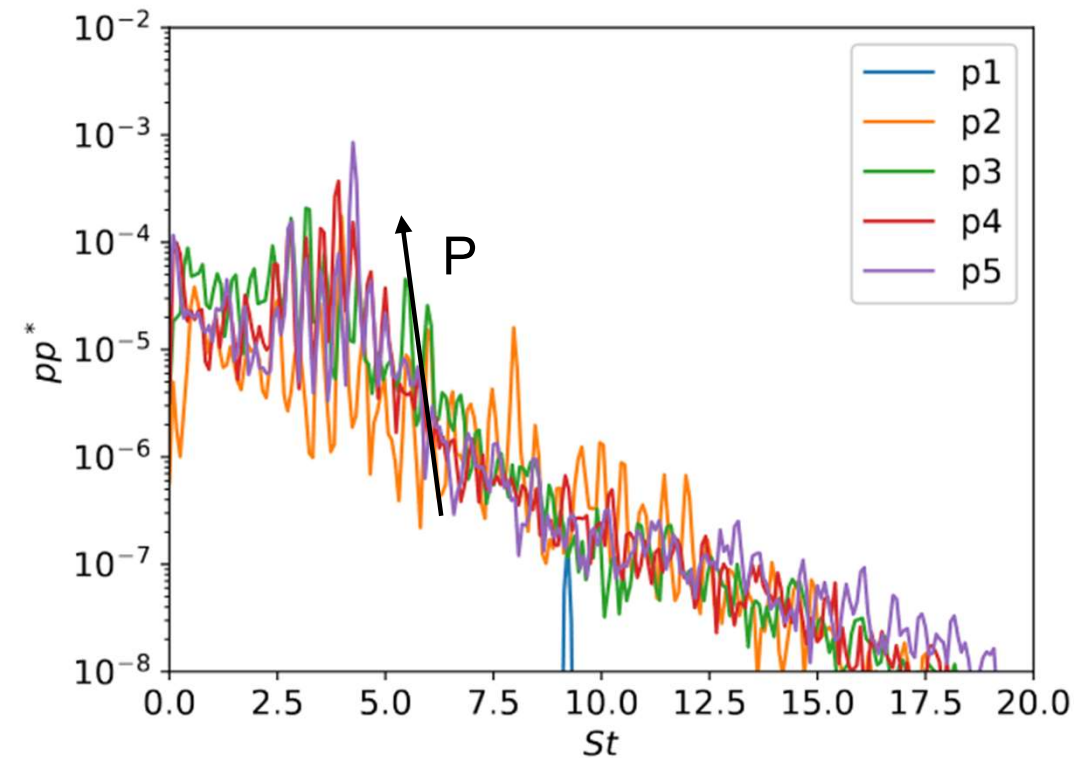
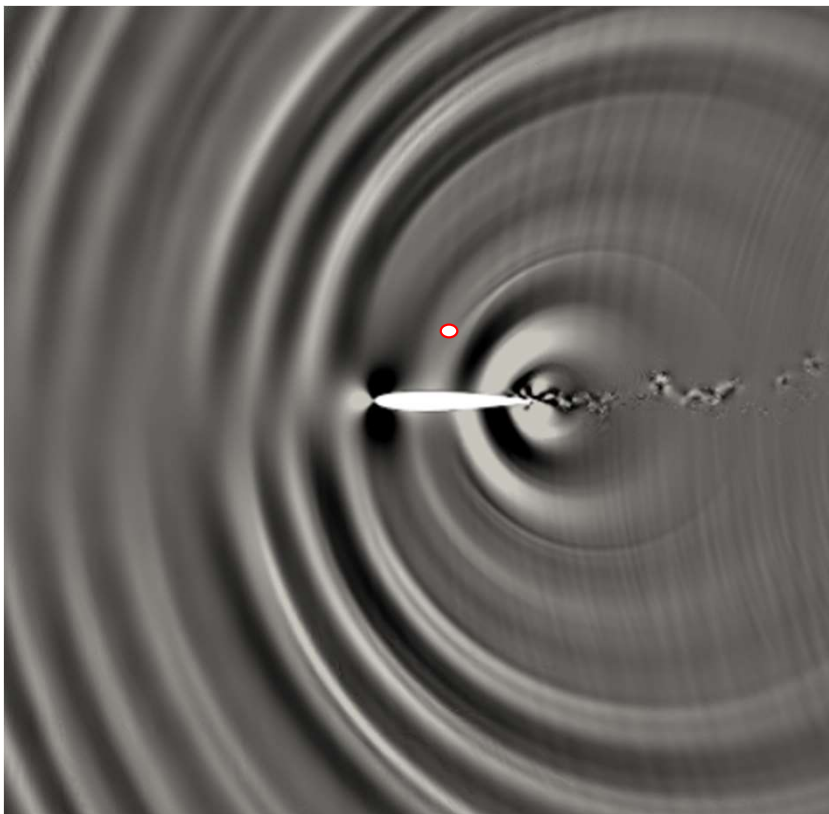
NACA0012 - $Re=800$ - Laminar flow



Horses: accuracy

NACA0012 airfoil at $Re = 105$, $M_0 = 0.4$ and $AoA = 0^\circ$

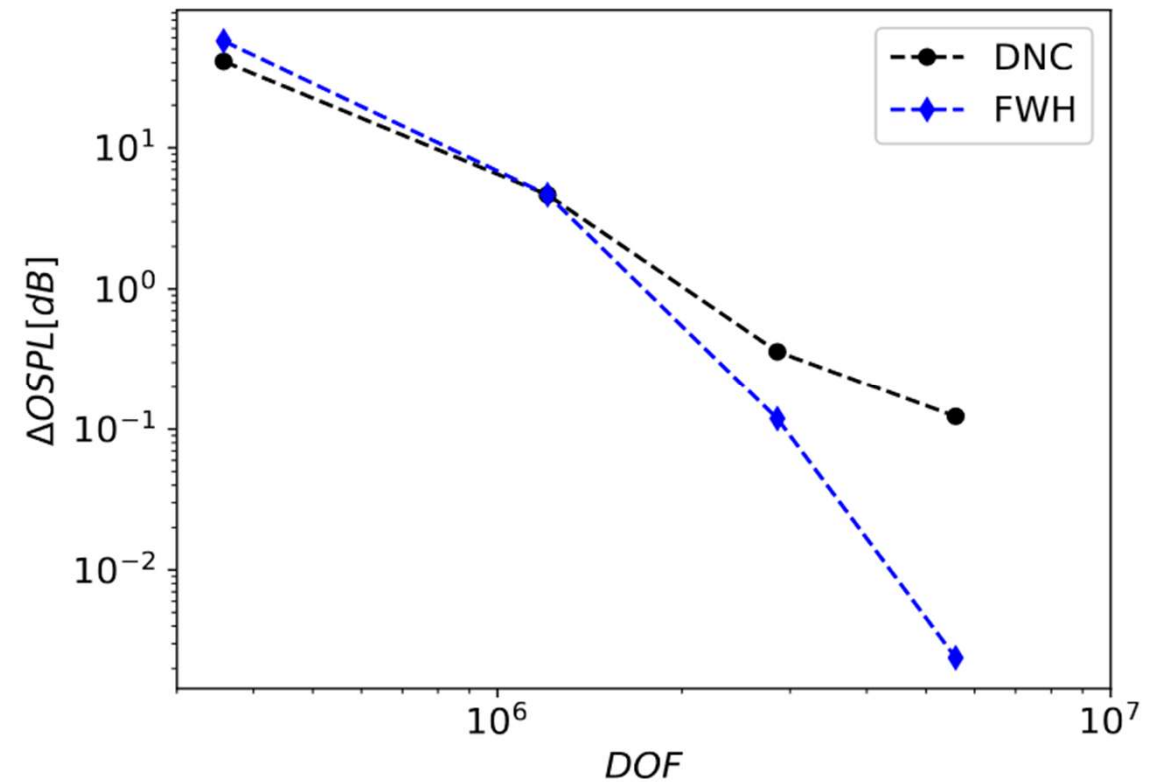
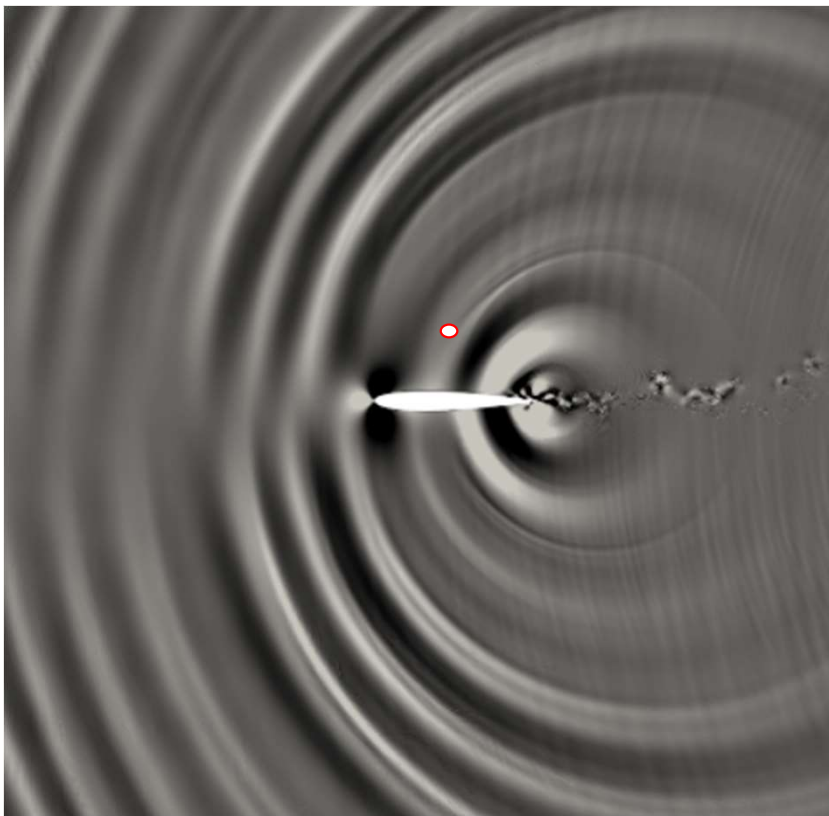
$P \uparrow$: Error decreases **exponentially**



Horses: accuracy

NACA0012 airfoil at $Re = 10^5$, $M_0 = 0.4$ and $AoA = 0^\circ$

$P \uparrow$: Error decreases **exponentially**

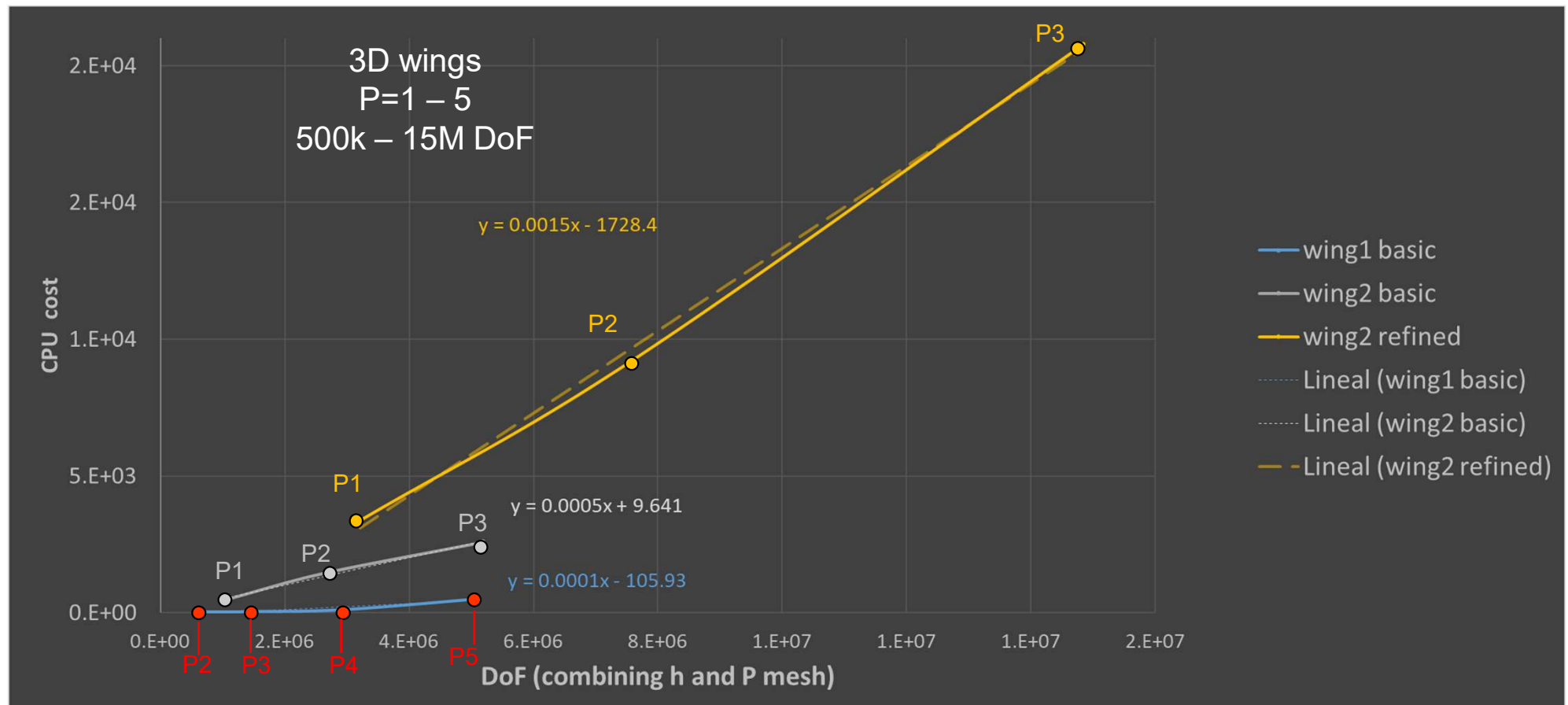


Horses: cost



$P \uparrow$: Error decreases **exponentially**

$P \uparrow$: Cost increases **linearly**



Summary

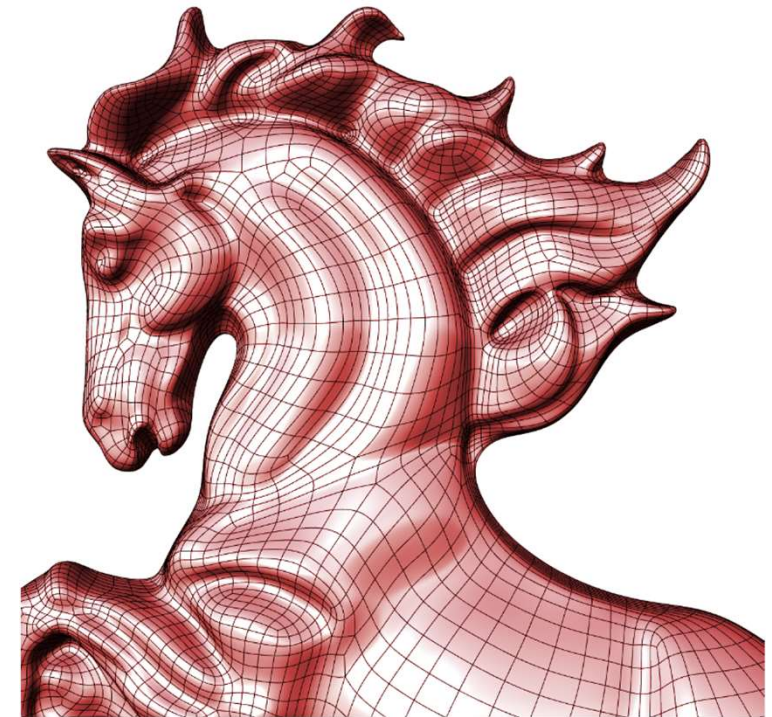
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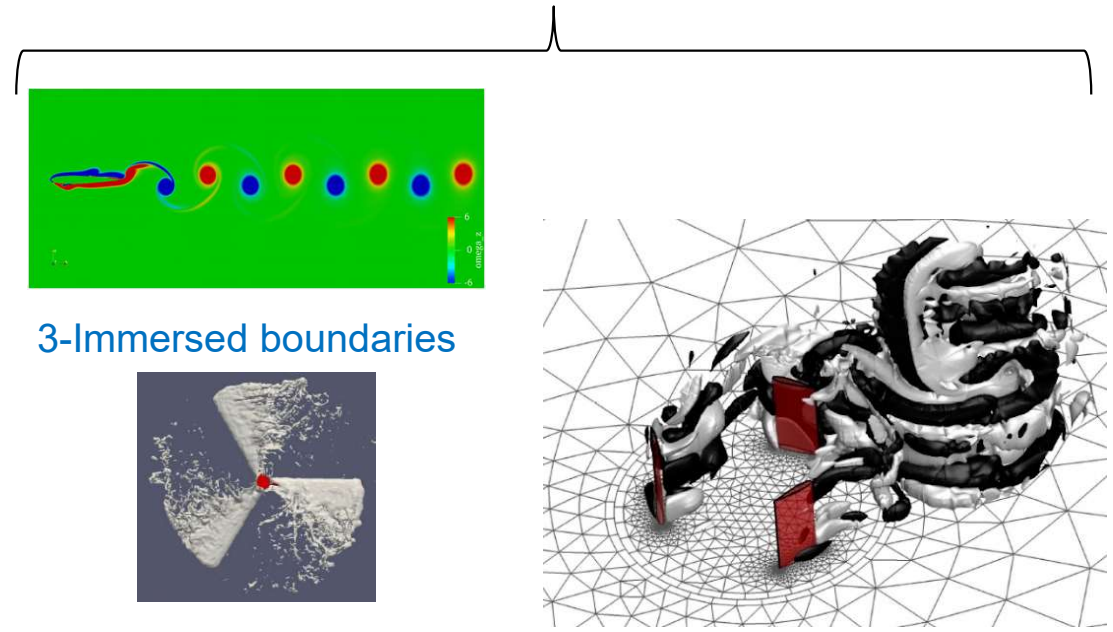
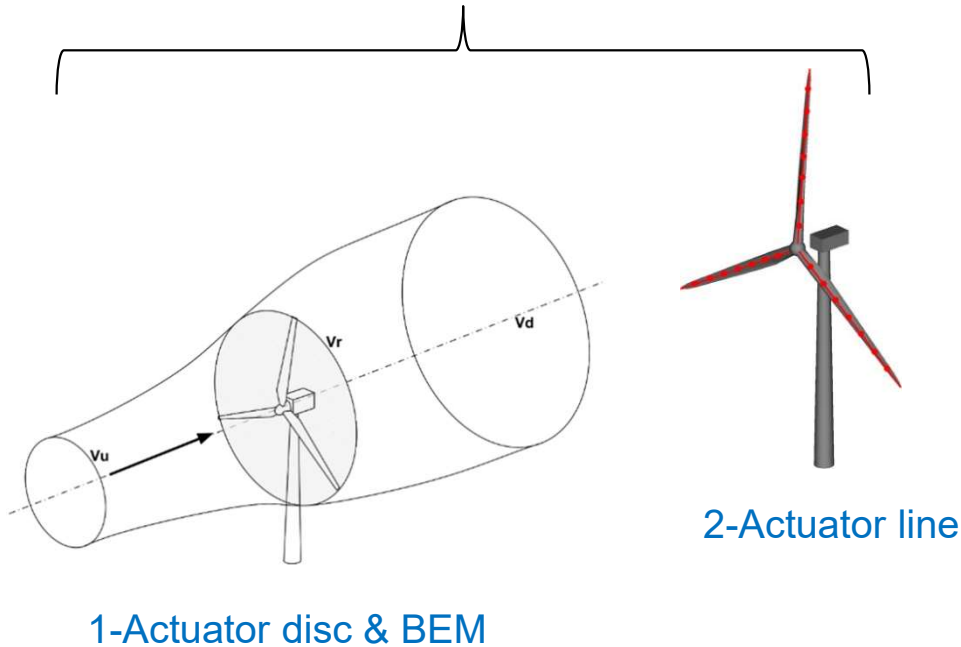
3. Machine Learning + CFD

- Mesh adaption
- NN acceleration
- RL for automation



Require 2D aerodynamic data

Explicit 3D geometry



Cost

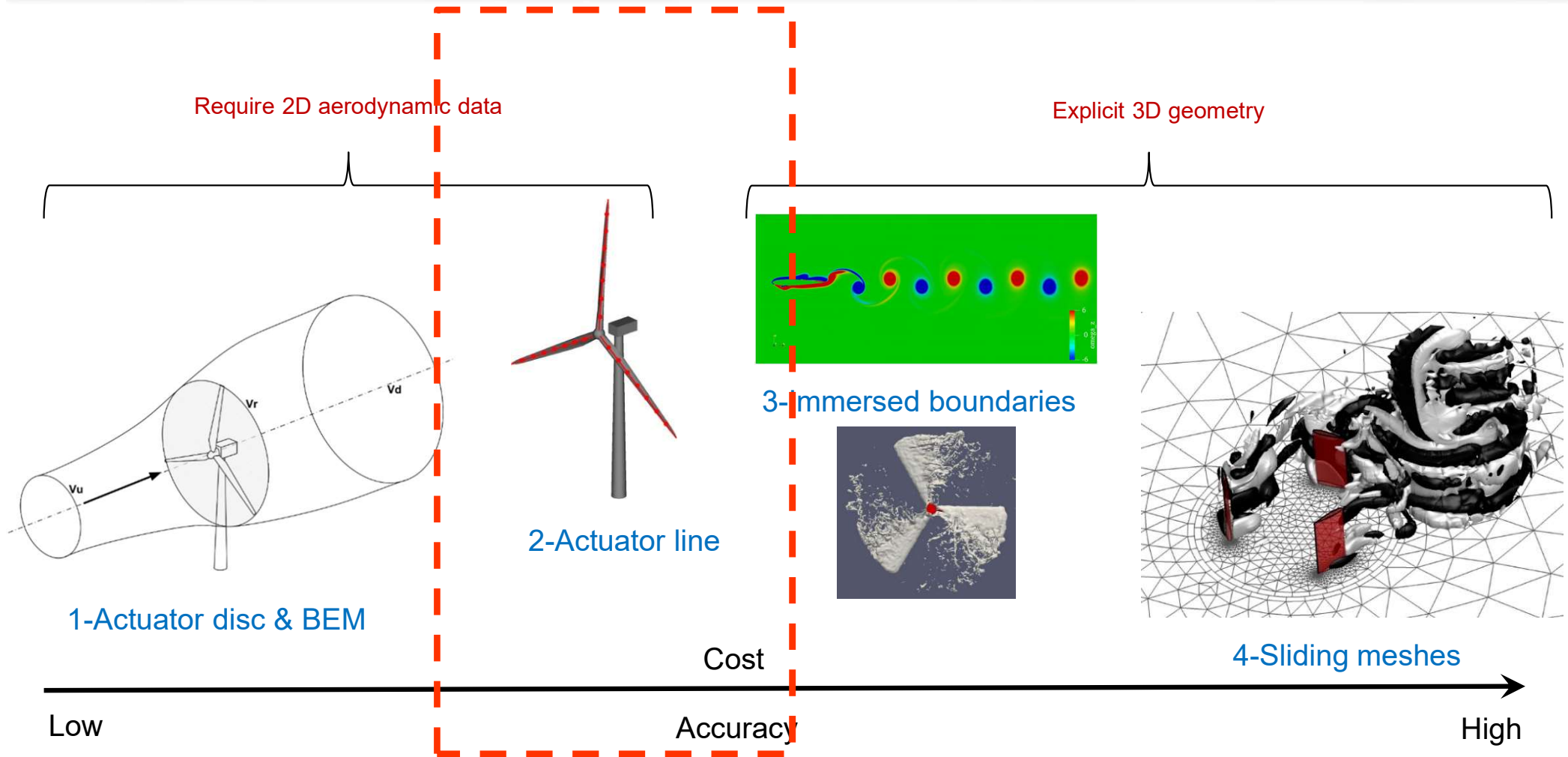
Accuracy

Low

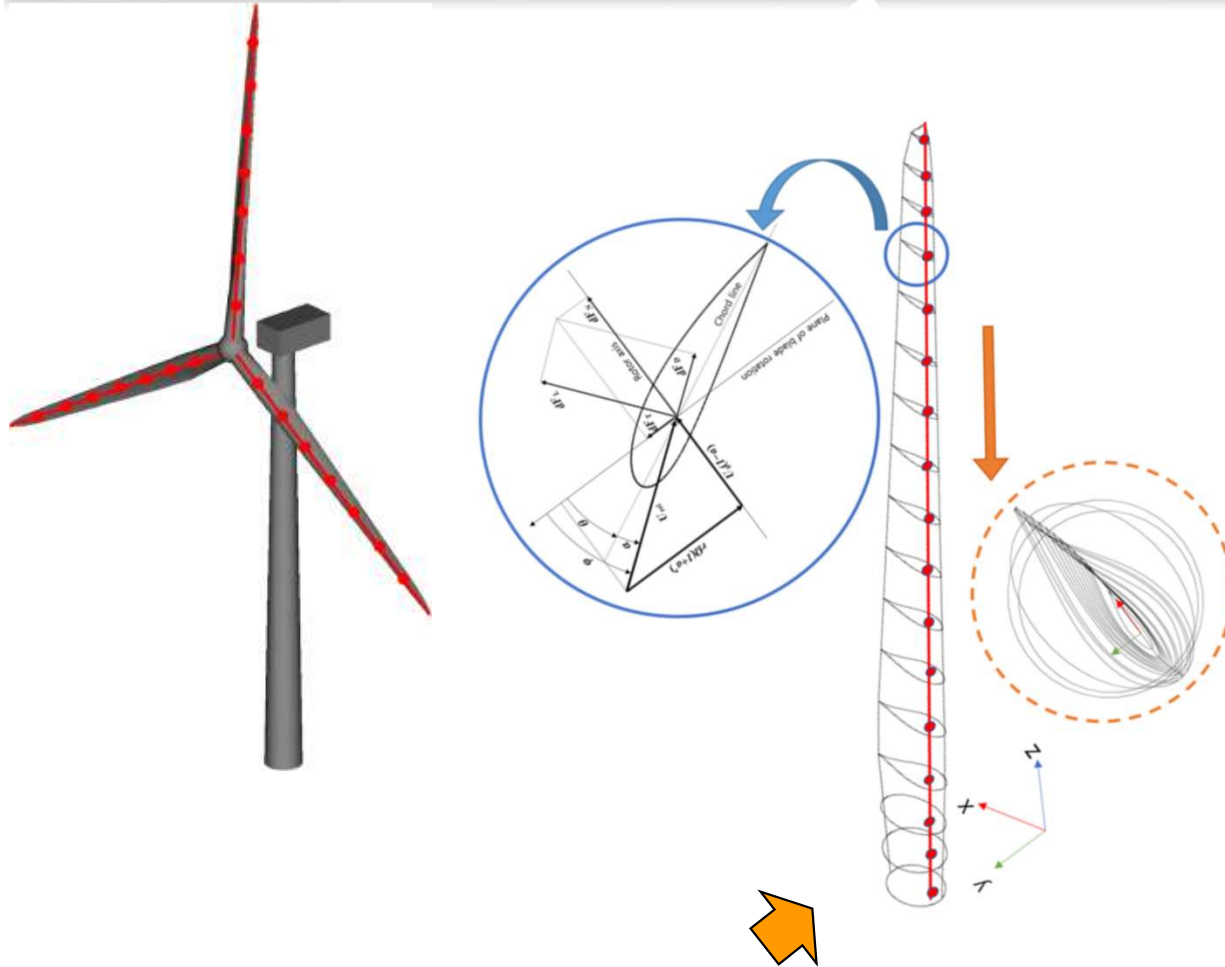
High

High

- 4- E Ferrer and RHJ Willden, A high order Discontinuous Galerkin - Fourier incompressible 3D Navier–Stokes solver with **rotating sliding meshes**, *Journal of Computational Physics*, 2012
- 4- E Ferrer, RHJ Willden, Blade–wake interactions in **cross-flow turbines**, *International Journal of Marine Energy*, 2015
- 3- J Kou, A Hurtado-de-Mendoza, S Joshi, S Le Clainche, E Ferrer, Eigensolution analysis of **immersed boundaries** for high-order schemes, *Journal of Computational Physics*, 2022
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- 1- E Ferrer, OMF Browne, E Valero, Sensitivity analysis to control the far–wake unsteadiness behind **turbines**, *Energies*, 2017



- 4- E Ferrer and RHJ Willden, A high order Discontinuous Galerkin - Fourier incompressible 3D Navier–Stokes solver with **rotating sliding meshes**, *Journal of Computational Physics*, 2012
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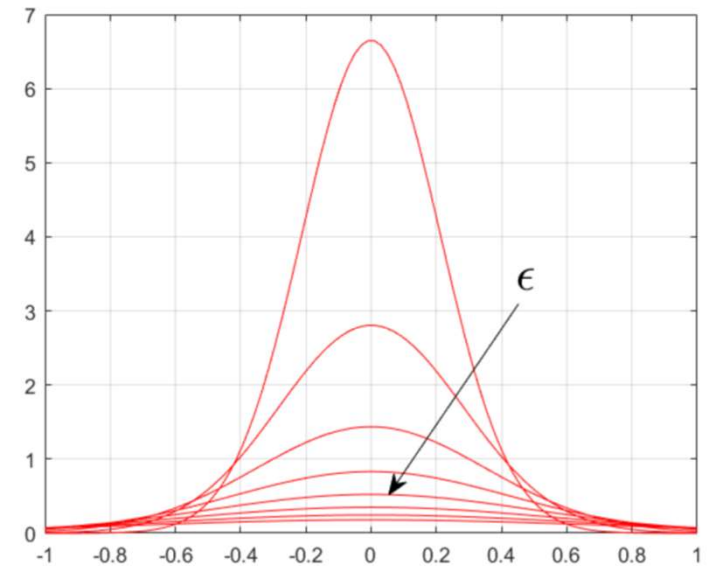


Tabulated data

$$f_L = \frac{1}{2} \rho U_{rel}^2 S C_l, \quad f_D = \frac{1}{2} \rho U_{rel}^2 S C_d,$$

$$\frac{d\mathbf{Q}}{dt} = \mathcal{R}(\mathbf{Q}, \nabla \mathbf{Q}) + \mathcal{S}(\mathbf{Q})$$

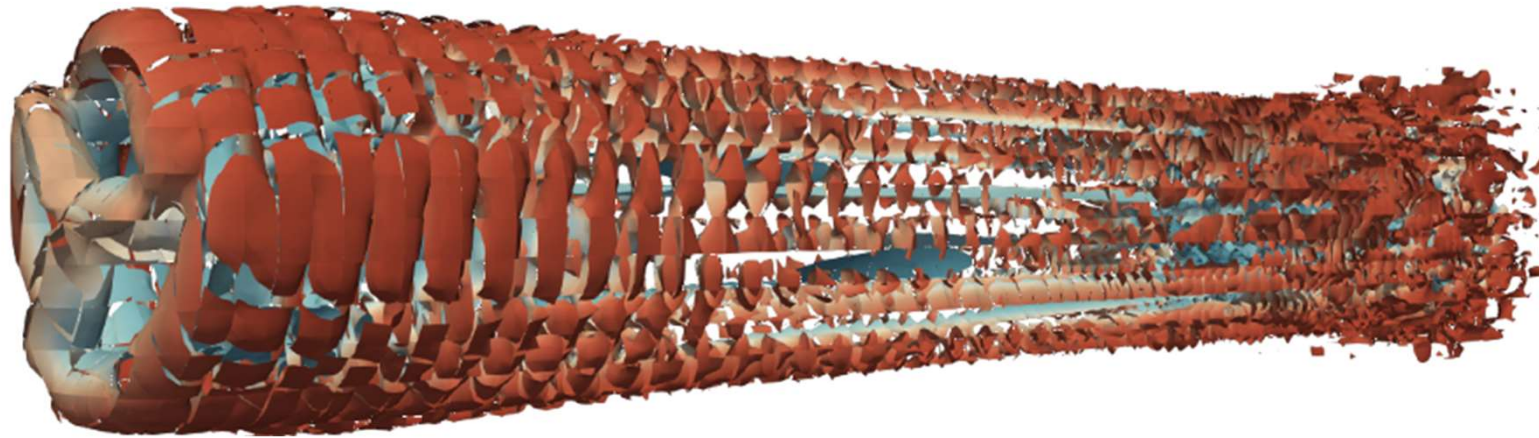
$$\mathcal{S}(\mathbf{Q}) = \eta_\epsilon \mathbf{F}$$



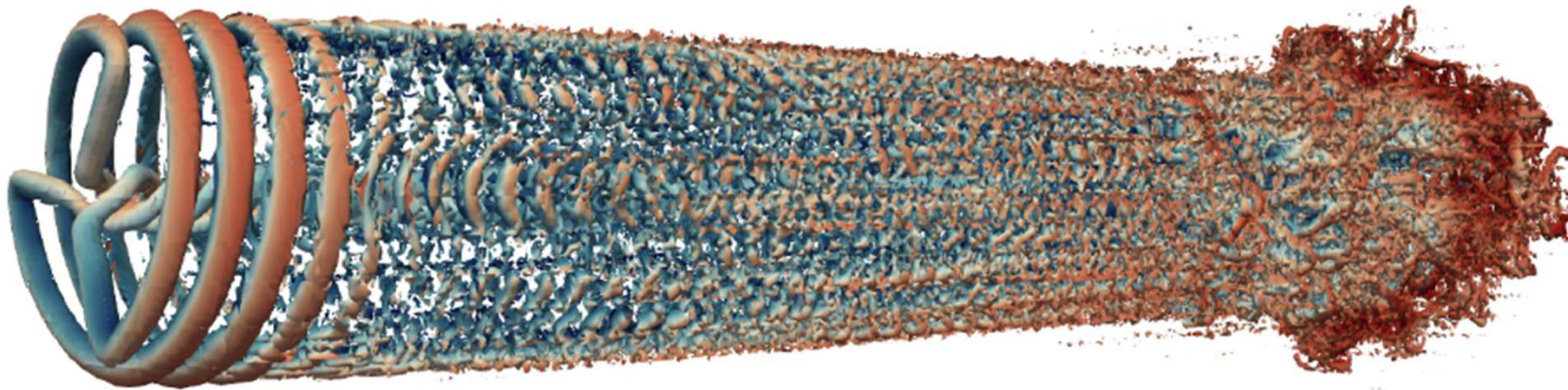
$$\eta_\epsilon = \frac{1}{\epsilon^3 \pi^{\frac{3}{2}}} e^{-\left(\frac{d}{\epsilon}\right)^2}$$

$$\epsilon_k = k \times \Delta_{grid} = k \times \frac{(\Delta_x \Delta_y \Delta_z)^{\frac{1}{3}}}{p+1}$$

Improved solution using the same h-mesh



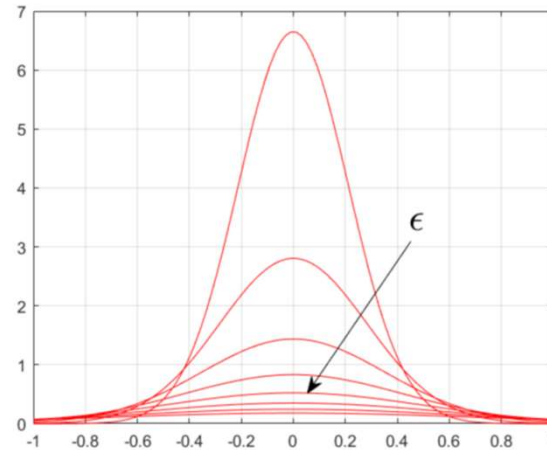
$P = 2$



$P = 5$



$$f_L = \frac{1}{2} \rho U_{rel}^2 S C_l, \quad f_D = \frac{1}{2} \rho U_{rel}^2 S C_d,$$

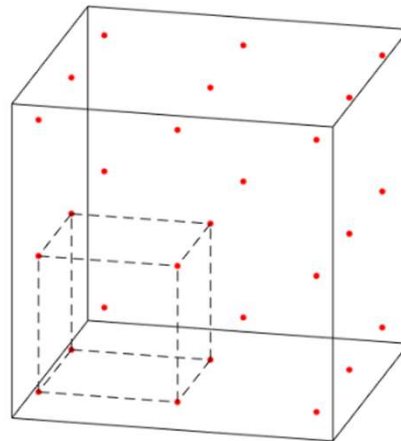


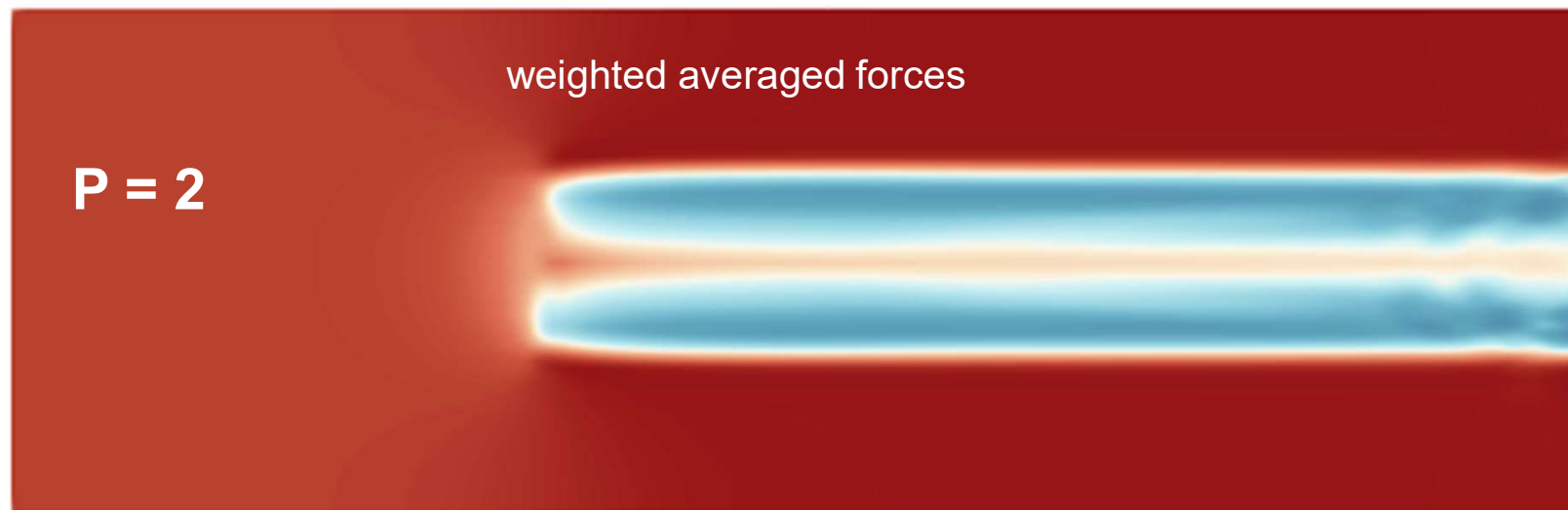
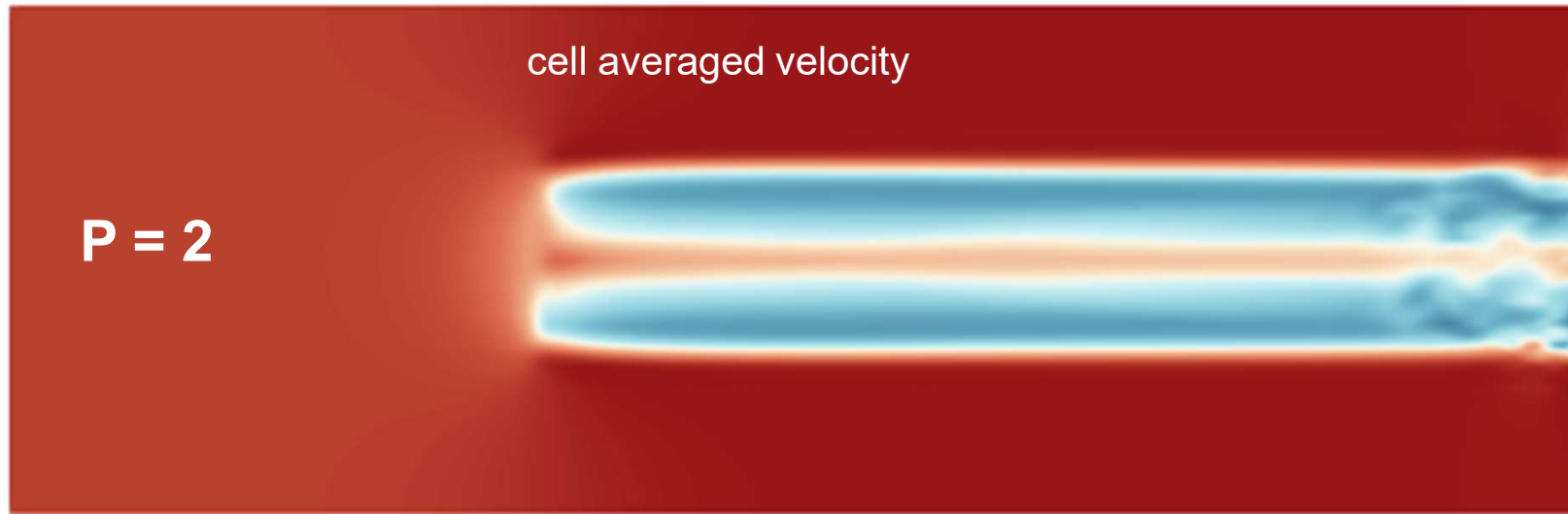
cell averaged velocity

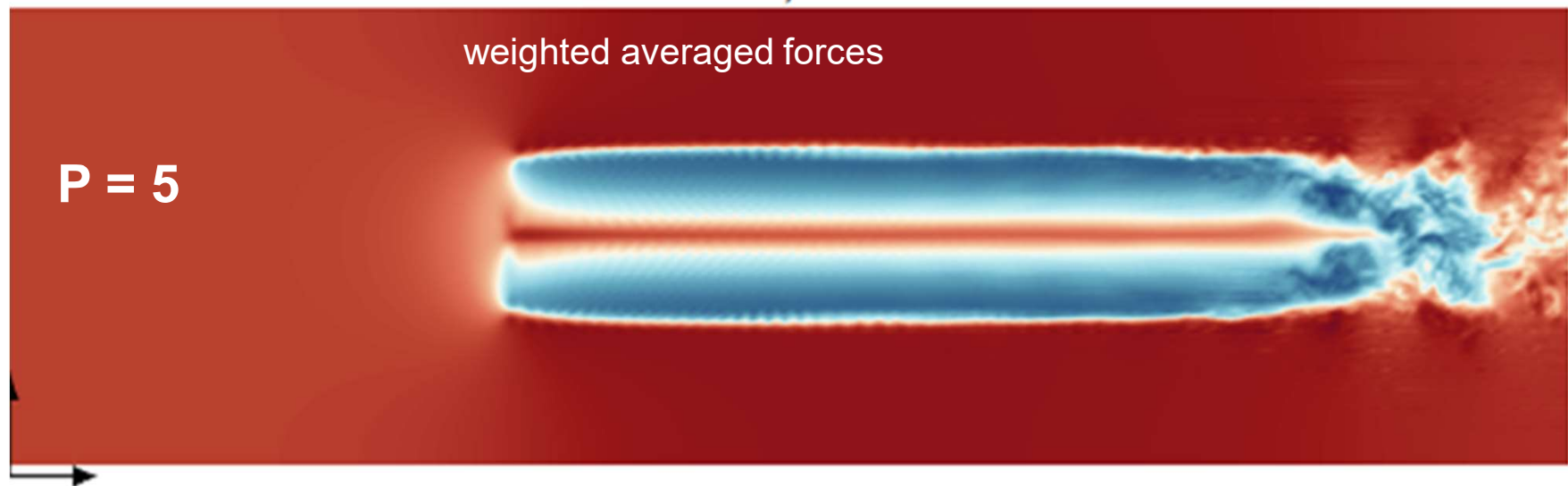
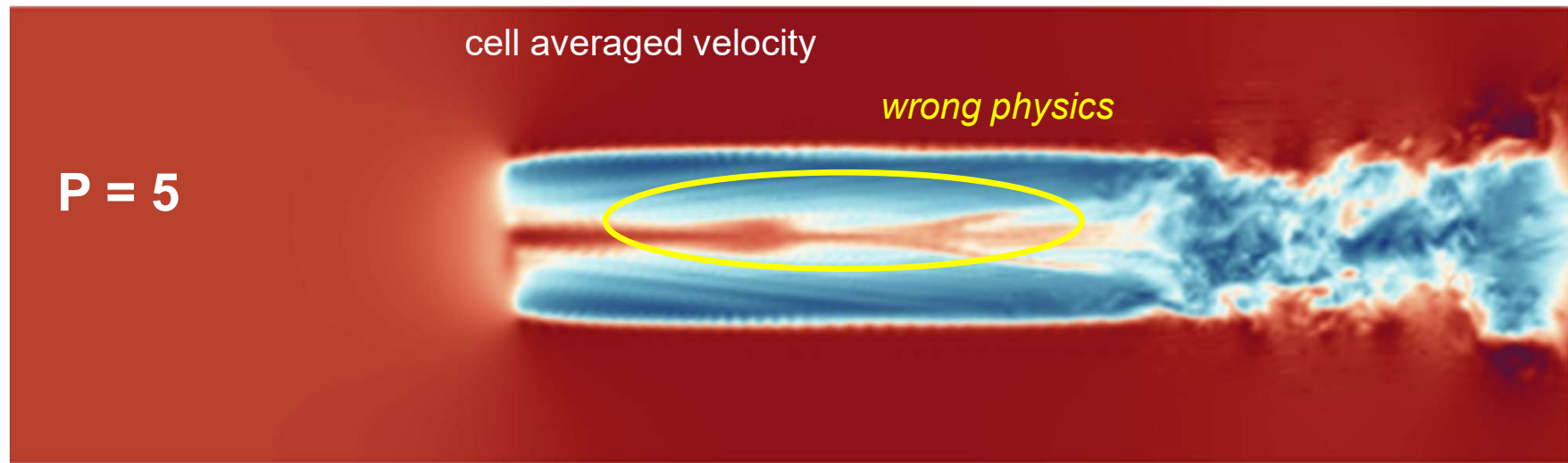
$$\bar{\mathbf{q}}_t = \frac{1}{N} \sum_{i=1}^N \mathbf{q}_{t_i}$$

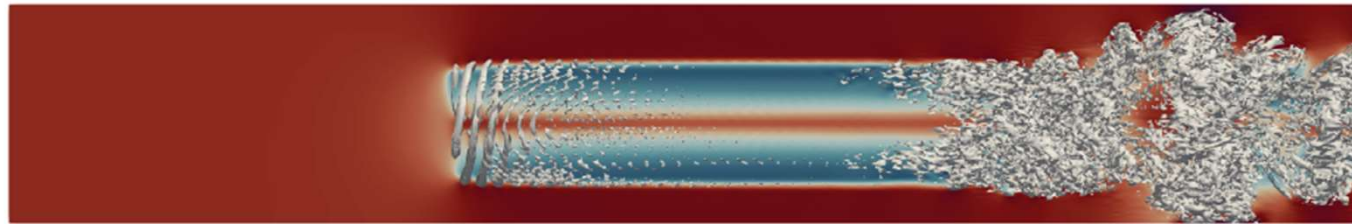
weighted averaged forces

$$\bar{\mathbf{f}}_j = \frac{\sum_{i=1}^N \eta_{ji}(d) \cdot \mathbf{f}_i}{\sum_{j=1}^{N_a} \sum_{i=1}^N \eta_{ji}(d)}$$

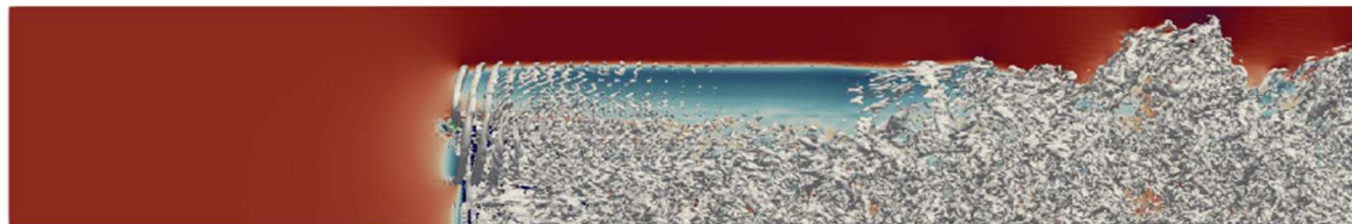




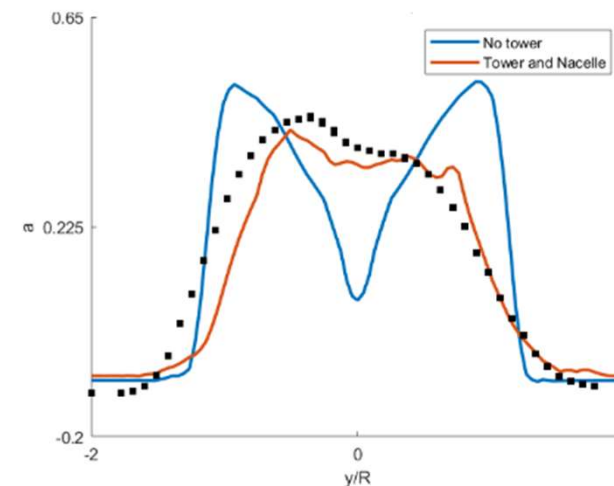
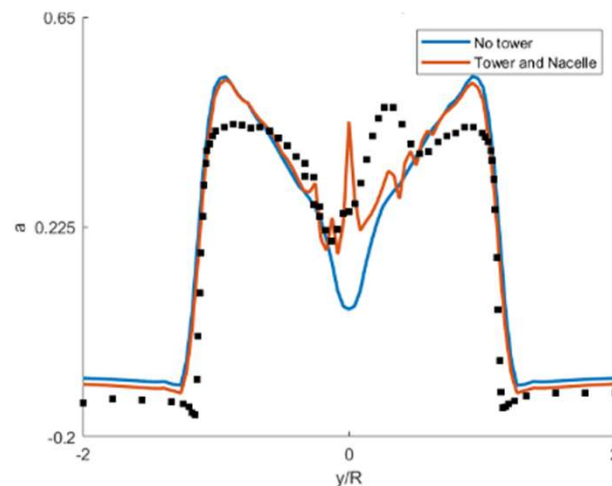




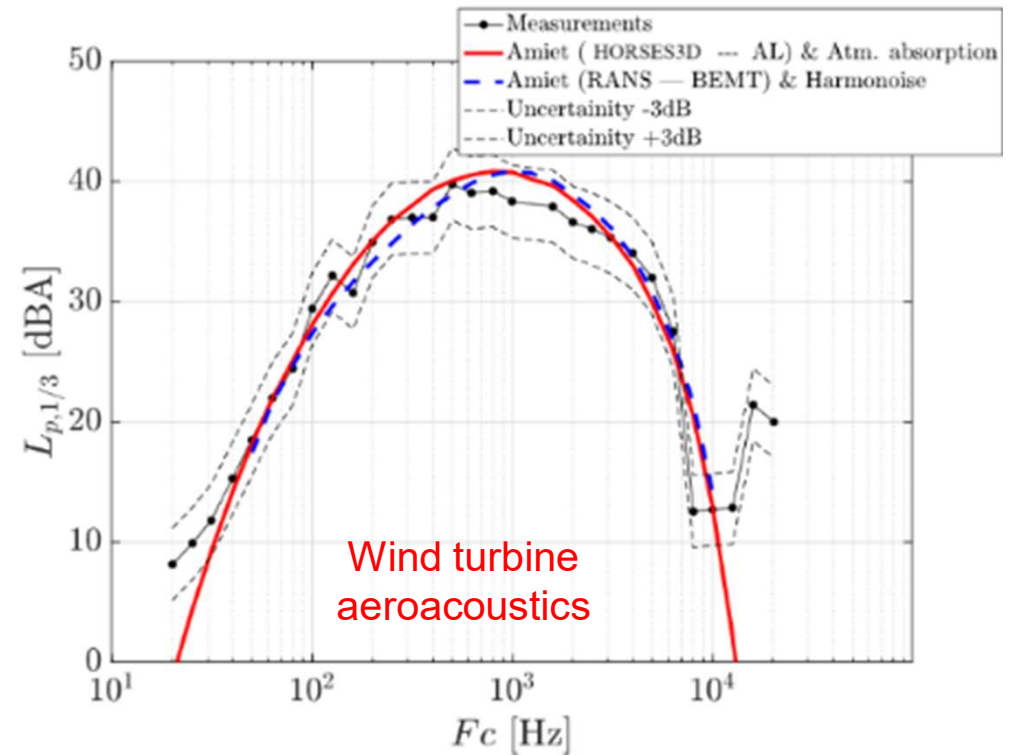
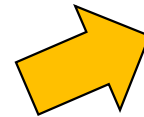
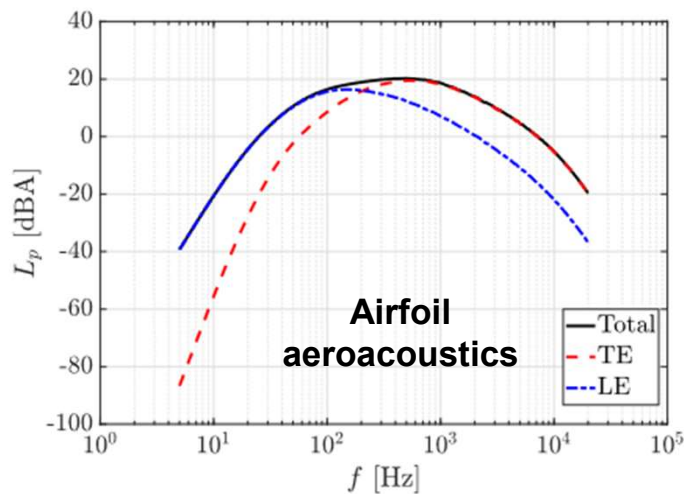
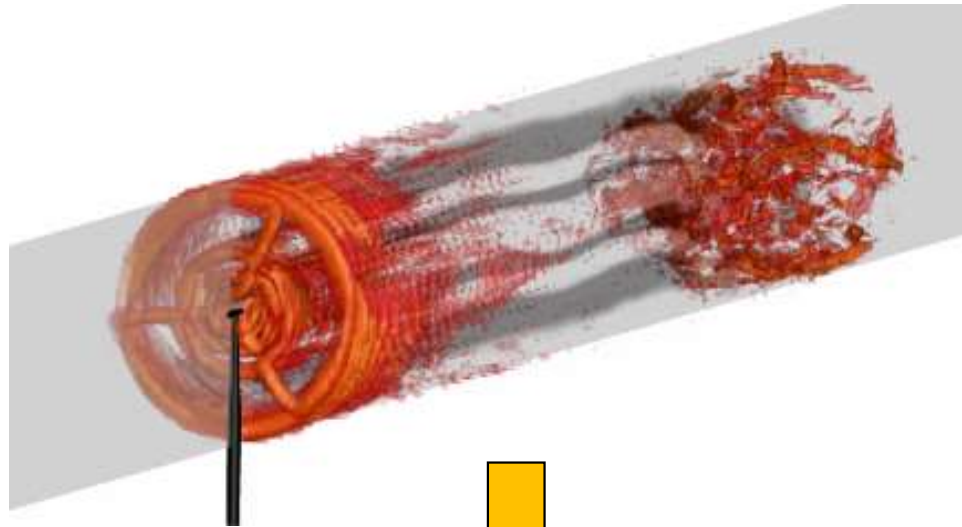
a) Actuator line without tower and nacelle.

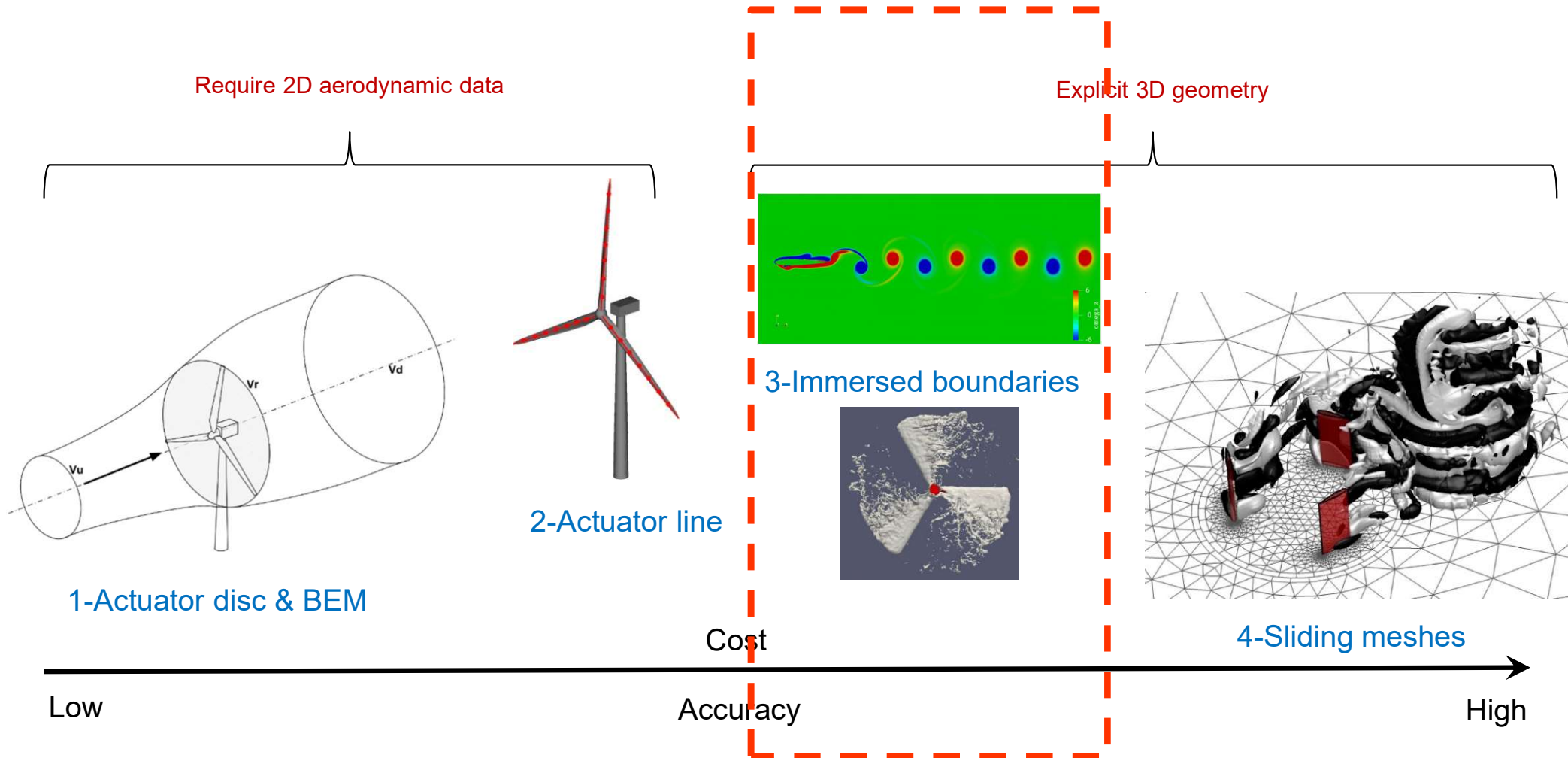


b) Actuator line with tower and nacelle, which are modeled using immersed boundaries.



Computing acoustics with actuator lines + Amiet

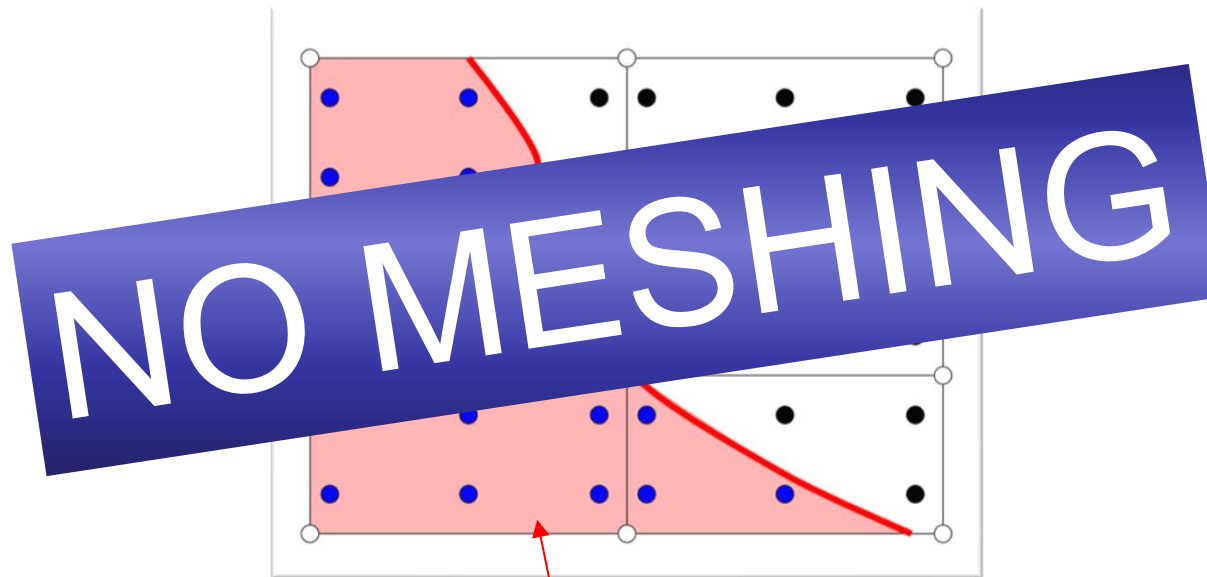




- 4- E Ferrer and RHJ Willden, A high order Discontinuous Galerkin - Fourier incompressible 3D Navier–Stokes solver with **rotating sliding meshes**, *Journal of Computational Physics*, 2012
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Immersed boundary method (penalty) → Mesh Free method

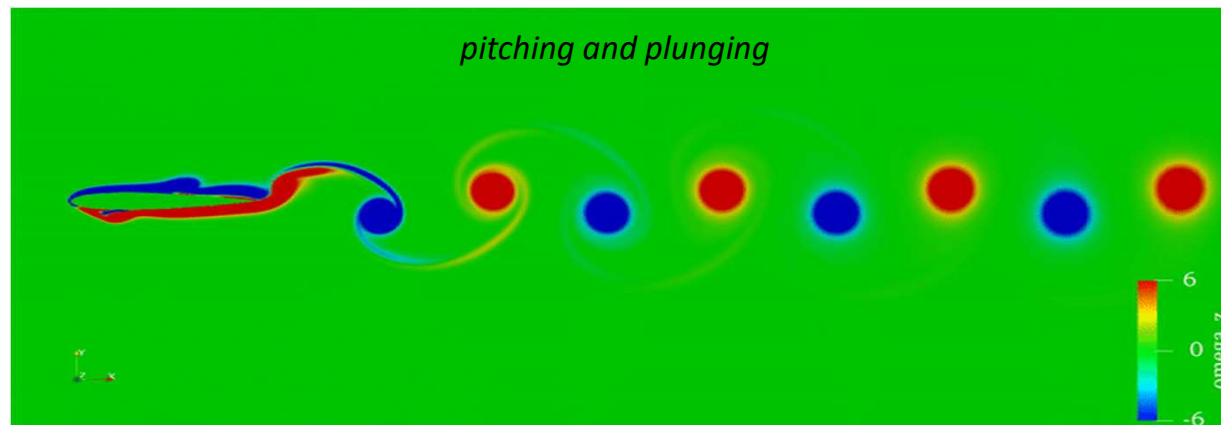
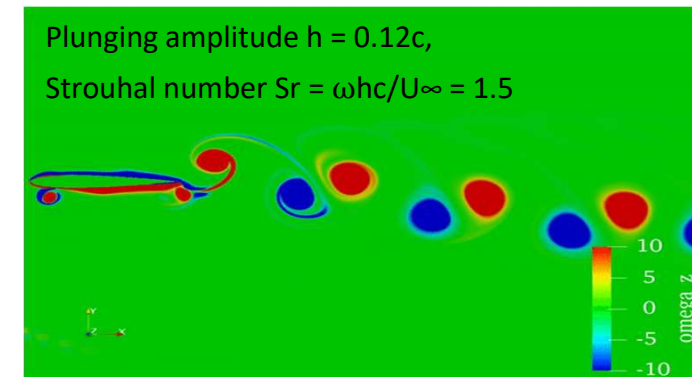
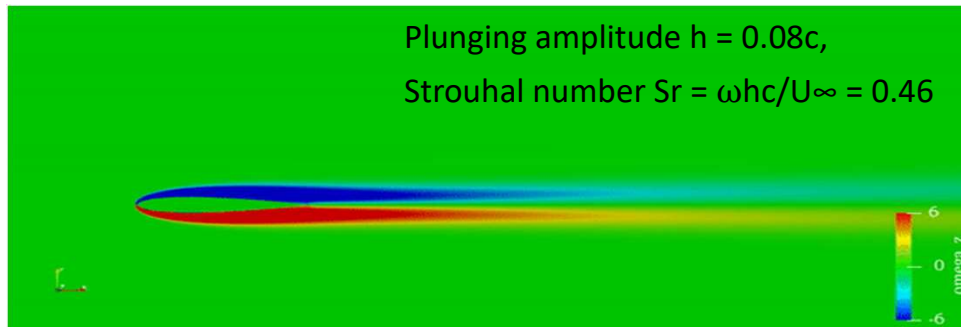
- Simple 'Cartesian' grids (with local P refinement)
- Complex geometries
- Moving geometries



$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \vec{\mathbf{F}}(\mathbf{U}) = \nabla \cdot \vec{\mathbf{G}}(\mathbf{U}, \nabla \mathbf{U}) + \mathbf{S}(\mathbf{U}).$$

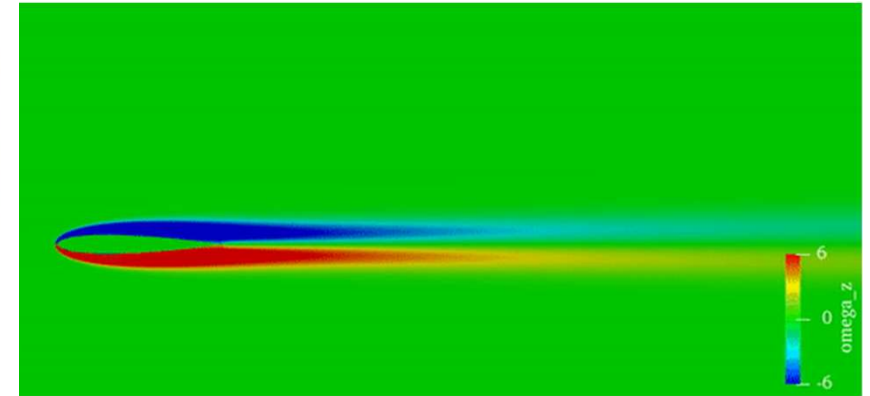
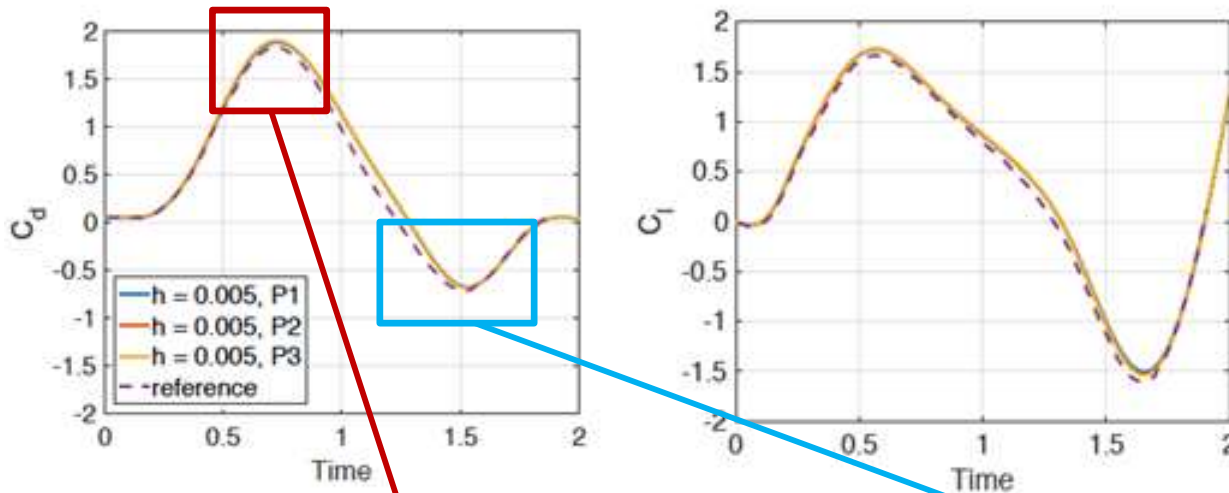
$$\mathbf{S}(\mathbf{U}) = a \times \begin{pmatrix} 0 \\ \rho u_0 - \rho u \\ \rho v_0 - \rho v \\ \rho w_0 - \rho w \\ \frac{\rho}{2}(u_0^2 + v_0^2 + w_0^2) - \frac{\rho}{2}(u^2 + v^2 + w^2) \end{pmatrix}$$

Moving NACA0012 at Reynolds number 1000, *pitching and plunging*:

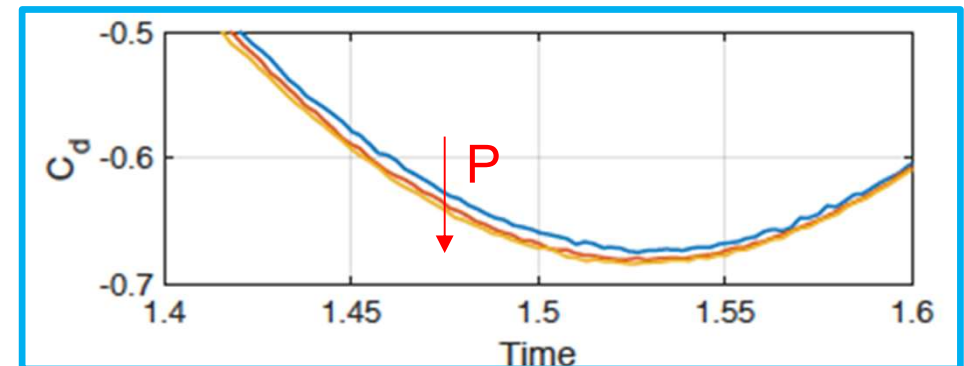
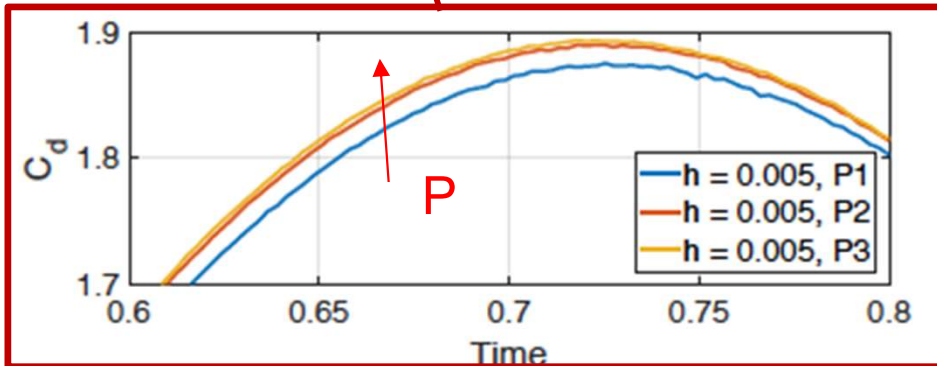


- J Kou, A Hurtado-de-Mendoza, S Joshi, S Le Clainche, **E Ferrer**, "Eigensolution analysis of immersed boundary method based on volume penalization: applications to high-order schemes", *Journal of Computational Physics*, Vol 449, 110817, 2022
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- J Kou, **E Ferrer**, "A combined volume penalization / selective frequency damping for immersed boundary methods applied to high-order schemes" *Journal of Computational Physics*, Vol 472, 111678, 2023

Moving NACA0012 at Reynolds number 1000, *pitching and plunging*:



P-adaption increases accuracy



- J Kou, A Hurtado-de-Mendoza, S Joshi, S Le Clainche, **E Ferrer**, "Eigensolution analysis of immersed boundary method based on volume penalization: applications to high-order schemes", *Journal of Computational Physics*, Vol 449, 110817, 2022
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Immersed boundary method (penalty)

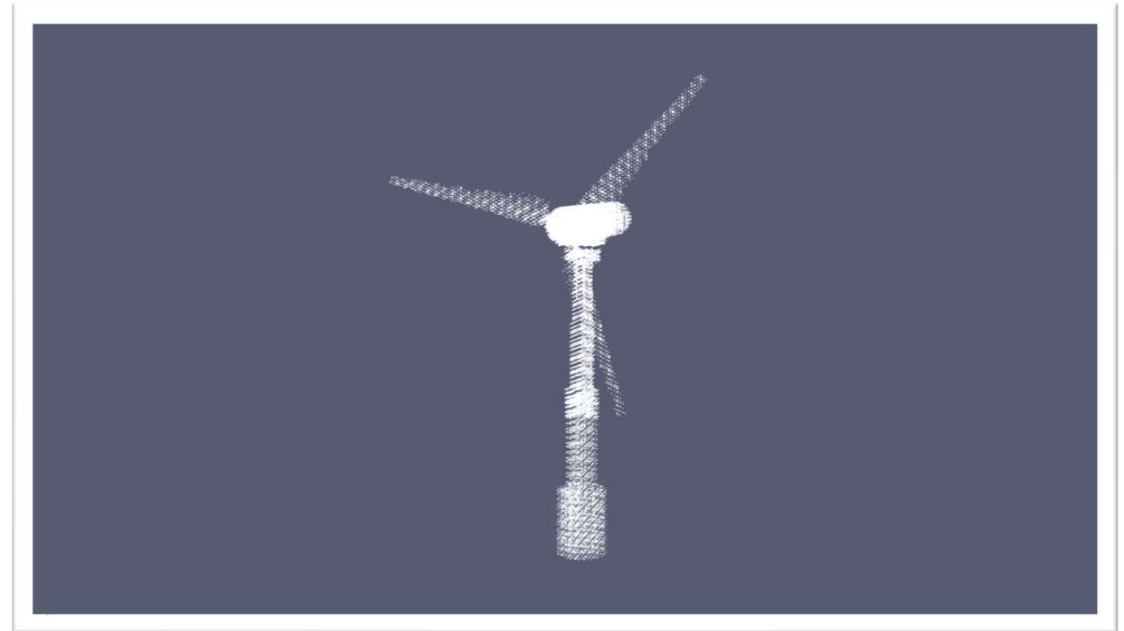
IB for rotating Wind turbine



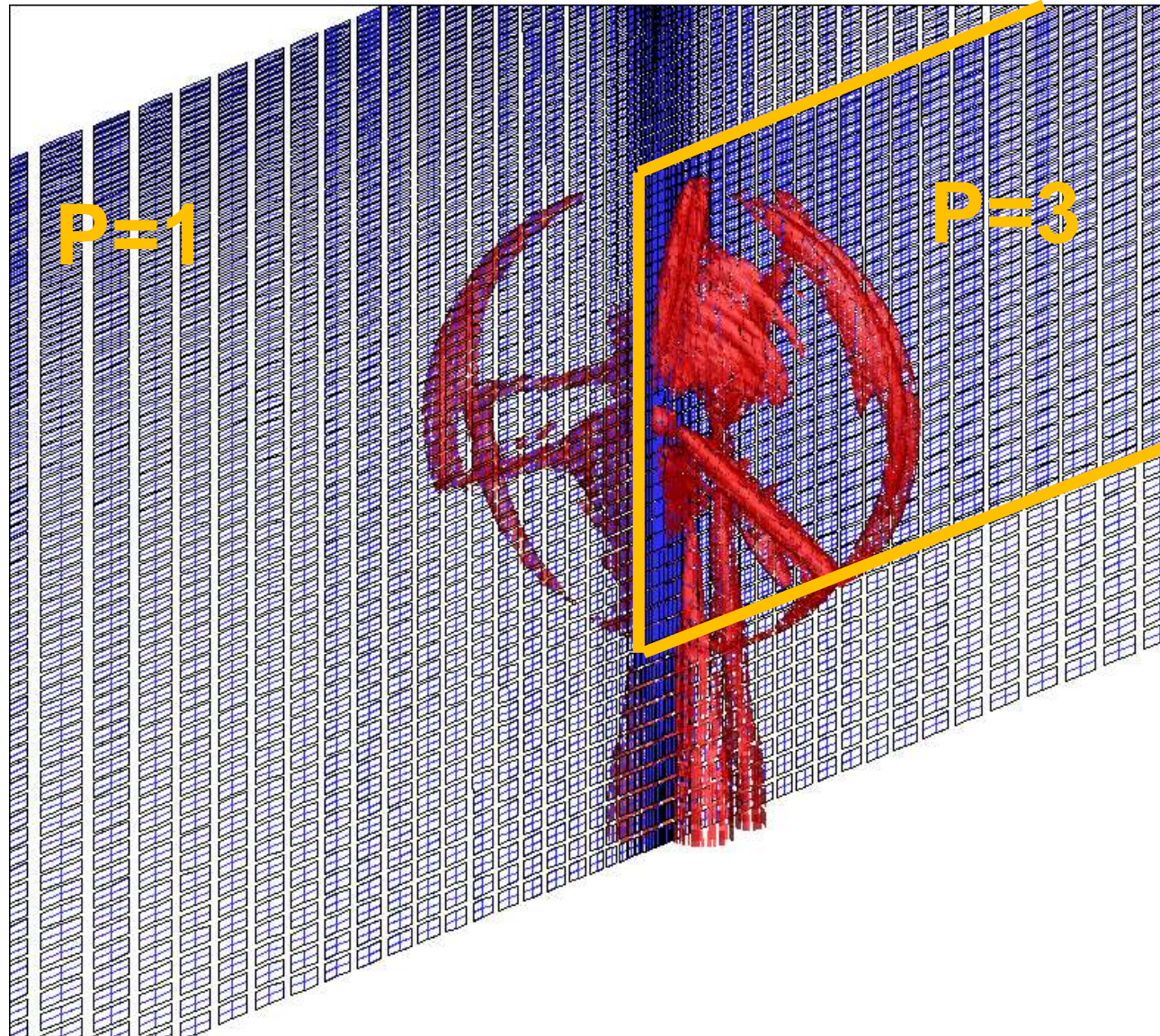
Only CAD '.stl' file
for the wind turbine

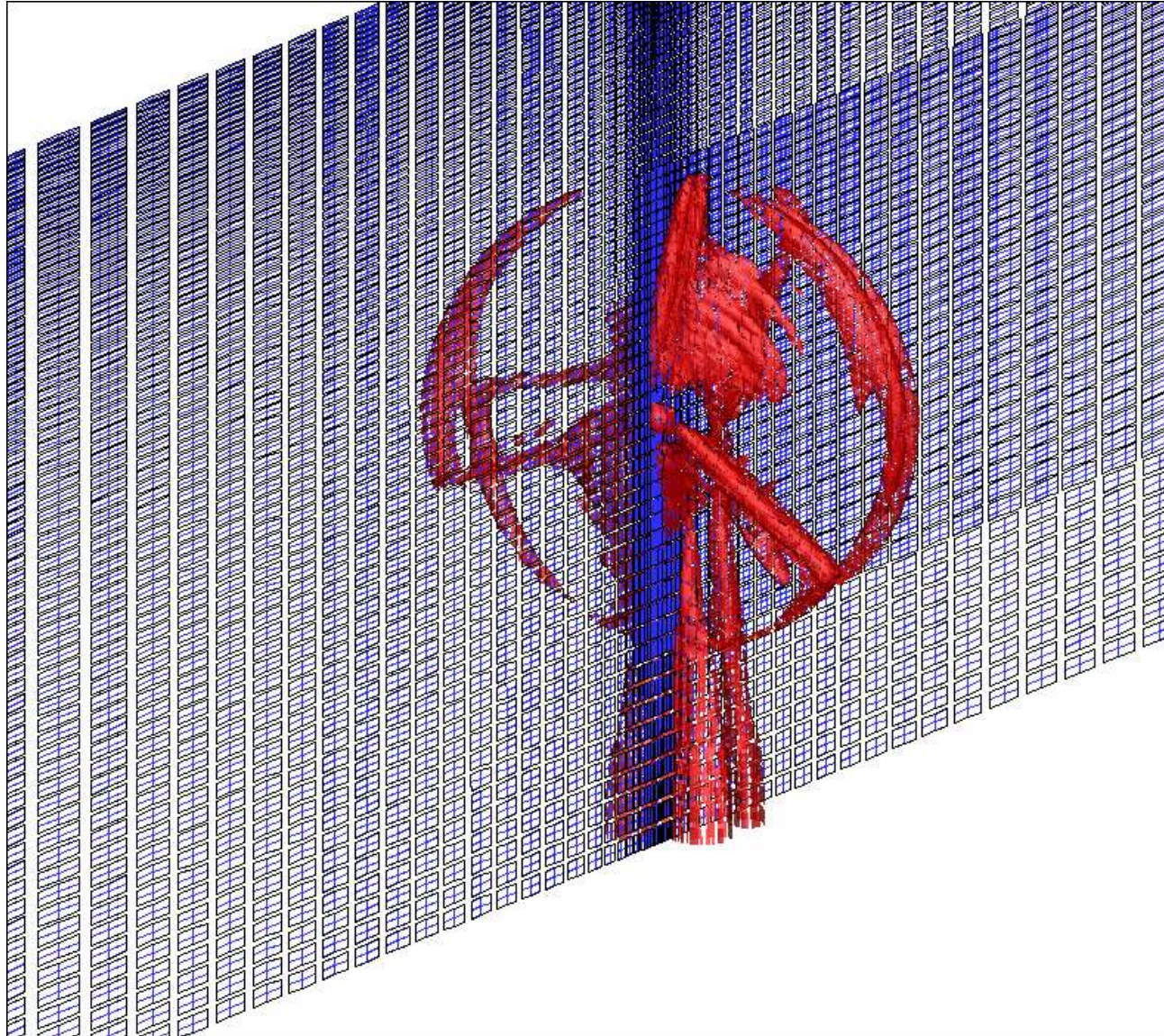
&

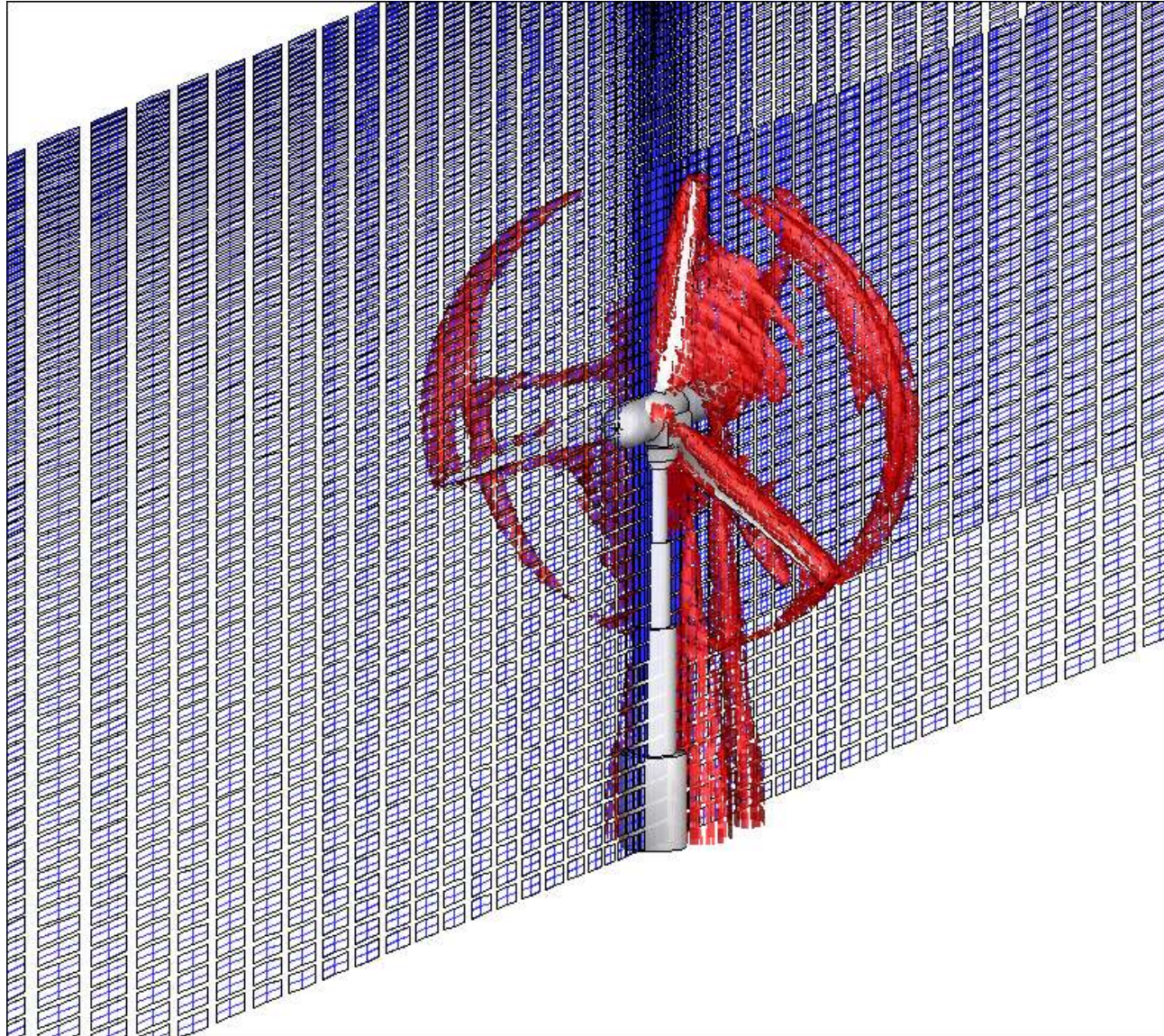
Simple Cartesian mesh

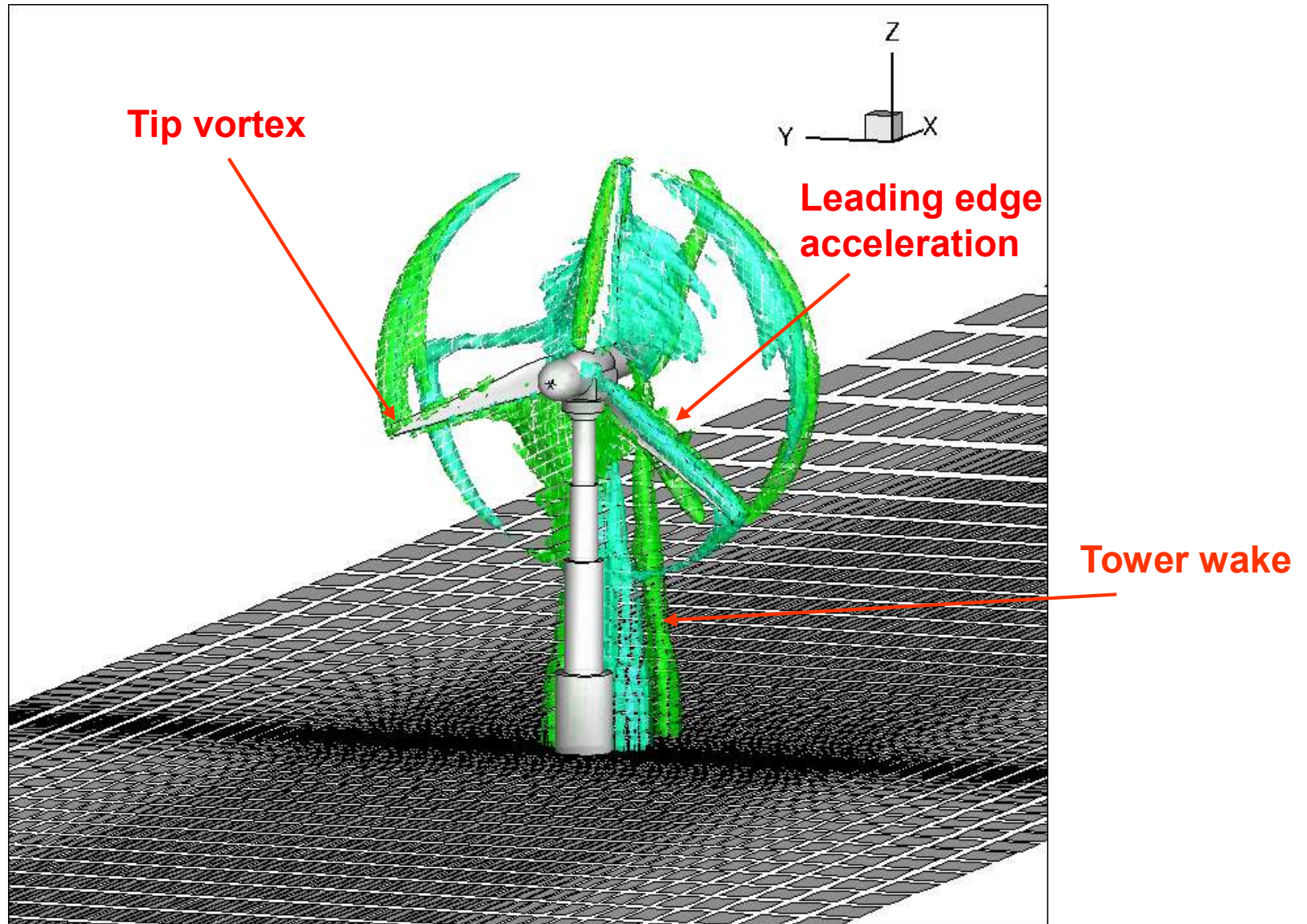


Penalty points for the wind turbine



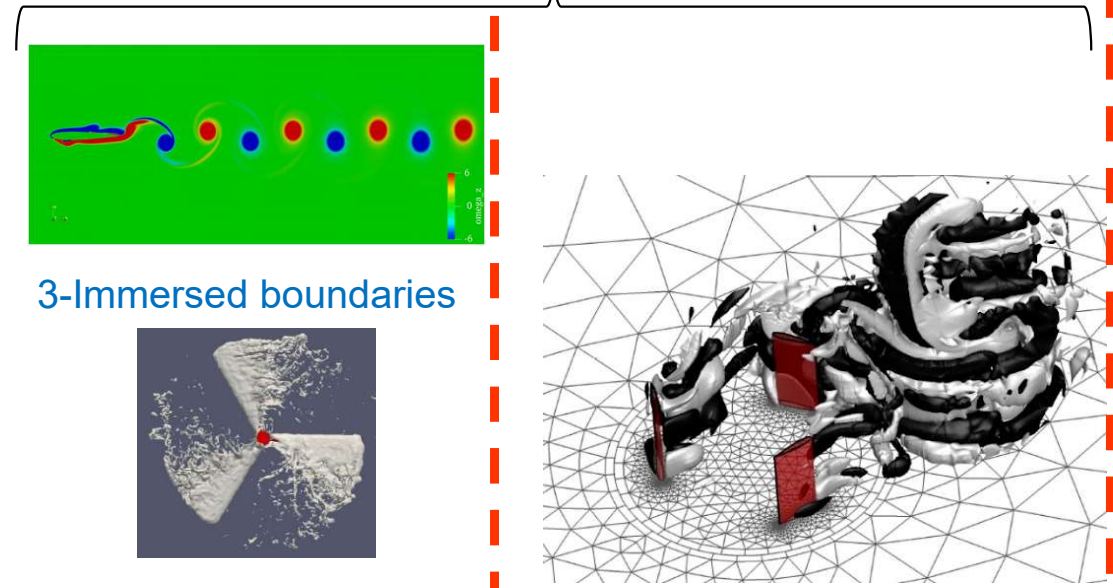
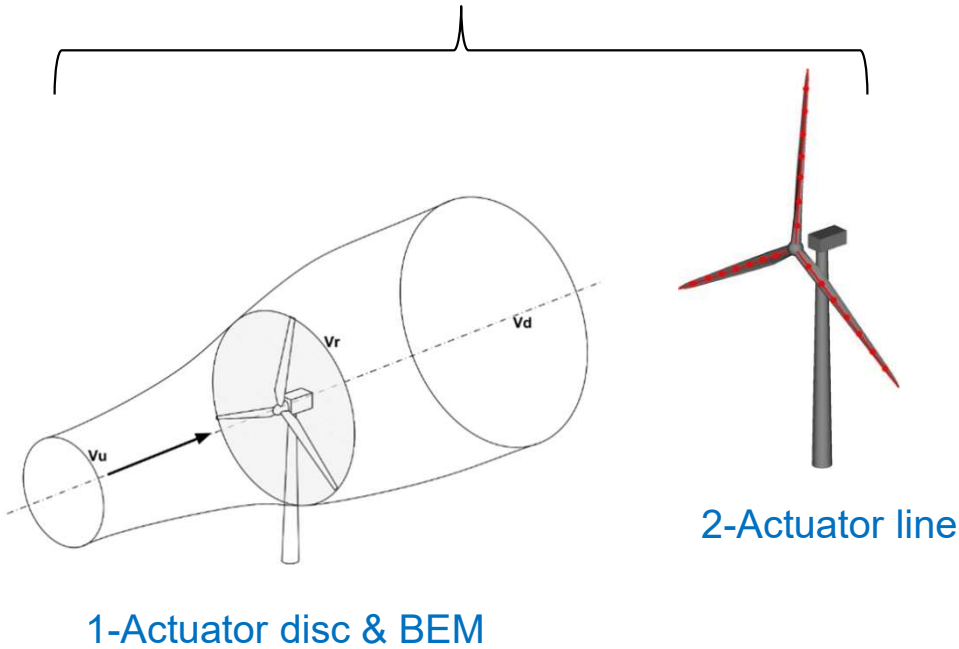






Require 2D aerodynamic data

Explicit 3D geometry



Cost

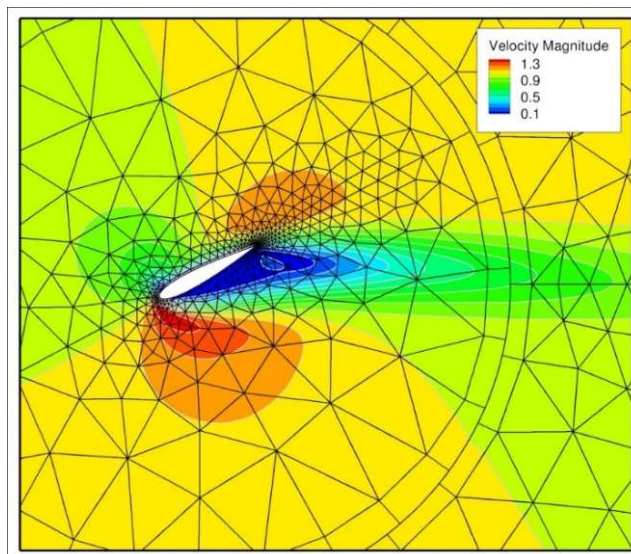
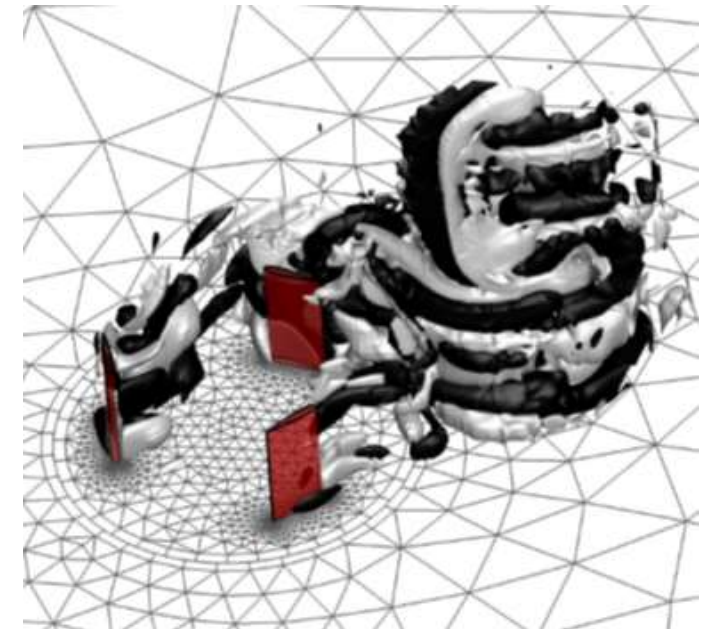
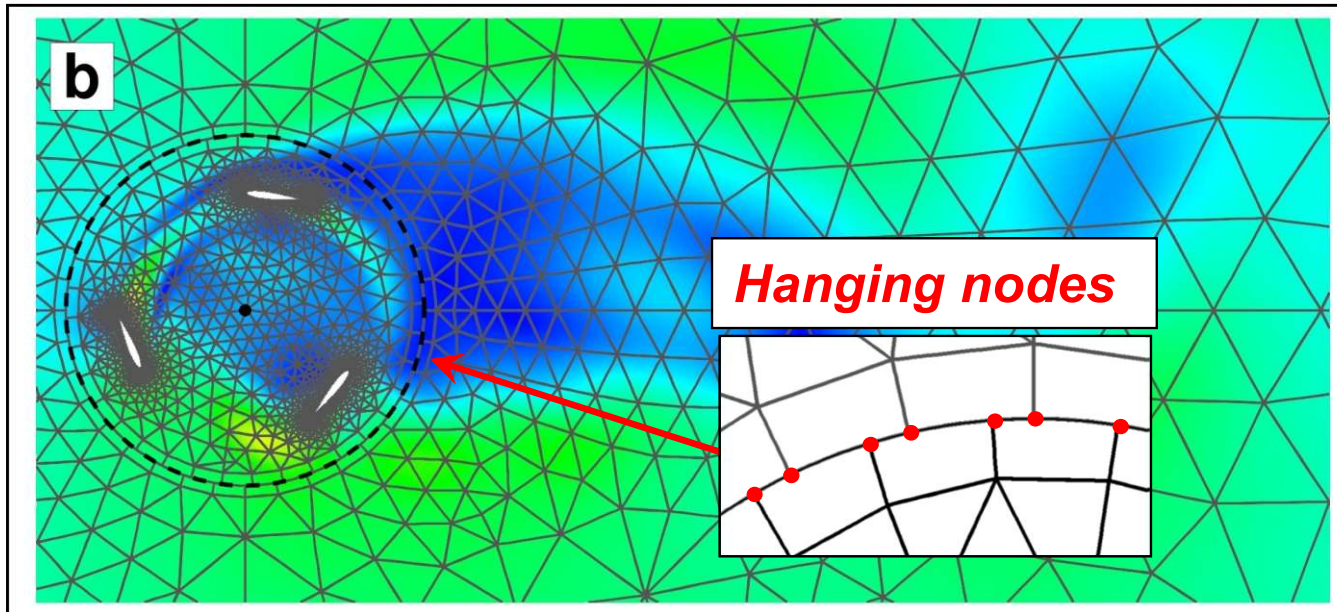
Accuracy

Low

High

- 4- E Ferrer and RHJ Willden, A high order Discontinuous Galerkin - Fourier incompressible 3D Navier–Stokes solver with **rotating sliding meshes**, *Journal of Computational Physics*, 2012
- 4- E Ferrer, RHJ Willden, Blade–wake interactions in **cross-flow turbines**, *International Journal of Marine Energy*, 2015
- 3- J Kou, A Hurtado-de-Mendoza, S Joshi, S Le Clainche, E Ferrer, Eigensolution analysis of **immersed boundaries** for high-order schemes, *Journal of Computational Physics*, 2022
- 3- J Kou, S Joshi, A Hurtado-de-Mendoza, K Puri, C Hirsch, E Ferrer, An **Immersed boundary** method for high–order flux reconstruction, *Journal of Computational Physics*, 2022
- 2 & 3- E Ferrer, S Colombo, O Marino, “Aeroacoustic predictions of wind turbines based on **actuator lines and immersed boundaries**”, *Under review at Wind Energy*
- 1- E Ferrer, S Le Clainche, **Simple models for cross flow turbines**, in *Recent advances in CFD for Wind and Tidal Offshore Turbines*, 2019
- 1- E Ferrer, OMF Browne, E Valero, Sensitivity analysis to control the far–wake unsteadiness behind **turbines**, *Energies*, 2017

High order sliding meshes



DG solution
Rotating NACA0015
Re=100
Rot speed=0.3
polynomial order
k=5

Summary

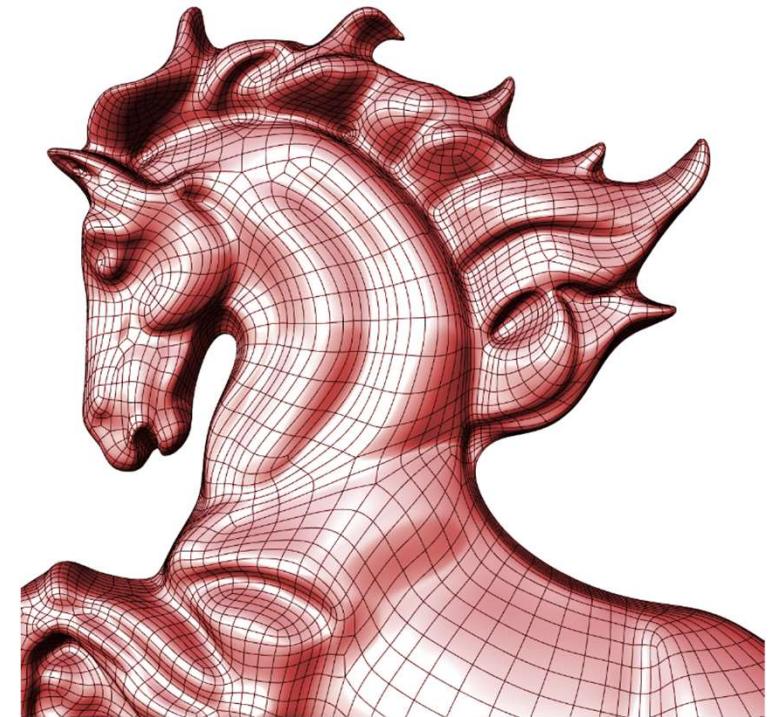
1- Introduction to DG & Horses3d

2- Multiphysics

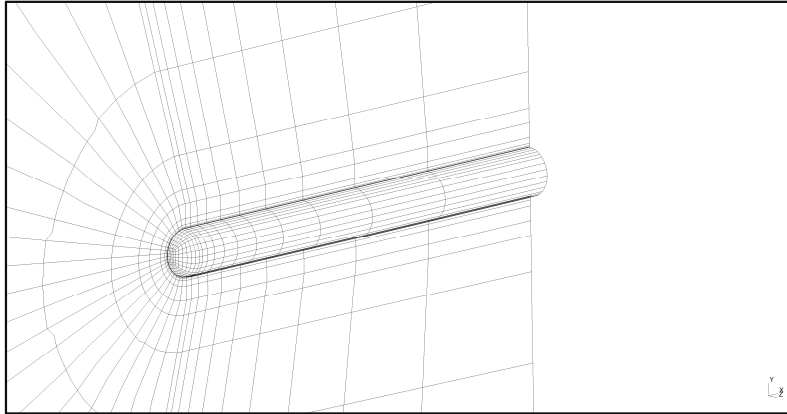
- Wind turbines
- Turbulence

3. Machine Learning + CFD

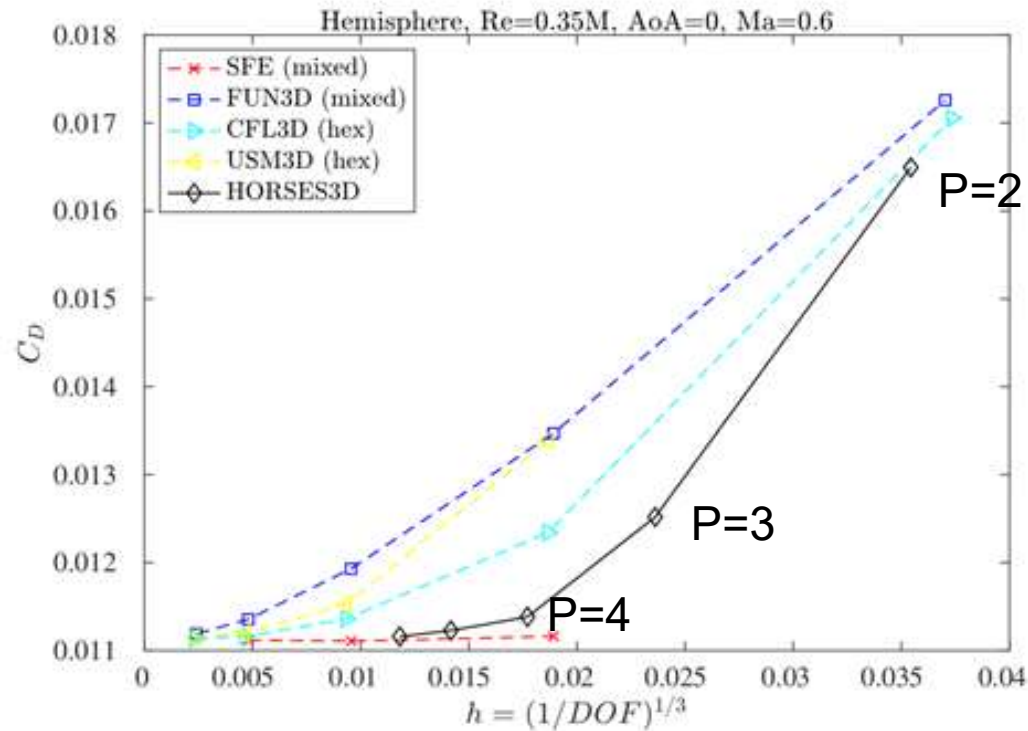
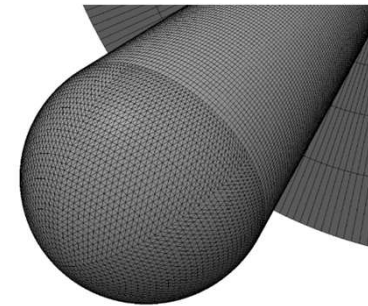
- Mesh adaption
- NN acceleration
- RL for automation



High order RANS (SAneg)

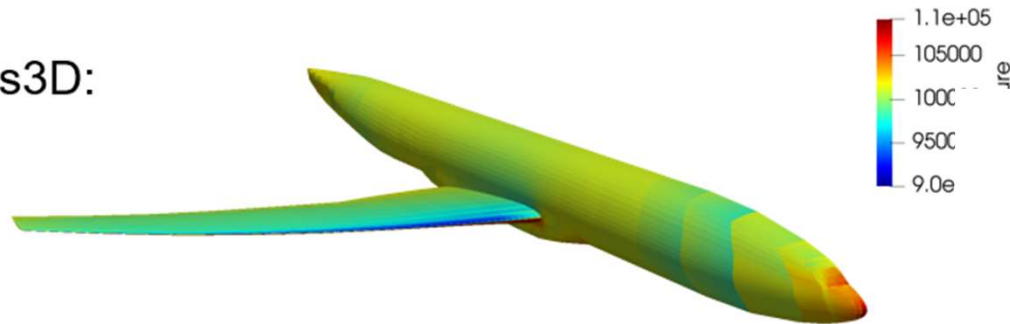


NASA workshop
https://turbmodels.larc.nasa.gov/hc3dnumerics_val.html

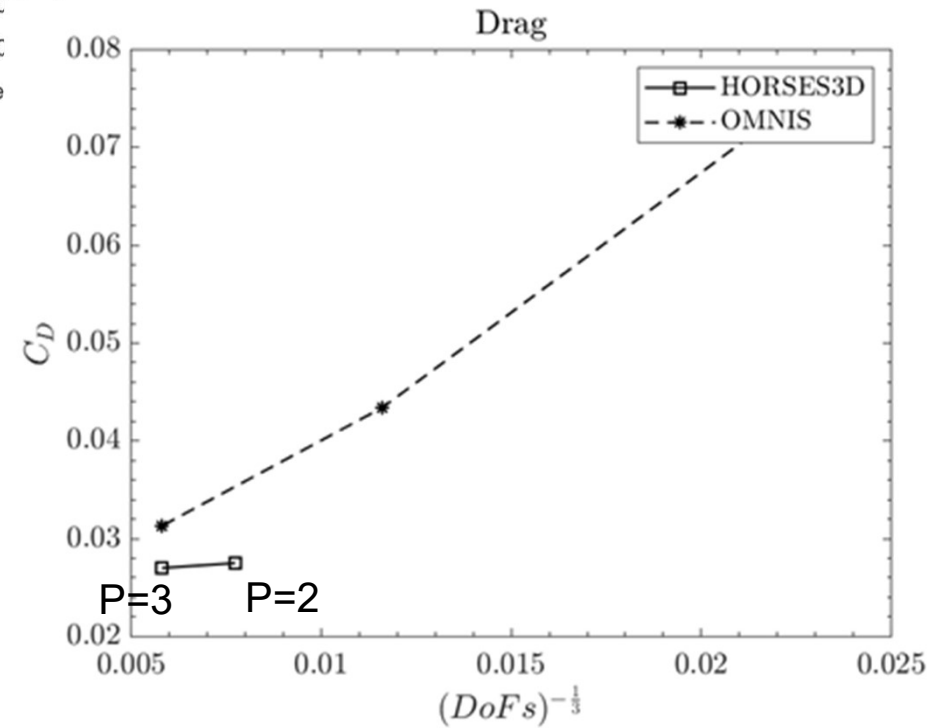
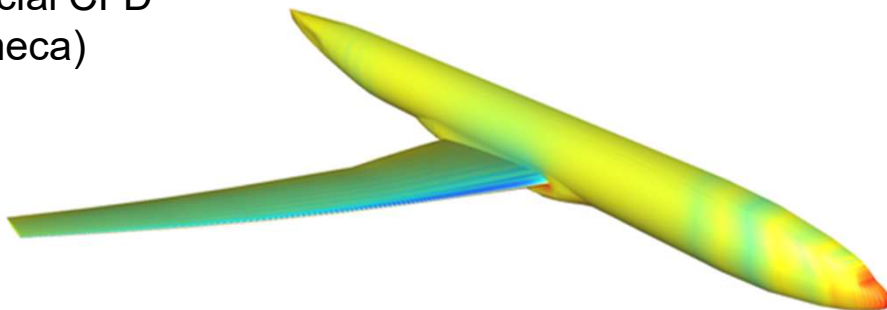


High order RANS (SAneg)

Horses3D:



Commercial CFD (Numeca)



CRM Family Of Models

From Left to Right: High-Speed CRM, High-Lift CRM, CRM with NLF wing and Icing Research Tunnel CRM.

Re=1.000.000
AoA = 0 deg



Re=1.000.000
AoA = 5 deg

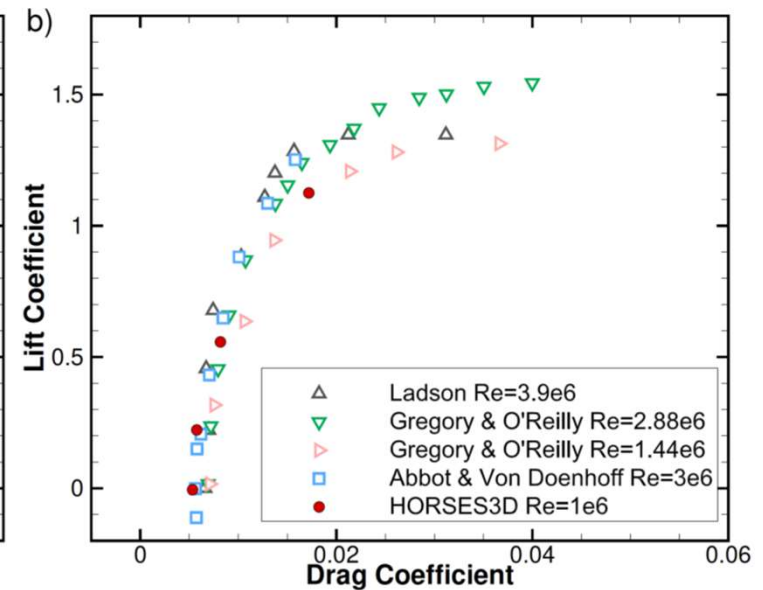
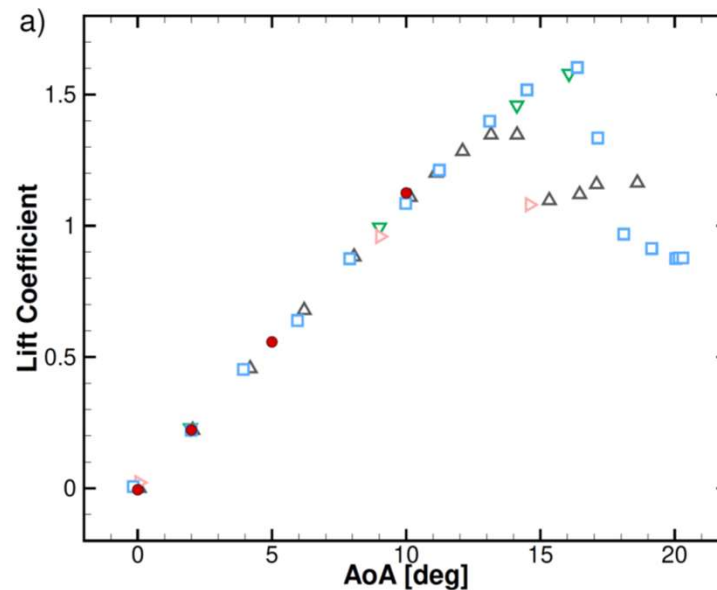


Implicit LES

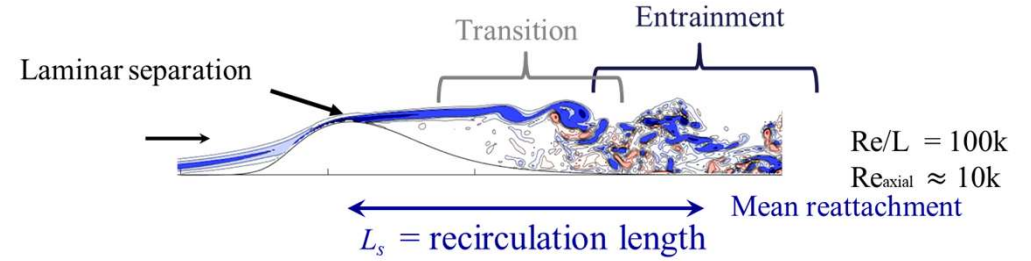
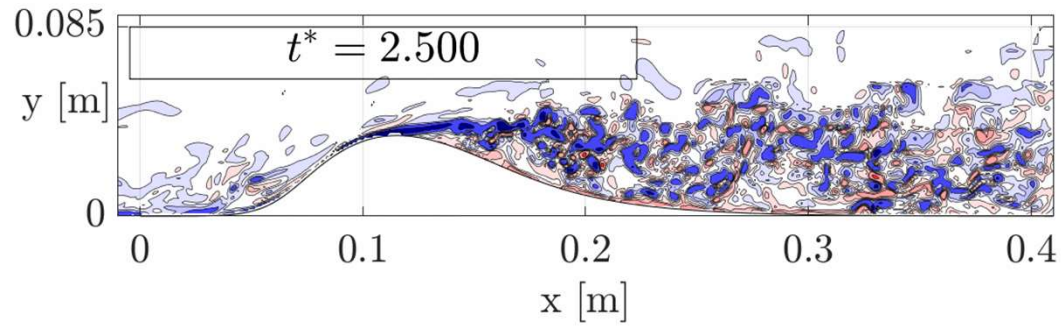
contours of velocity: [0.85; 1.2]

NACA0012 at various AoAs

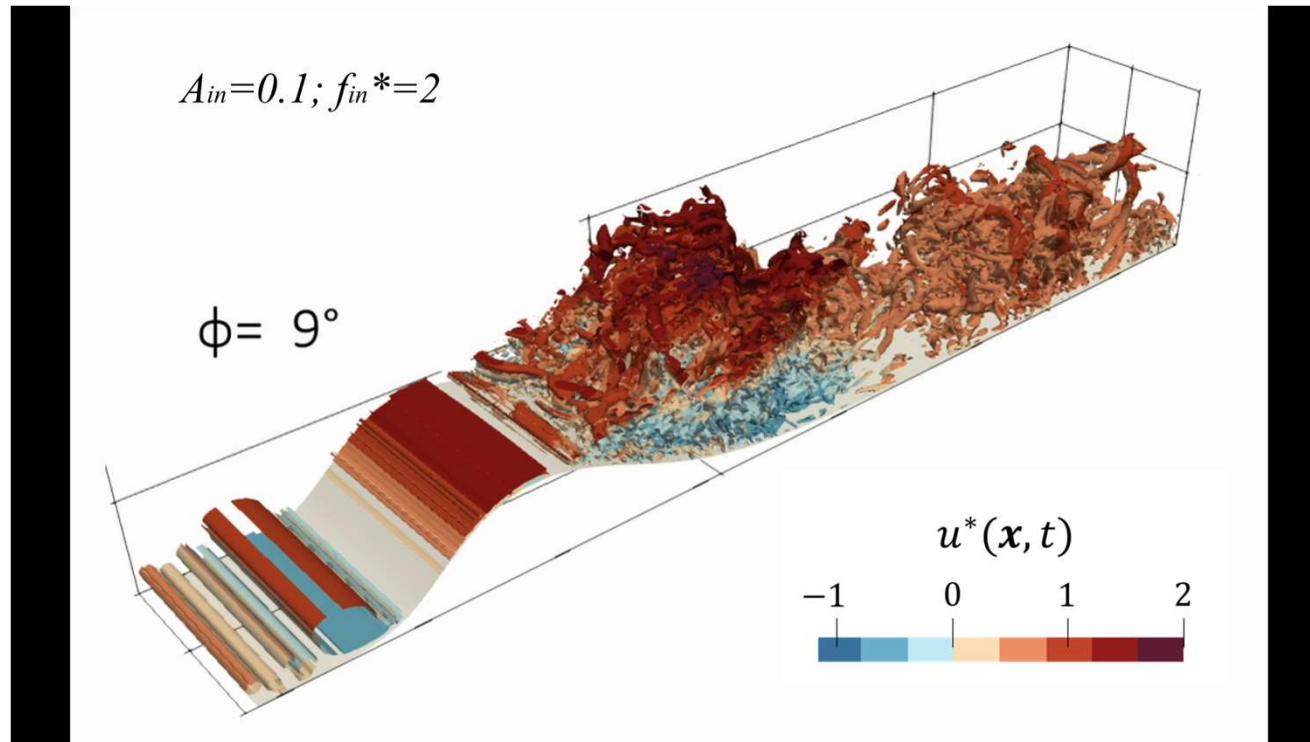
Re=1.000.000
AoA = 10 deg



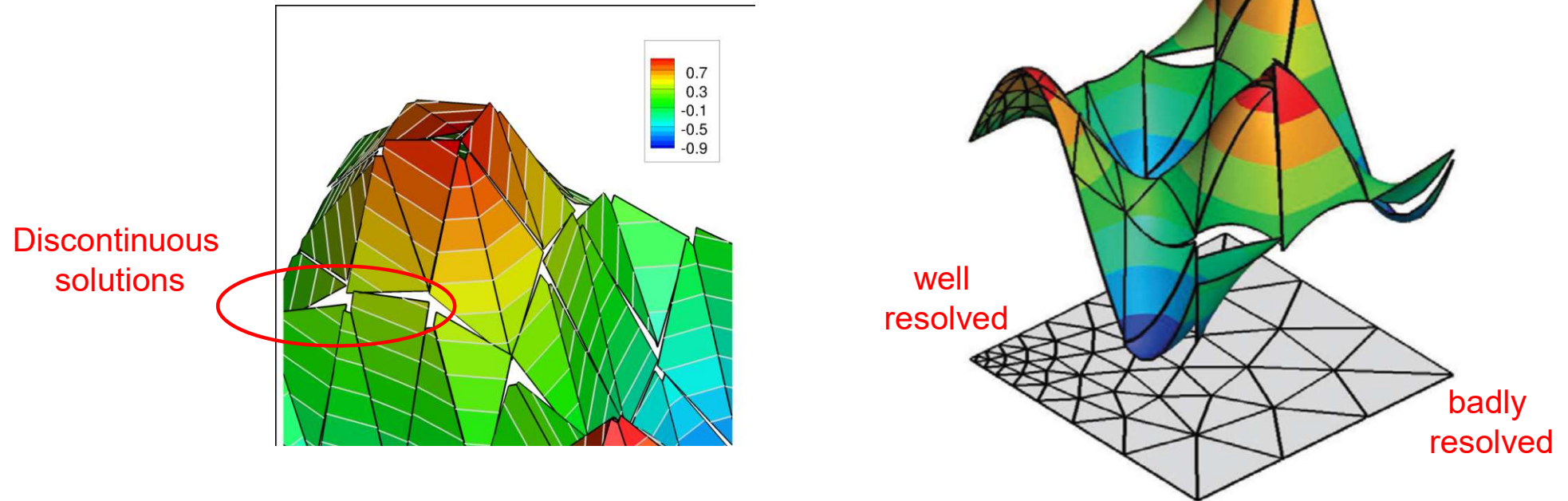
HORSES3D: Compressible DGSEM – energy-stable - SBP-SAT & Roe fluxes & BR1



Implicit LES



New turbulent models for discontinuous Galerkin



Viscosity proportional to jumps (associated to under-resolution)

Solution: $\frac{\tau_s}{Re} \int_{\partial\Omega_n} [[\tilde{\mathbf{q}}]] \phi_i \cdot \mathbf{n}$ Ferrer 2017

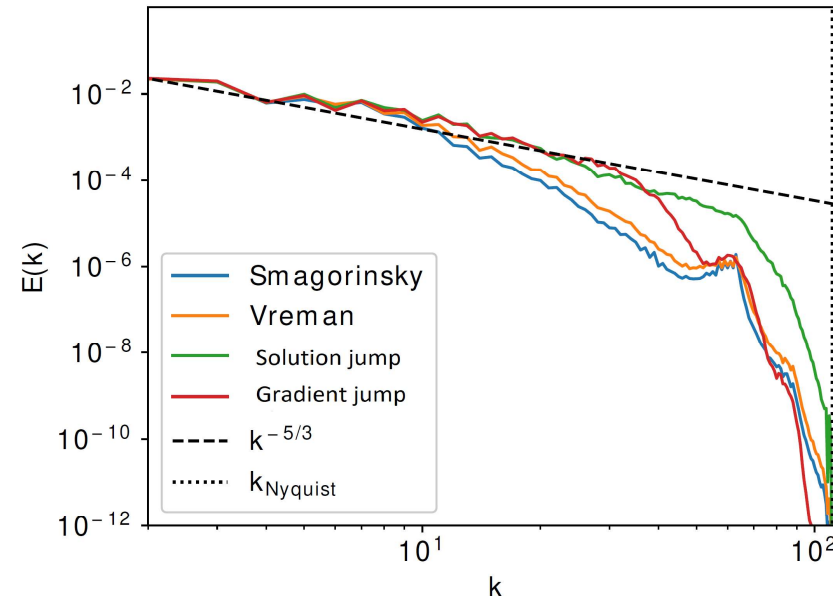
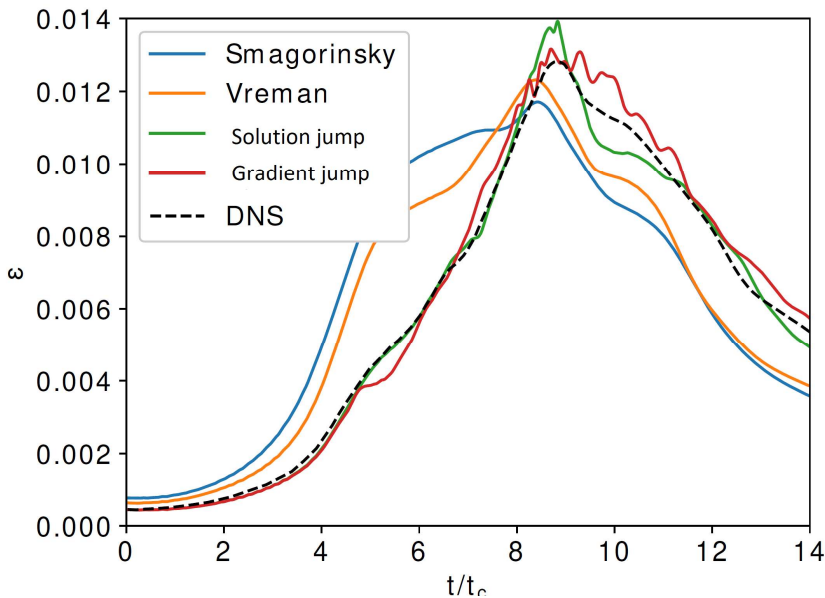
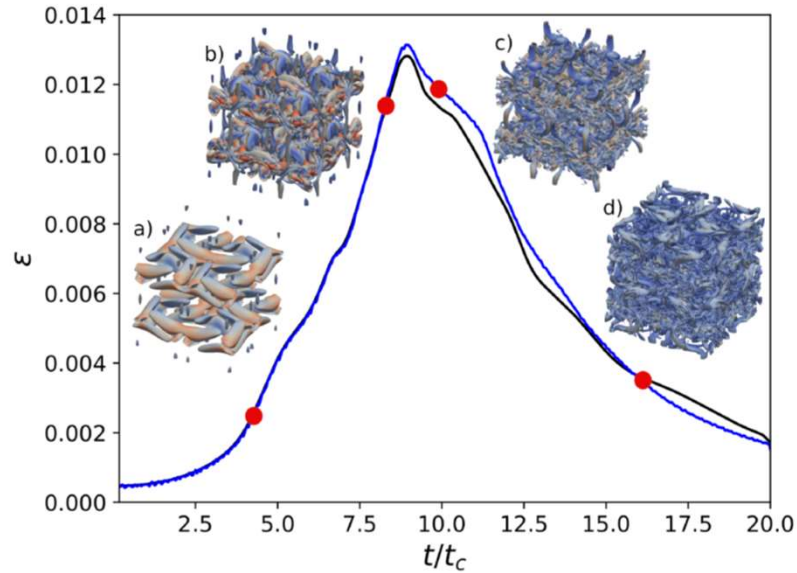
Gradients: $-\frac{\tau_g h^2}{Re} \int_{\partial\Omega_n} [[\nabla \tilde{\mathbf{q}}]] \nabla \phi_i \cdot \mathbf{n}$ Burman et al 2010
Moura et al 2022

J Kou, OA Marino, **E Ferrer**, "Jump penalty stabilisation techniques for under-resolved turbulence in DG schemes" *Journal of Computational Physics*, Vol 491, 112399, 2023

E Ferrer, "An interior penalty stabilised incompressible DG–Fourier solver for implicit Large Eddy Simulations", *Journal of Computational Physics*, Vol 348, 2017

New turbulent models for discontinuous Galerkin

Taylor Green Vortex Re=1600



Summary

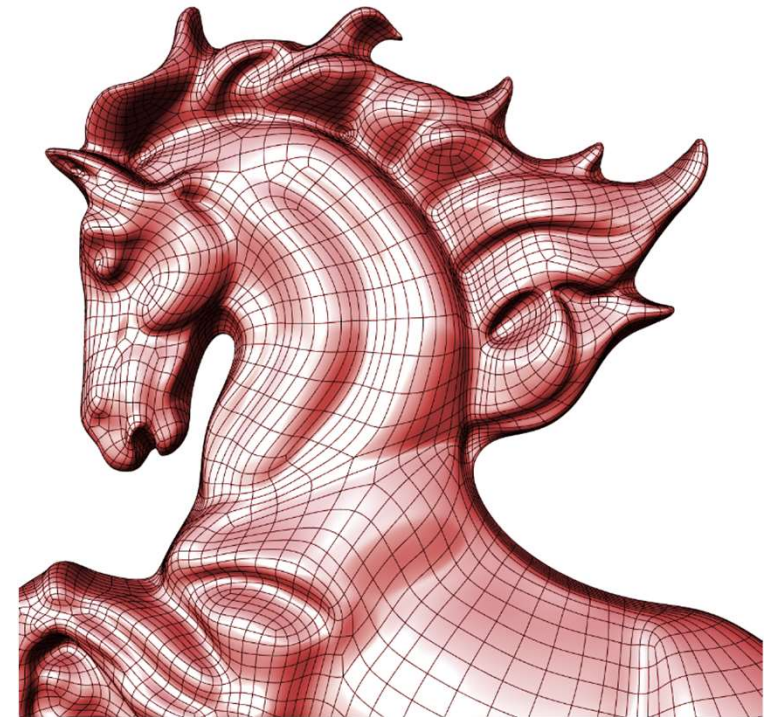
1- Introduction to DG & Horses3d

2- Multiphysics

- Wind turbines
- Turbulence

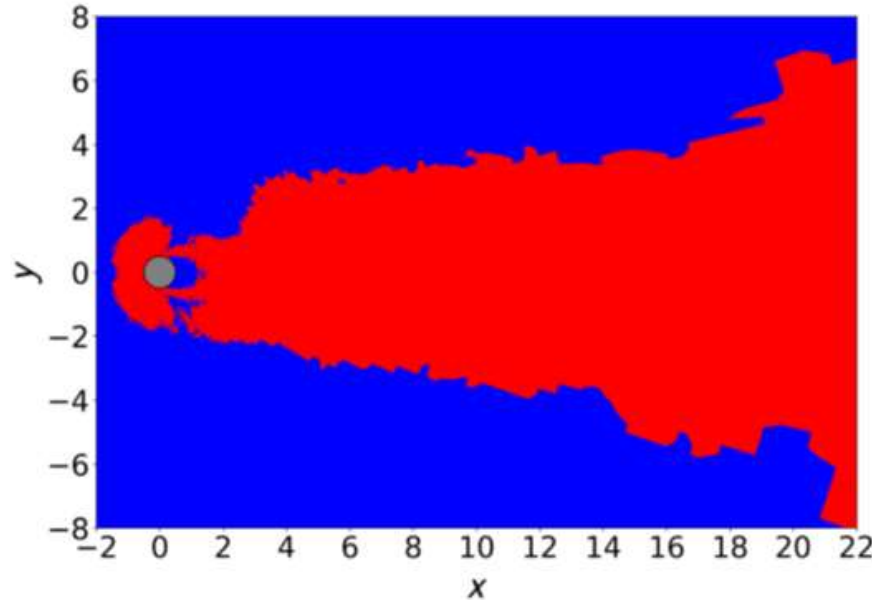
3. Machine Learning + CFD

- Mesh adaption
- NN acceleration
- RL for automation

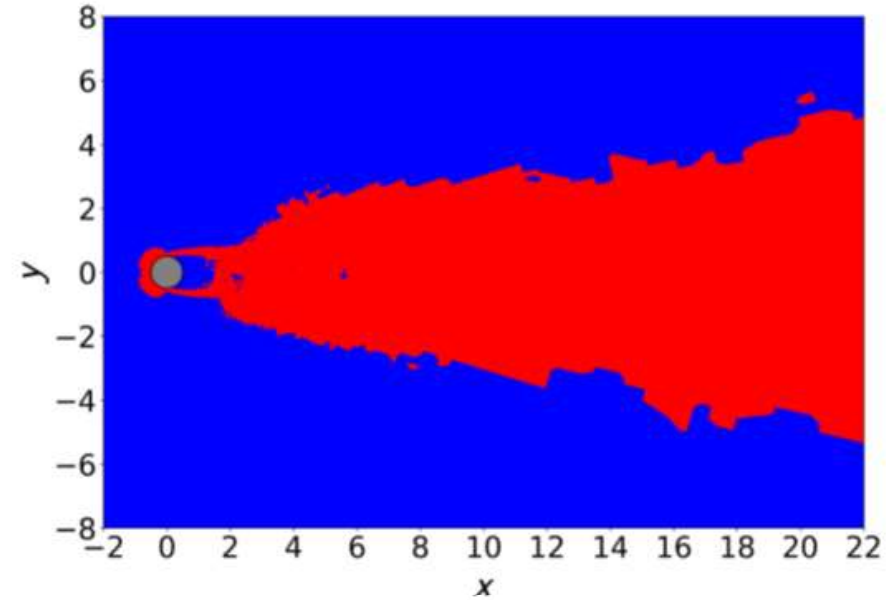


Machine Learning to detect flow regions

$F=1.25$

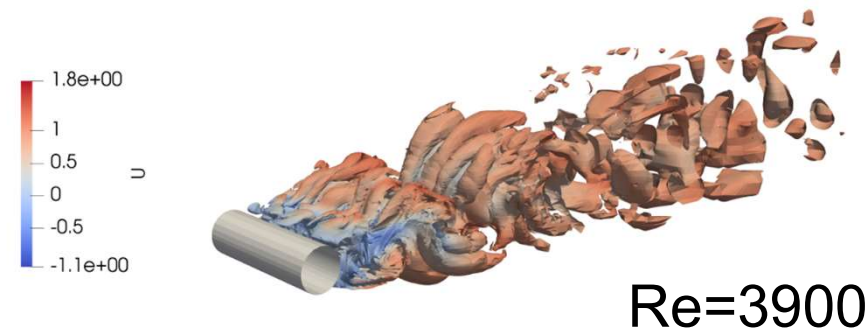


$F=1.5$



Feature based sensors
Eddy viscosity sensor

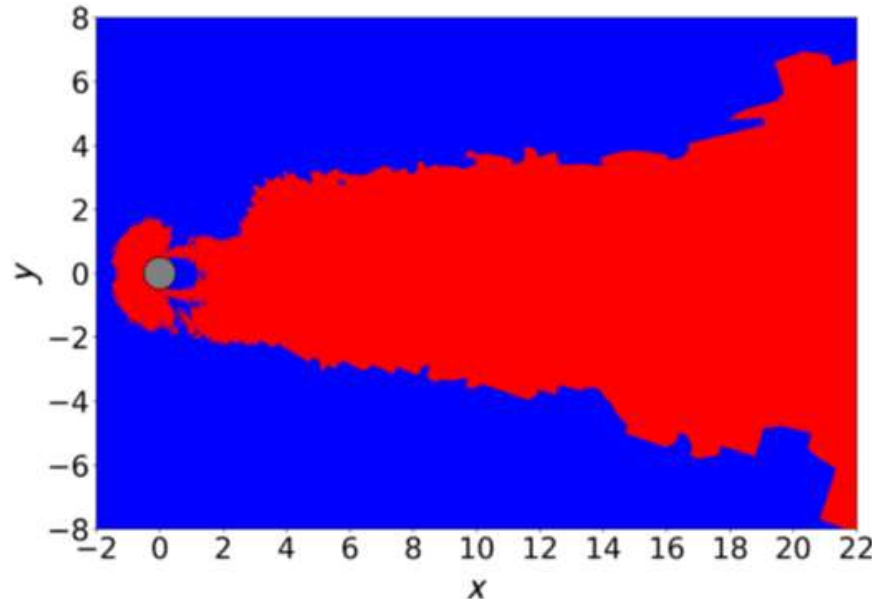
$$F_{\mu_t} = \frac{\mu + \mu_t}{\mu}$$



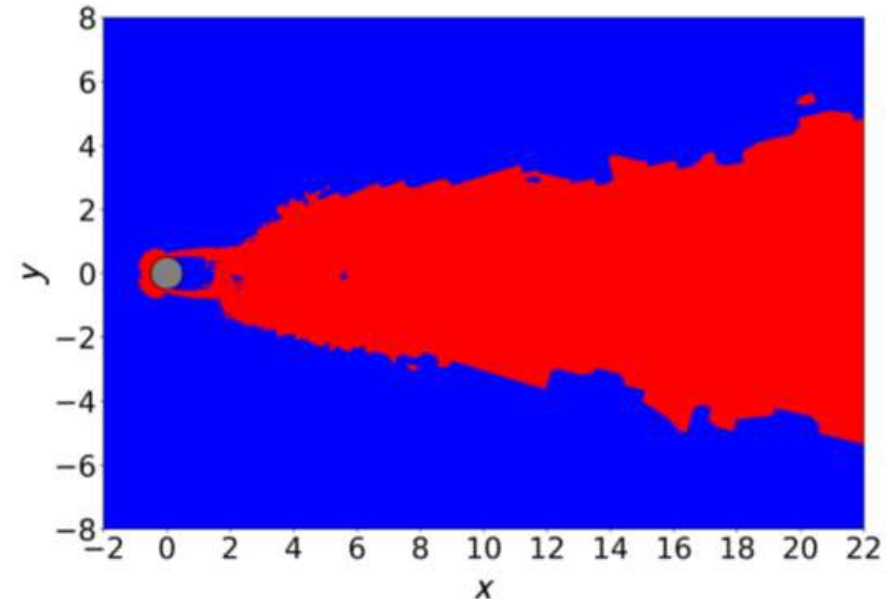
Machine Learning to detect flow regions

Re=3900

$F=1.25$



$F=1.5$



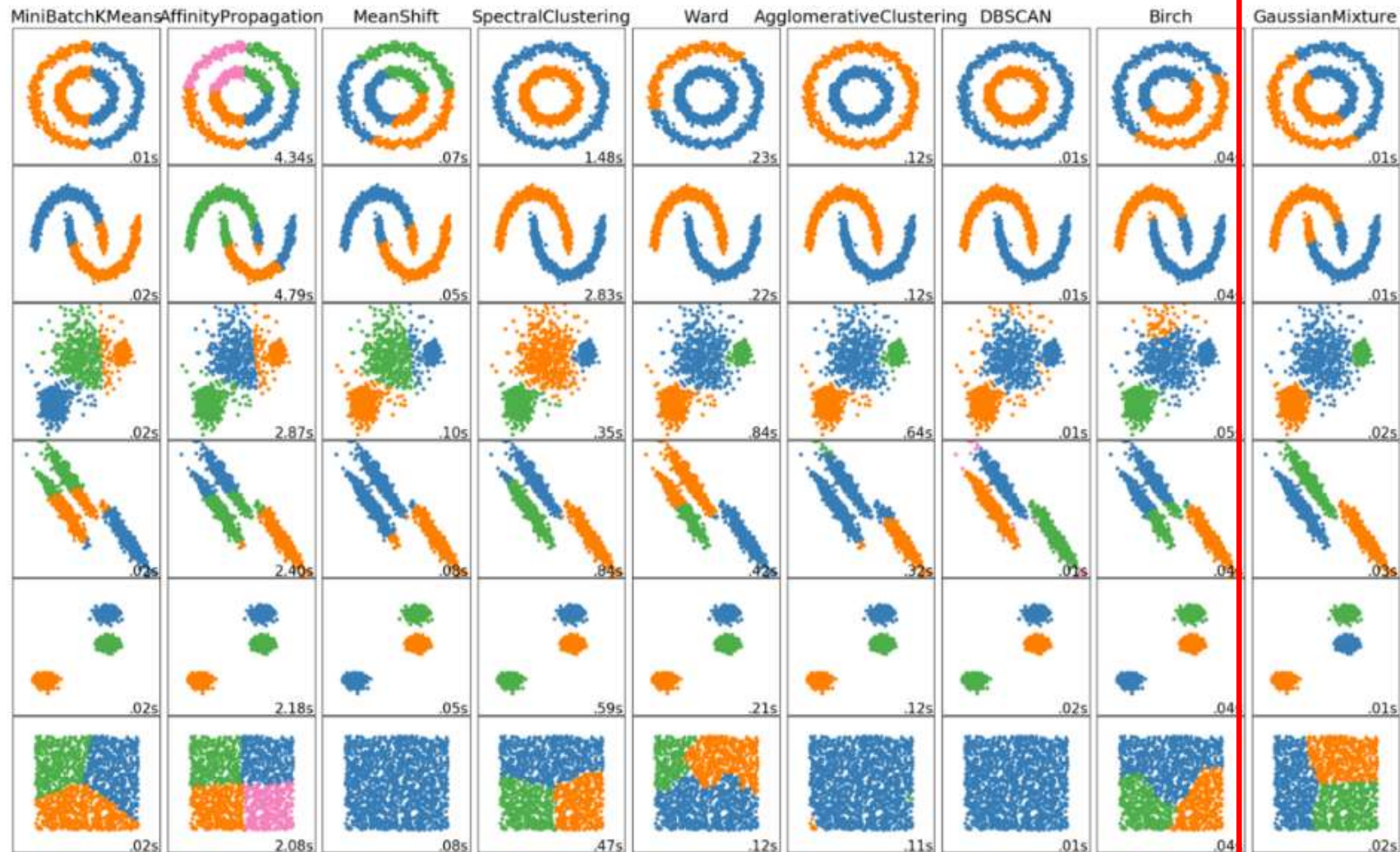
Feature based sensors
Eddy viscosity sensor

$$F_{\mu_t} = \frac{\mu + \mu_t}{\mu}$$

- Very sensitive to **threshold**
- Cannot detect **mixed regions**
(e.g. laminar-turbulent)

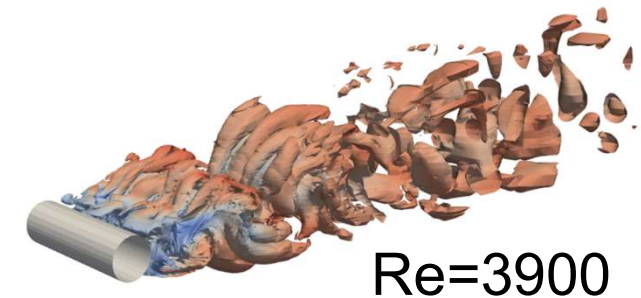
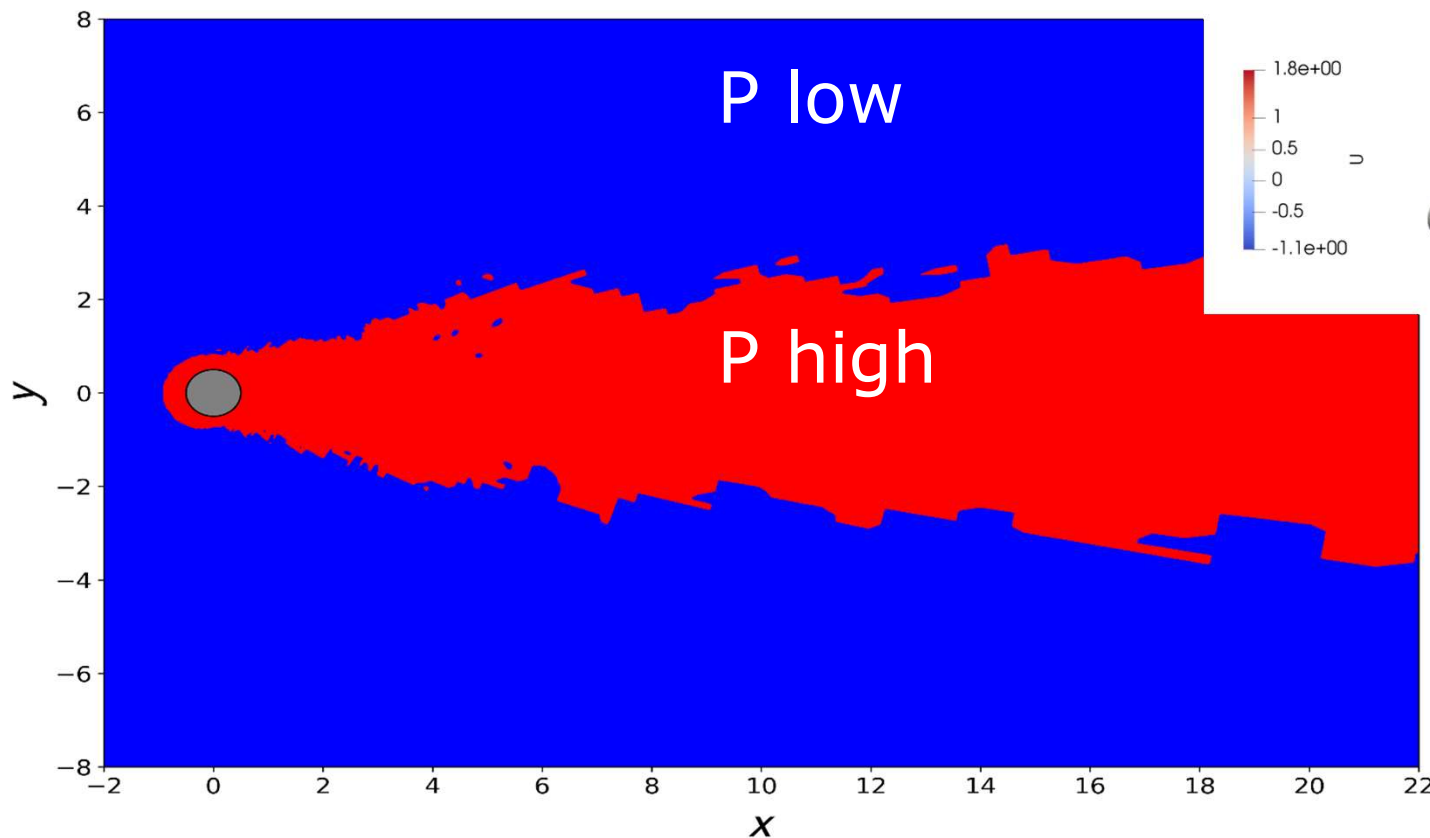
Machine Learning to detect flow regions

Clustering (classify data): Gaussian mixture model



Machine Learning to detect flow regions

Clustering (classify data): Gaussian mixture model



Automate the detection
(no thresholds)

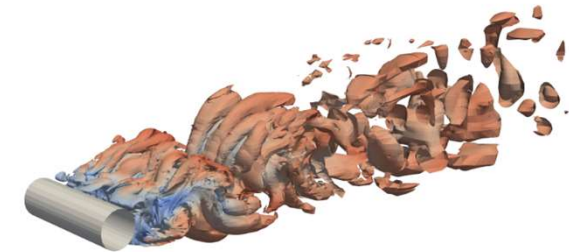
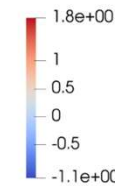
**Use a robust feature space
for a variety of Re**

Invariants of strain and rotational
rate tensors

Machine Learning to detect flow regions

Clustering: Gaussian mixture model

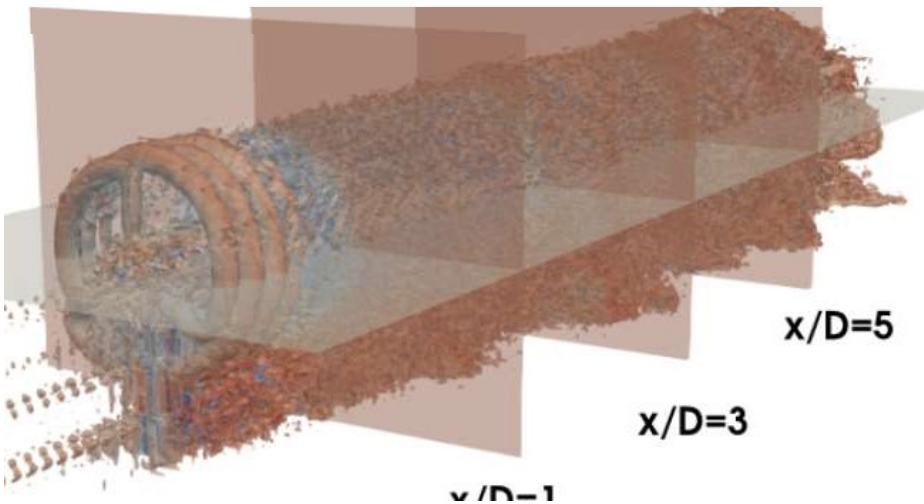
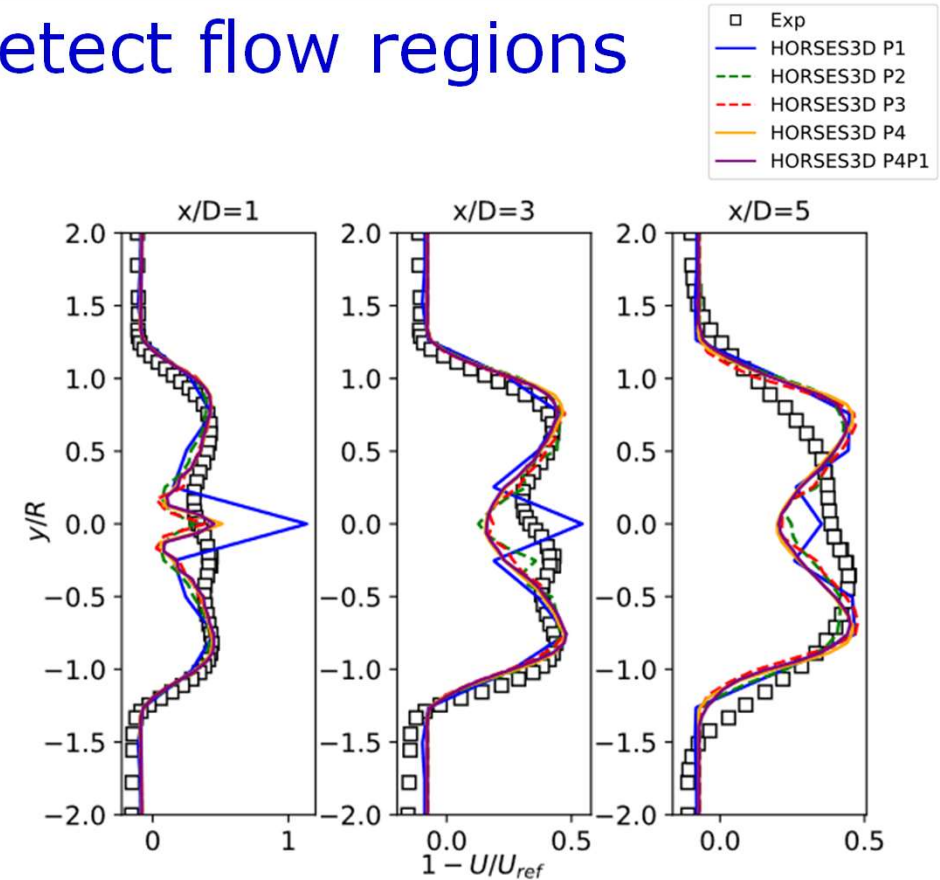
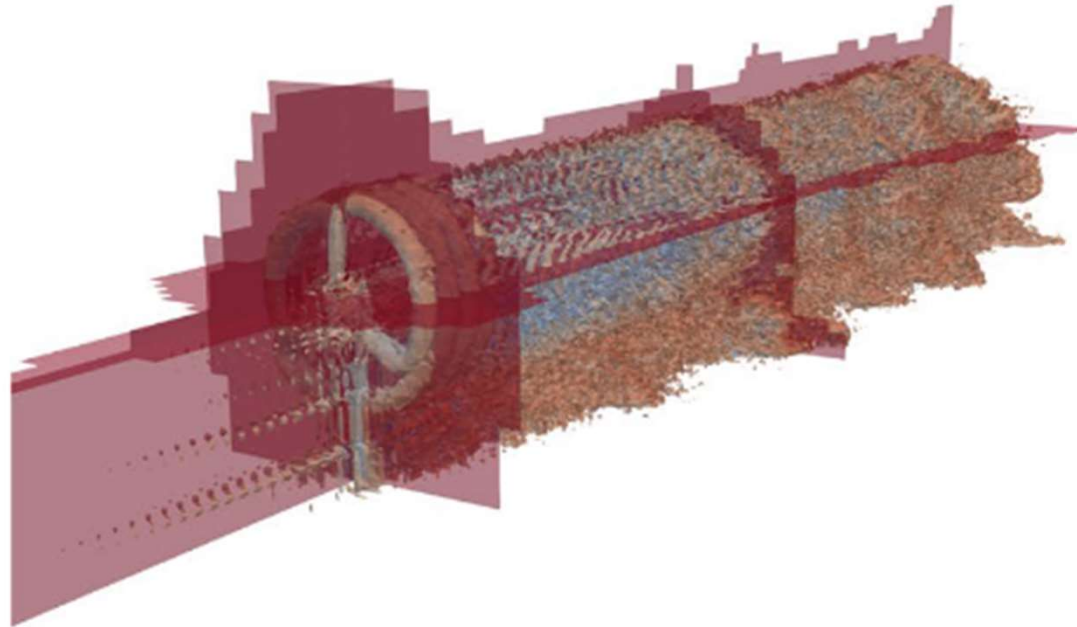
	St	C_d	L_r	$L_z \setminus D$
Uniform P3	0.202	0.7844	1.36	π
Uniform P4	0.203	0.9513	1.64	π
Cluster-Adapt P4-P2	0.204	0.9506	1.63	π
Parnadeau et al. [40]	0.208	-	1.56	π
Snyder and Degrez [45]	0.207	1.09	1.30	π
Kravchenko and Moin [46]	0.210	1.04	1.35	π
Breuer [47]	-	1.07	1.20	π
Franke and Frank [48]	0.209	0.98	1.64	π
(DNS) Ma et al. [41]	0.219	1.59	-	π
Ouvrard et al. [49]	0.223	0.94	1.56	π



Re=3900

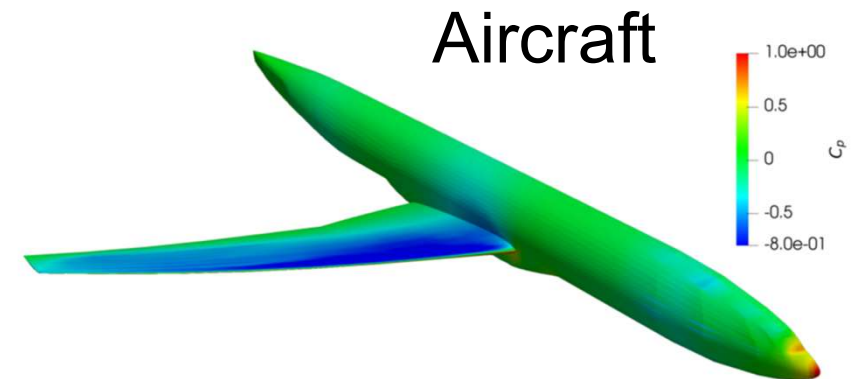
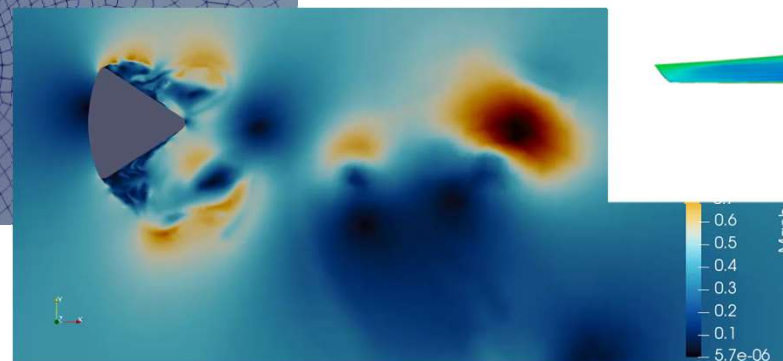
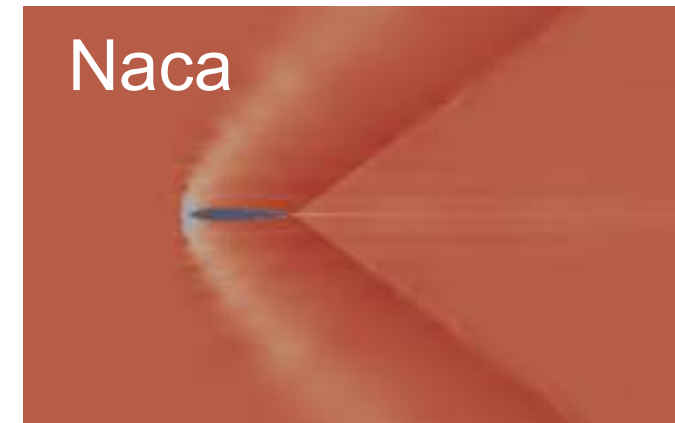
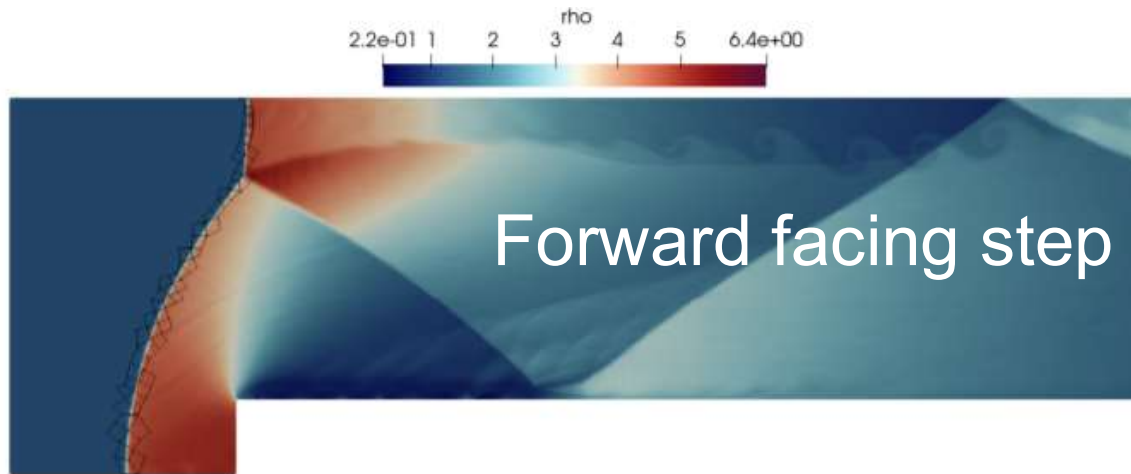
	DoFs	reduction of DoFs	reduction of comp. time
Cluster-Adapt P4-P2	1.55M	41%	33%

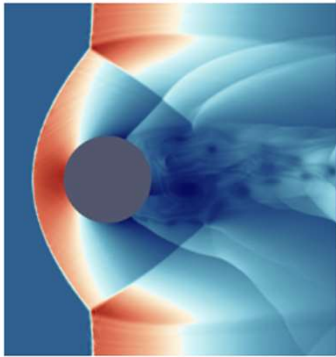
Machine Learning to detect flow regions



	DoFs	reduction of DoFs	reduction of comp. time
Uniform P1	0.59M	93.6%	92.7%
Uniform P2	1.99M	78.2%	86.5%
Uniform P3	4.72M	48.7%	54.1%
Uniform P4	9.22M	-	-
Cluster-Adapt P4-P1	3.58M	61%	43%

Supersonic & Shock capturing





(a)

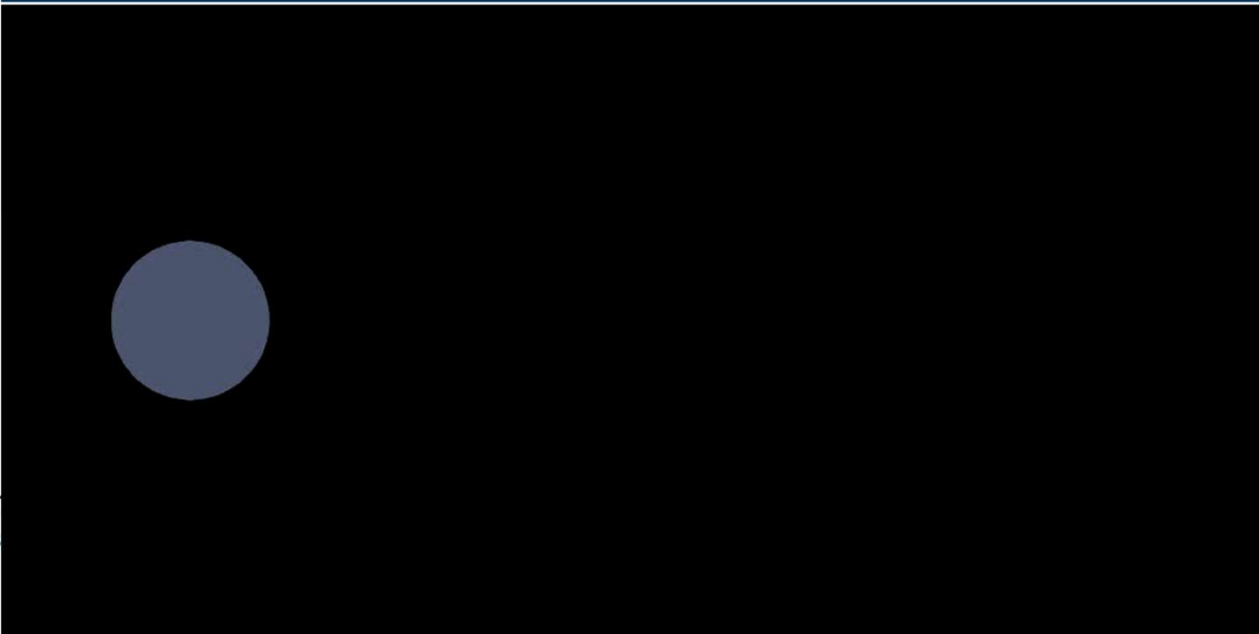
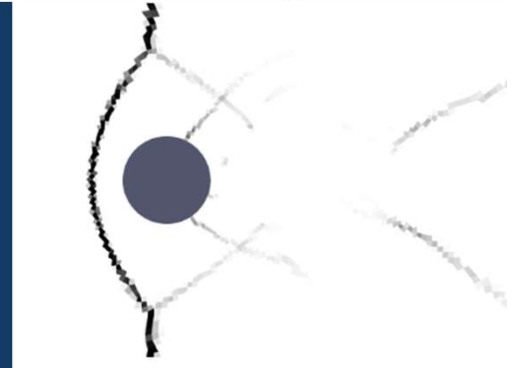


Figure 7
six clus



(d)



ity field, b) sensor with

Summary

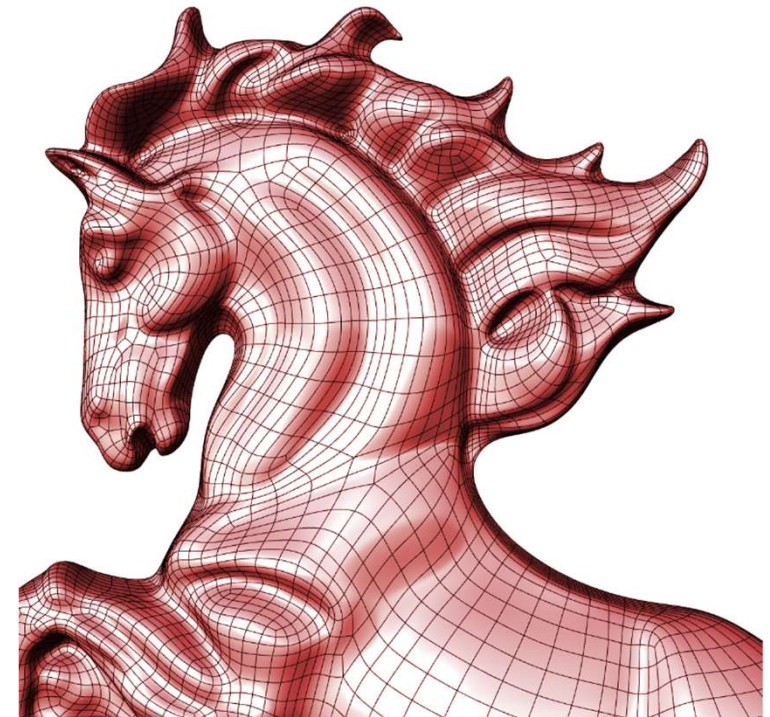
1- Introduction to DG & Horses3d

2- Multiphysics

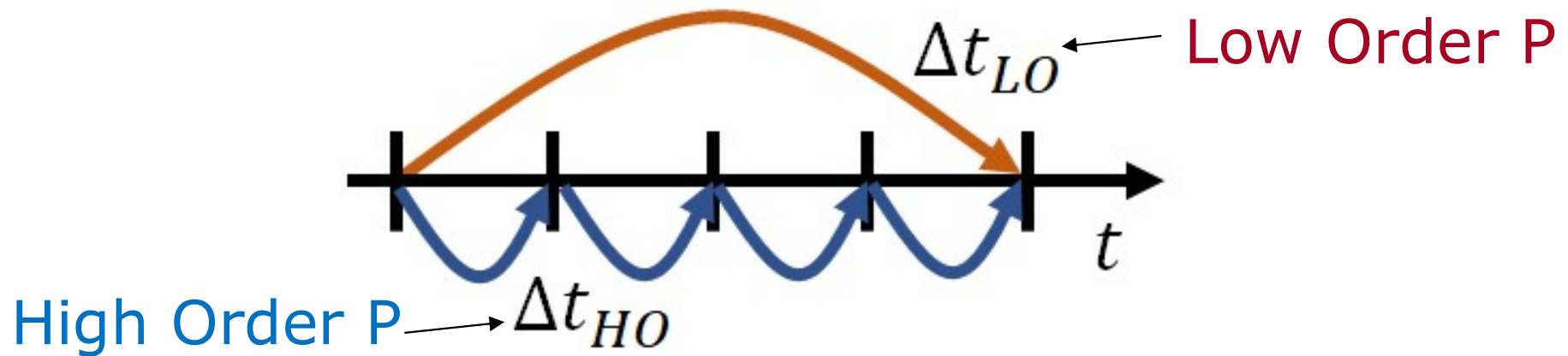
- Wind turbines
- Turbulence

3. Machine Learning + CFD

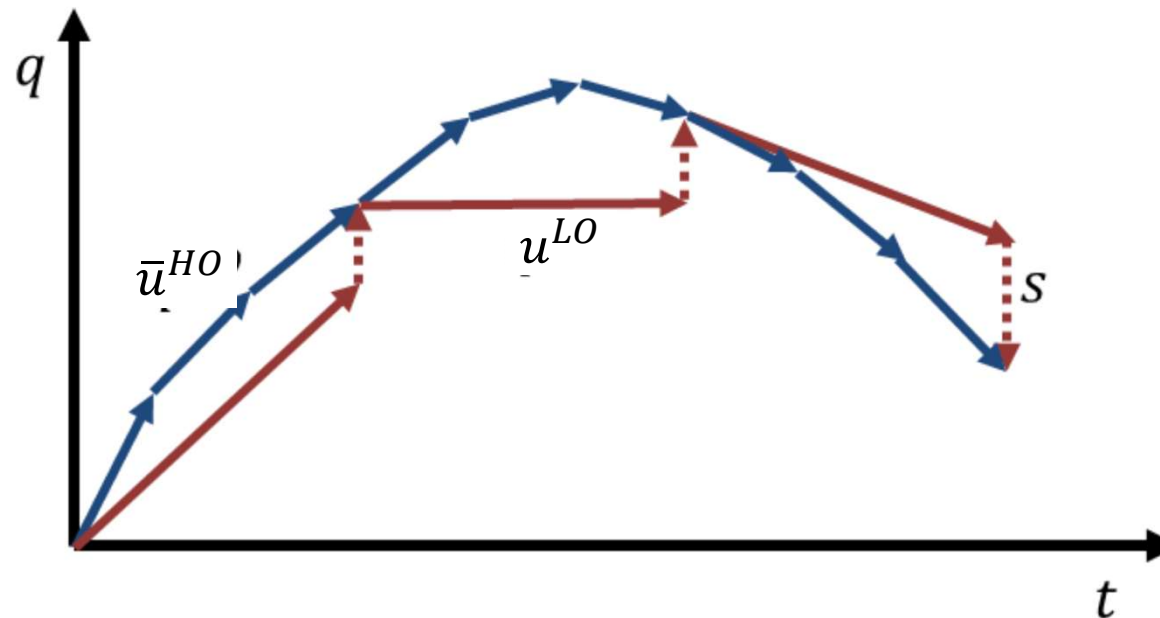
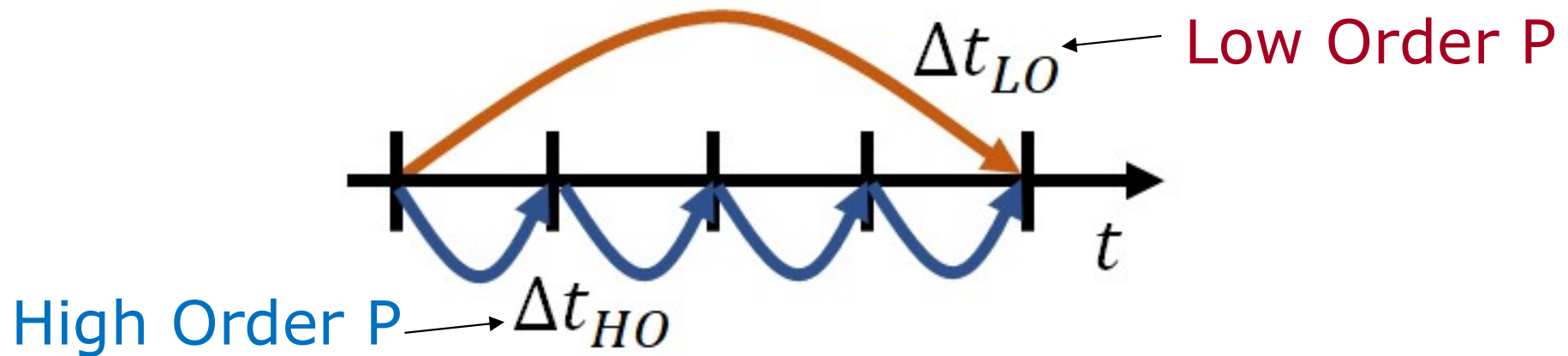
- Mesh adaption
- NN acceleration
- RL for automation



Machine Learning to accelerate CFD



Machine Learning to accelerate CFD



Machine Learning to accelerate CFD

LO evolution:

$$u_{n+1}^{LO} = u_n^{LO} + \Delta t_n q^{LO}(u_n^{LO}; t_n)$$

Filtered HO:

$$\bar{u}_{n+1}^{HO} = \bar{u}_n^{HO} + \Delta t_n \bar{q}^{HO}(u_n^{HO}; t_n)$$

Machine Learning to accelerate CFD

LO evolution:

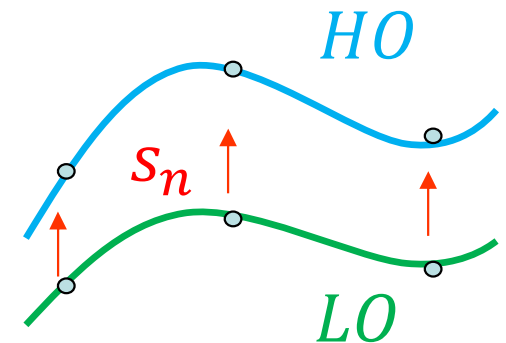
$$u_{n+1}^{LO} = u_n^{LO} + \Delta t_n q^{LO}(u_n^{LO}; t_n)$$

Filtered HO:

$$\bar{u}_{n+1}^{HO} = \bar{u}_n^{HO} + \Delta t_n \bar{q}^{HO}(u_n^{HO}; t_n)$$

LO-NN corrected:

$$u_{n+1}^{NN} = u_n^{NN} + \Delta t_n [q^{LO}(u_n^{NN}; t_n) + s_n]$$



Machine Learning to accelerate CFD

LO evolution:

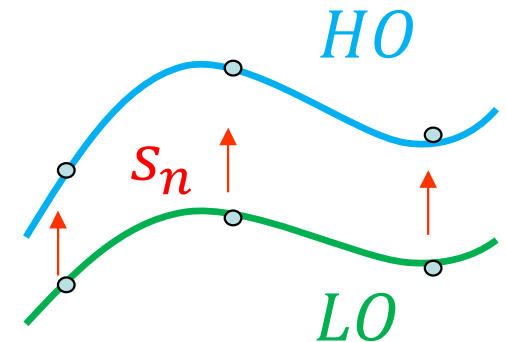
$$u_{n+1}^{LO} = u_n^{LO} + \Delta t_n q^{LO}(u_n^{LO}; t_n)$$

Filtered HO:

$$\bar{u}_{n+1}^{HO} = \bar{u}_n^{HO} + \Delta t_n \bar{q}^{HO}(u_n^{HO}; t_n)$$

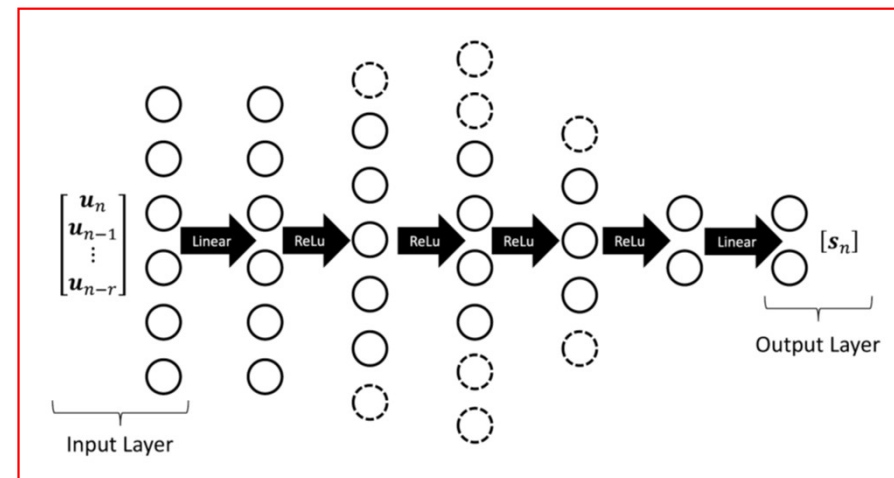
LO-NN corrected:

$$u_{n+1}^{NN} = u_n^{NN} + \Delta t_n [q^{LO}(u_n^{NN}; t_n) + s_n]$$



$$s_n = f(u_n^{NN}, u_{n-1}^{NN}, u_{n-r}^{NN}, \bar{u}^{HO})$$

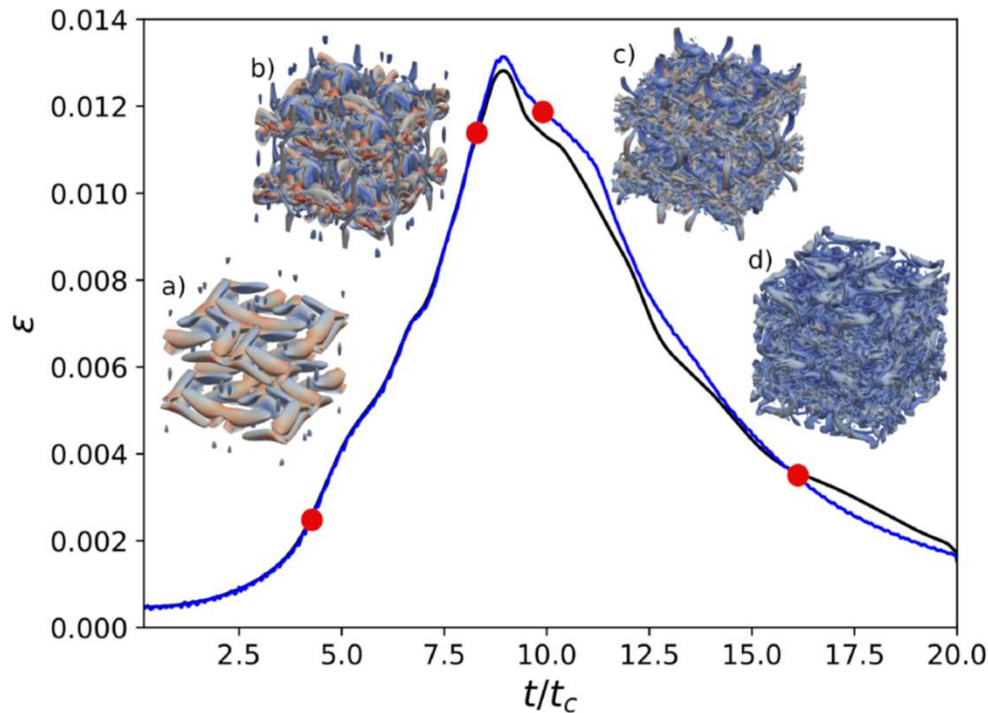
Trained to give HO solution



Machine Learning to accelerate CFD

3D Navier-Stokes - LES

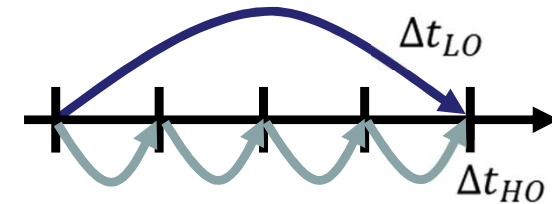
Taylor-Green – Reynolds 1600



Machine Learning to accelerate CFD

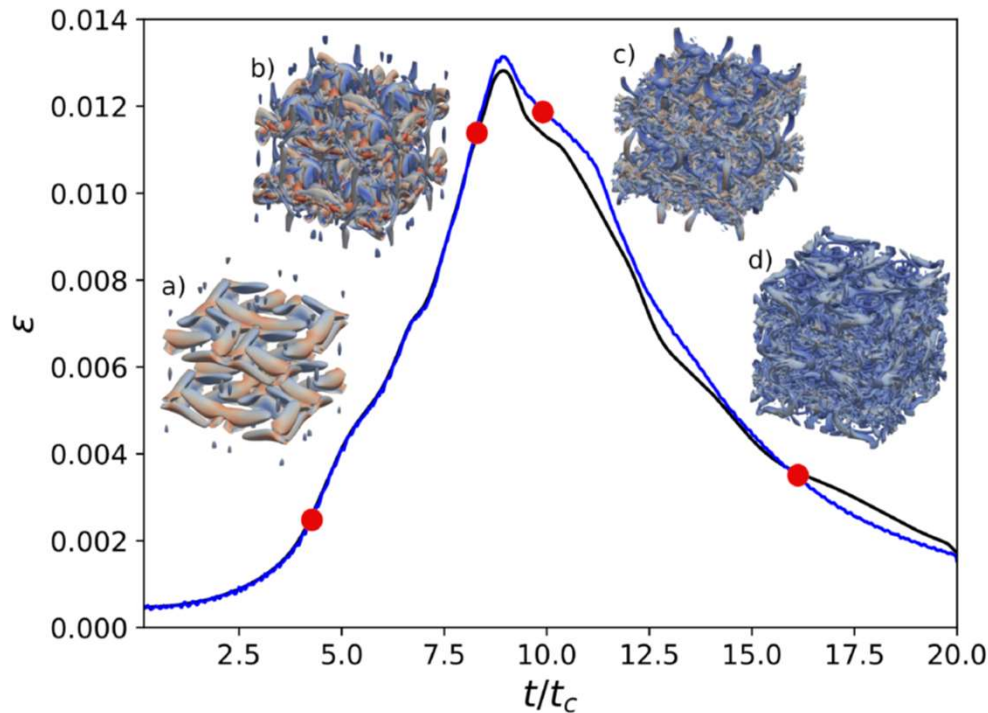
3D Navier-Stokes - LES

Taylor-Green – Reynolds 1600



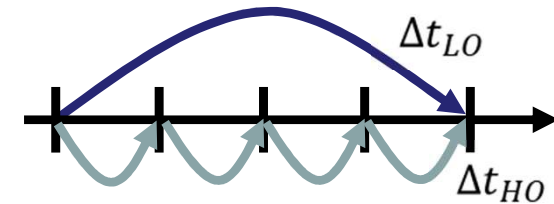
$$P8 \rightarrow P3$$

$$\Delta t_{LO} / \Delta t_{HO} = 3$$



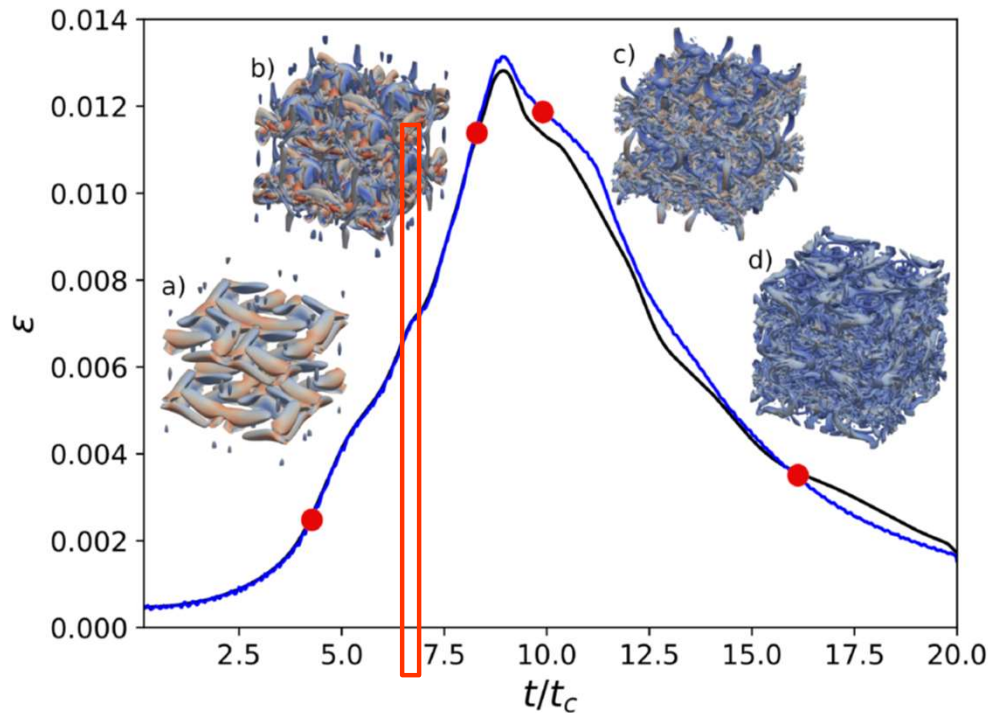
Machine Learning to accelerate CFD

3D Navier-Stokes - LES Taylor-Green – Reynolds 1600



$$P8 \rightarrow P3$$

$$\Delta t_{LO} / \Delta t_{HO} = 3$$

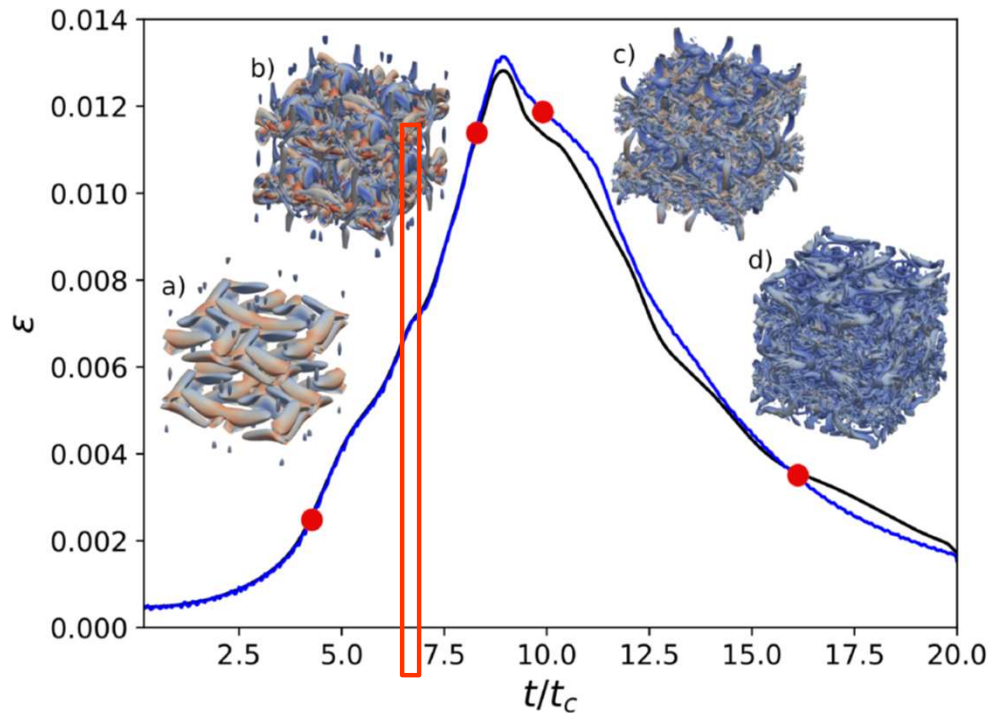
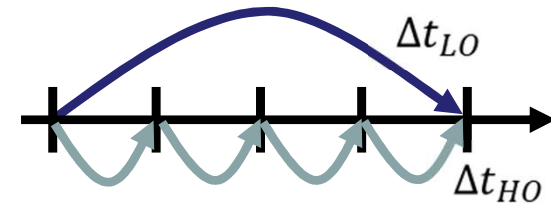


training

Machine Learning to accelerate CFD

3D Navier-Stokes - LES

Taylor-Green – Reynolds 1600

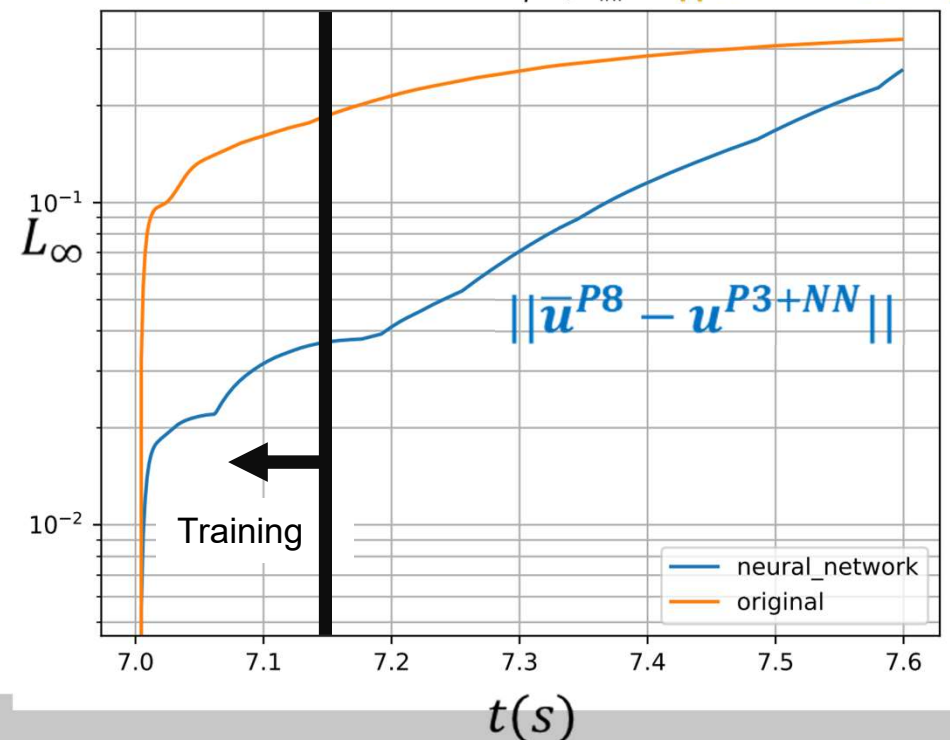


12 times faster

$$P8 \rightarrow P3$$

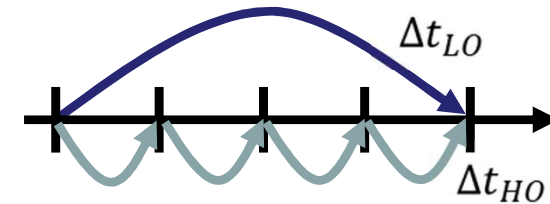
$$\Delta t_{LO} / \Delta t_{HO} = 3$$

error in ρw , L_{inf} $\|\bar{u}^{P8} - u^{P3}\|$



Machine Learning to accelerate CFD

3D Navier-Stokes - LES
Taylor-Green – Reynolds 1600



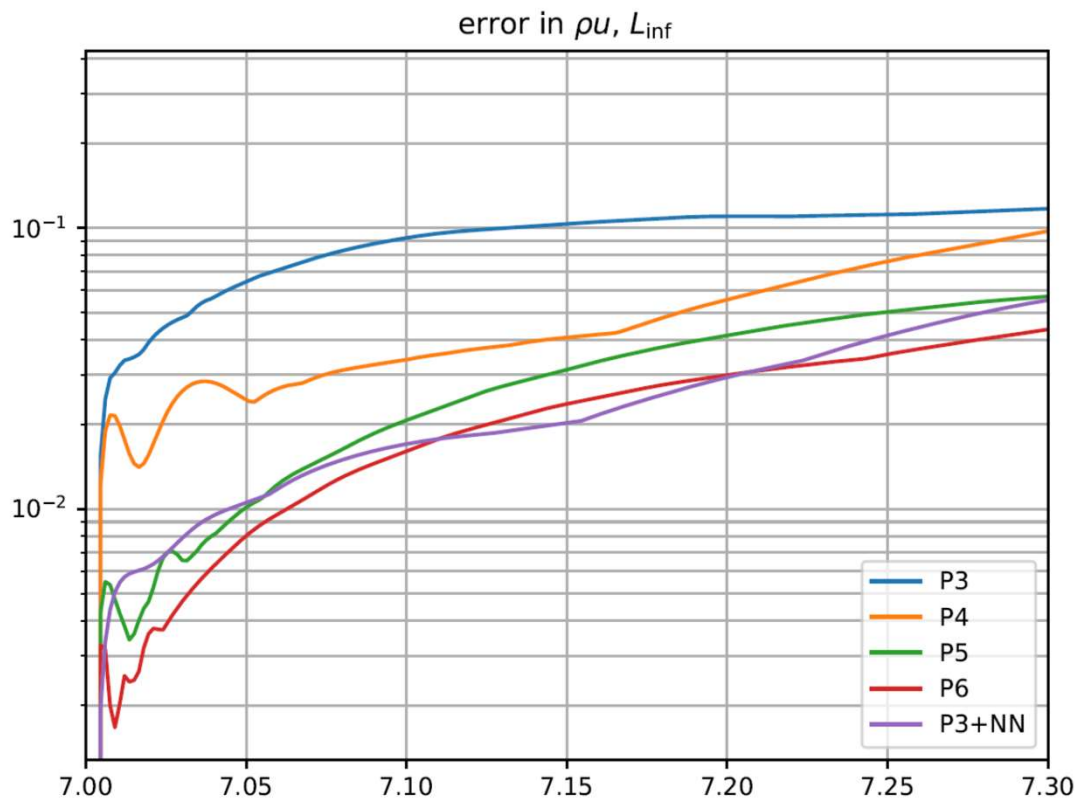
$$P8 \rightarrow P3$$

$$\Delta t_{LO} / \Delta t_{HO} = 3$$

What is the real accuracy?

Probably P=6

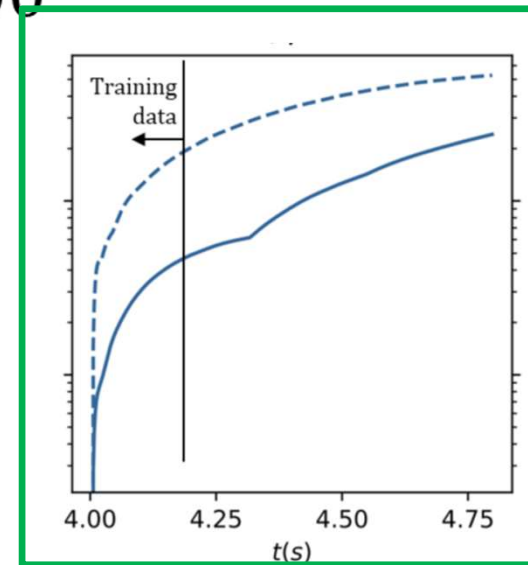
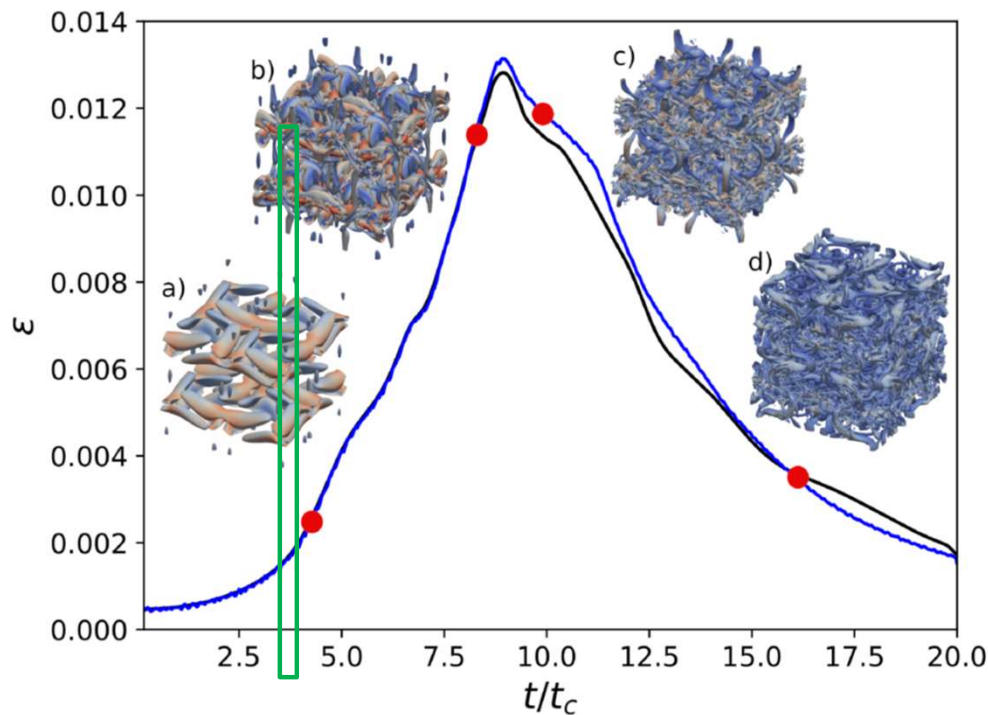
P3+NN is 4-5 times faster
(compared to P6)



Machine Learning to accelerate CFD

3D Navier-Stokes - LES

Taylor-Green – Reynolds 1600

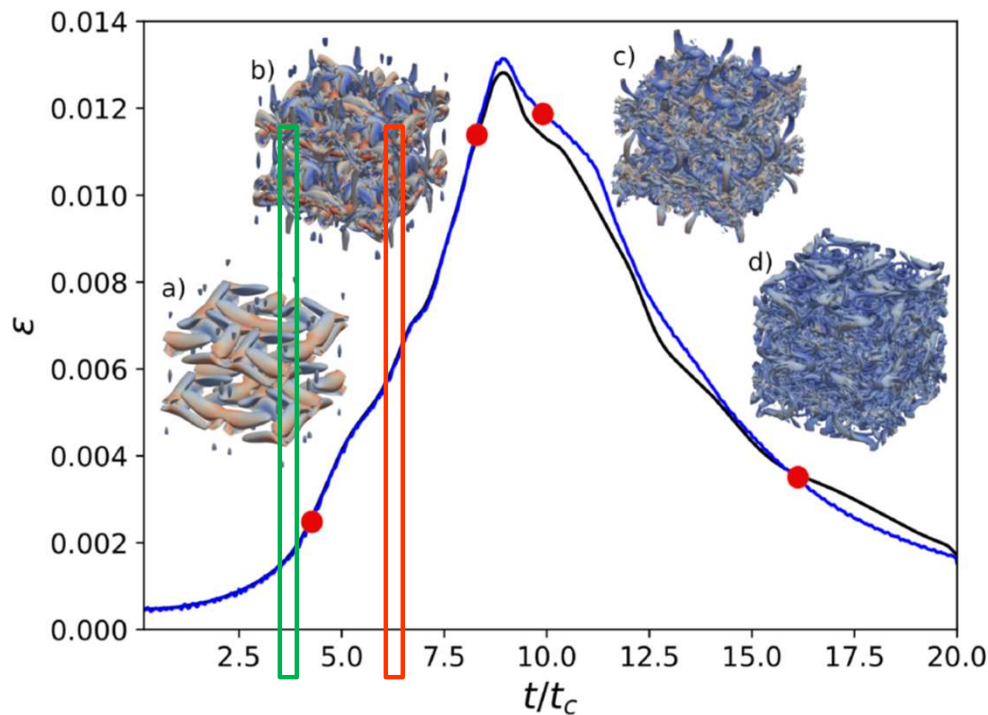


training

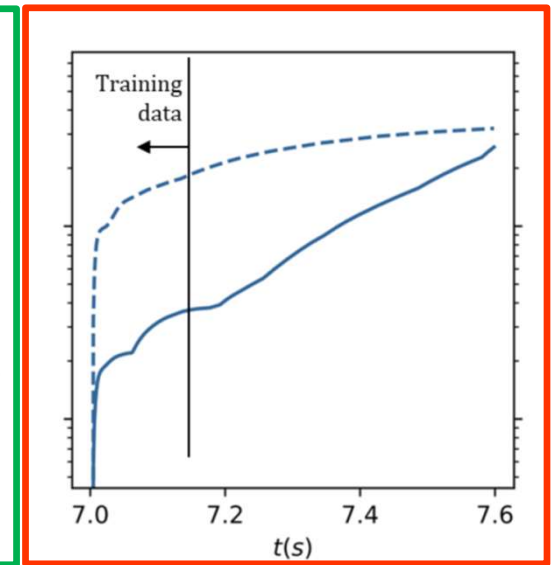
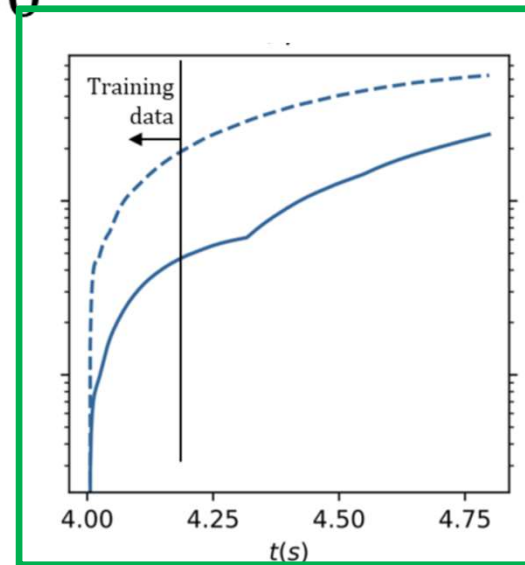
Machine Learning to accelerate CFD

3D Navier-Stokes - LES

Taylor-Green – Reynolds 1600



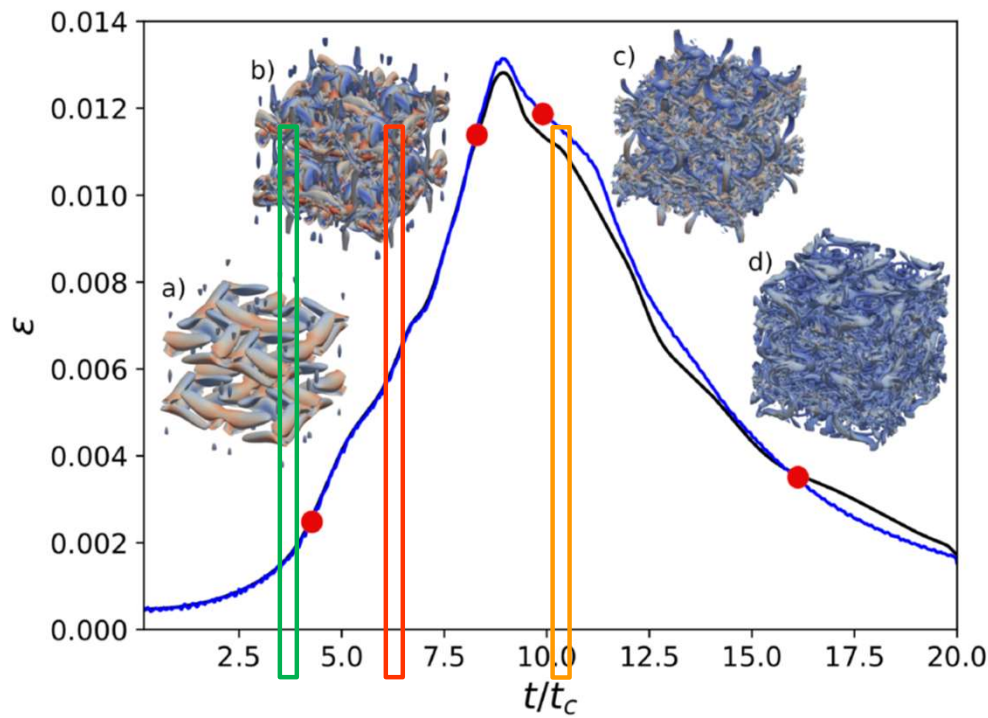
training



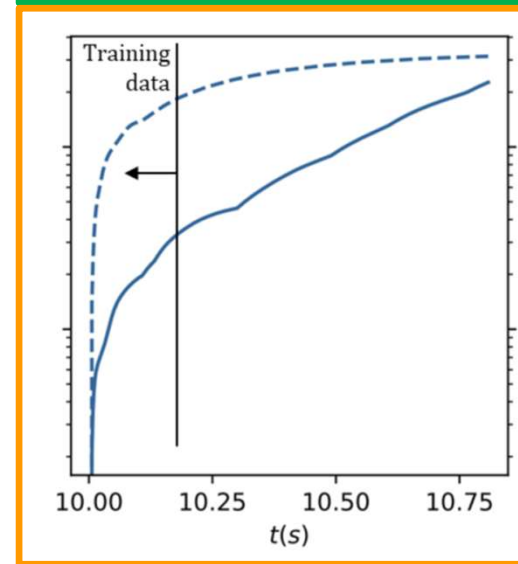
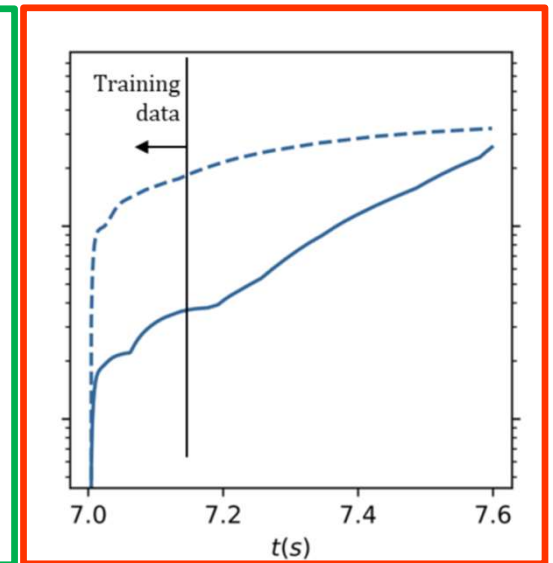
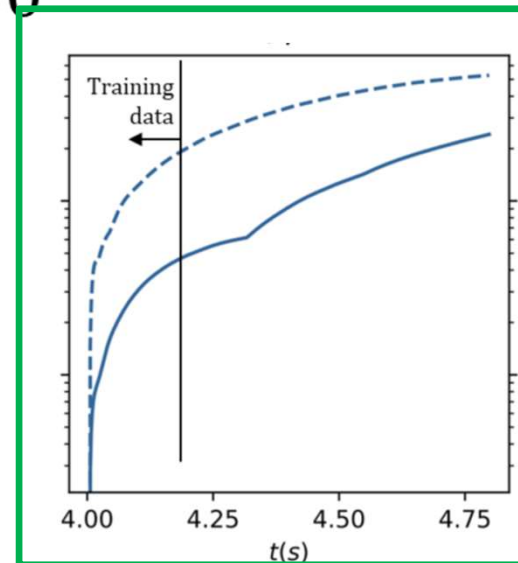
Machine Learning to accelerate CFD

3D Navier-Stokes - LES

Taylor-Green – Reynolds 1600



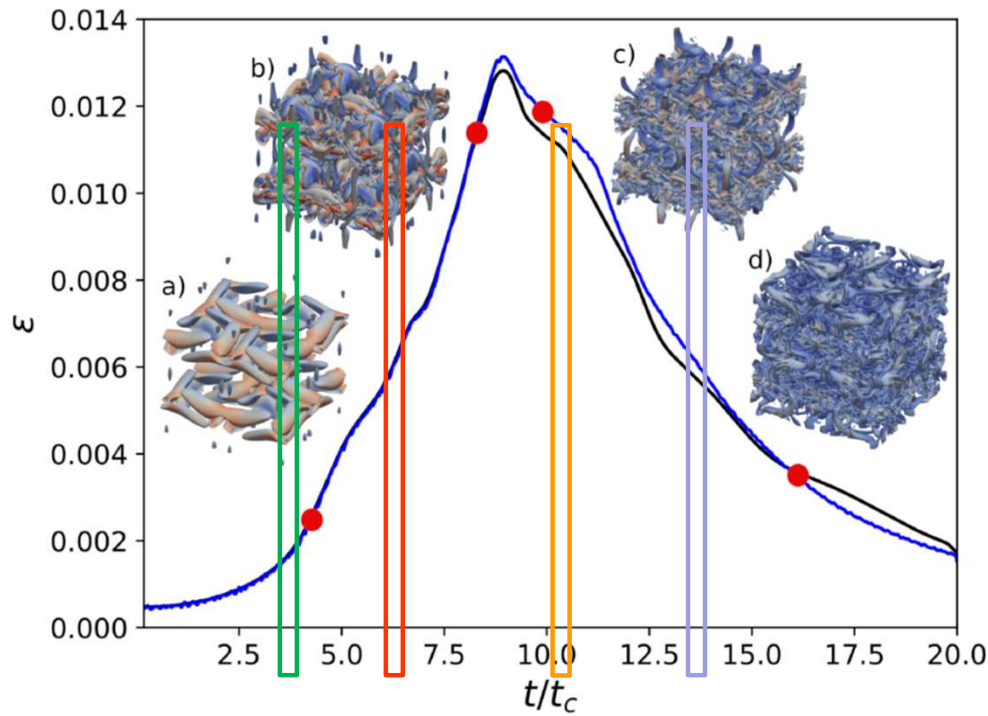
training



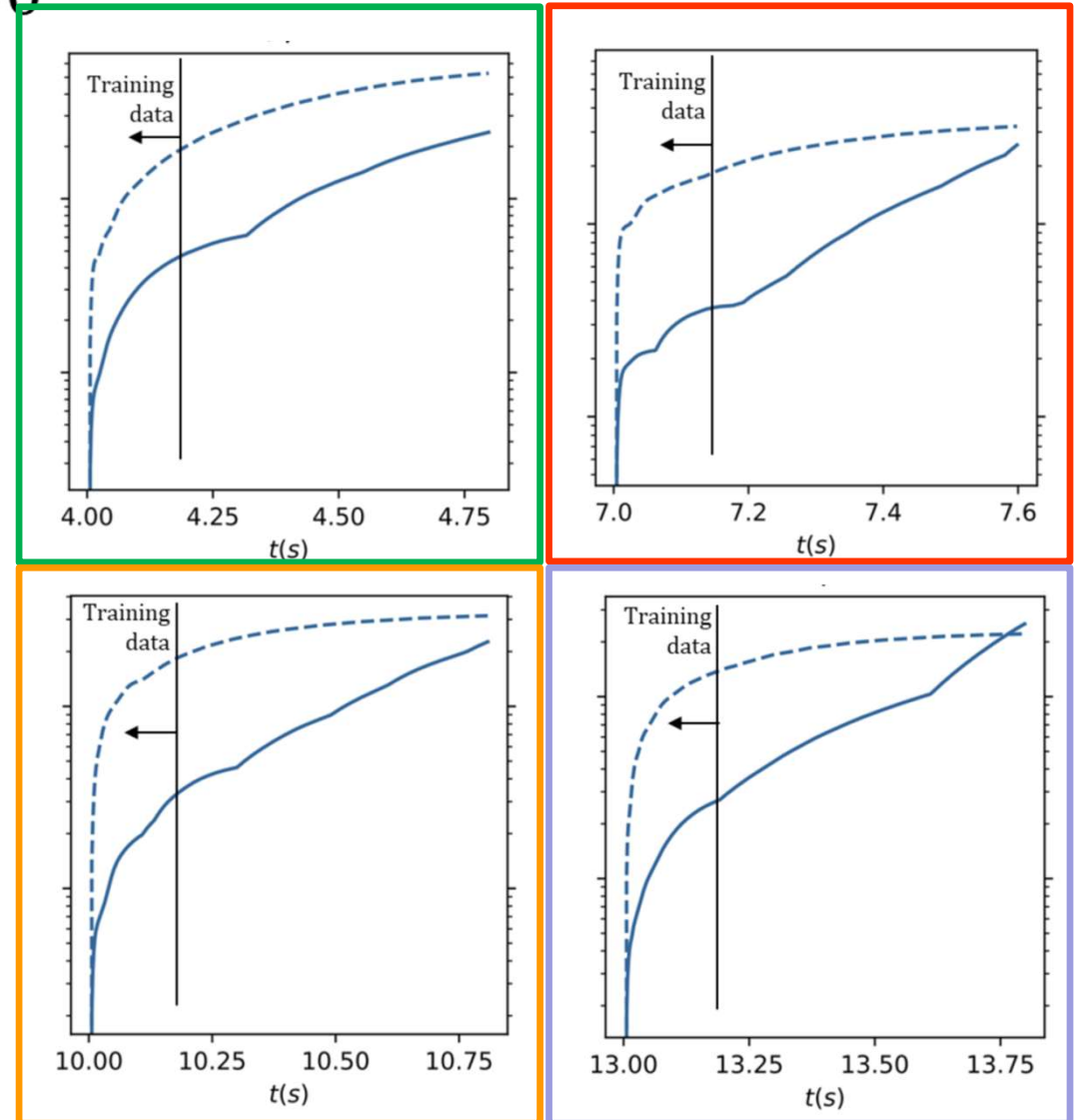
Machine Learning to accelerate CFD

3D Navier-Stokes - LES

Taylor-Green – Reynolds 1600



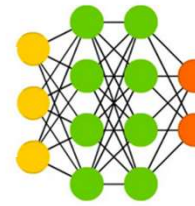
training



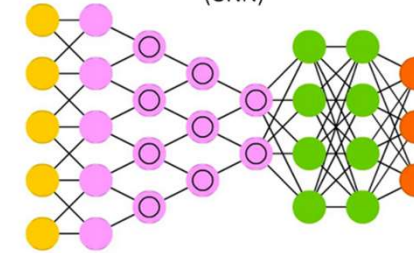
Machine Learning to accelerate CFD

3D Navier-Stokes - LES Taylor-Green – Reynolds 1600

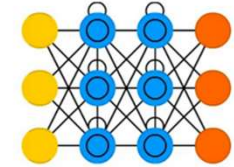
Deep Neural Network (DNN)



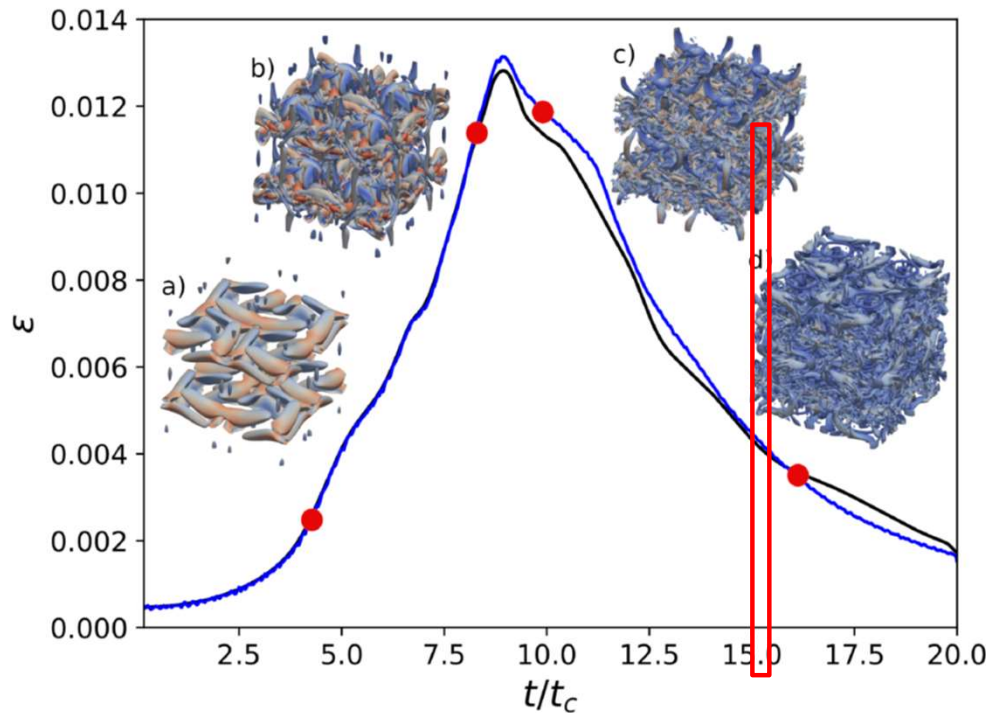
Convolutional Neural Network (CNN)



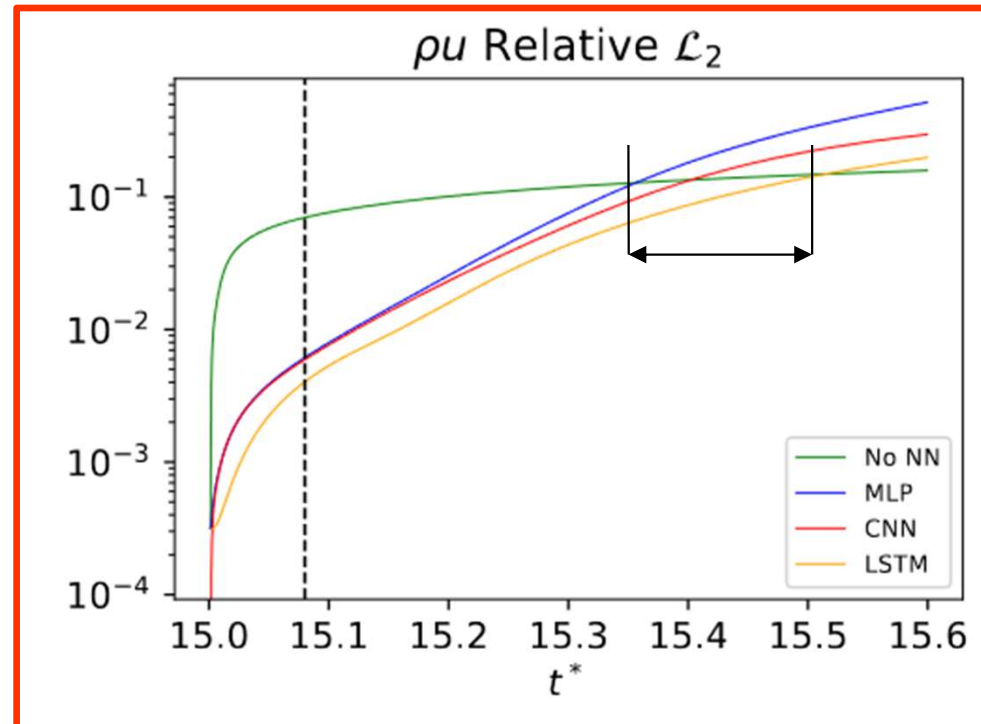
Long/Short Term Memory (LSTM)



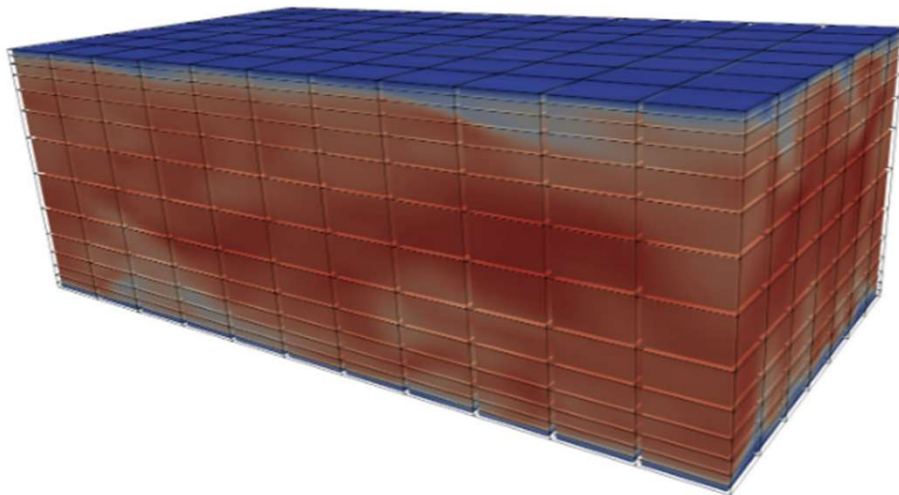
● Input Cell
 ● Hidden Cell
 ● Output Cell
 ● Kernel
 ● Convolution
 ● Memory Cell



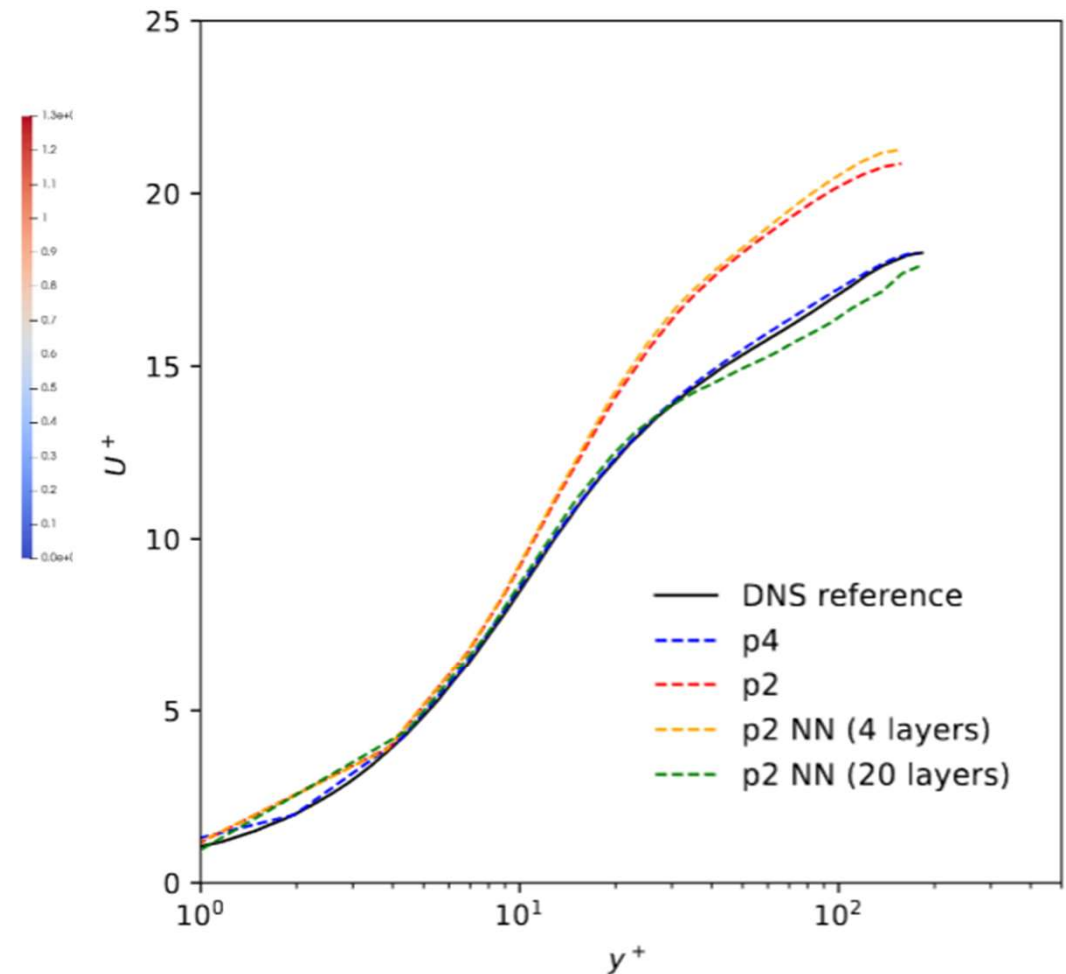
training



Machine Learning to accelerate CFD: *Wall bounded flows*



$Re_T = 182$



Summary

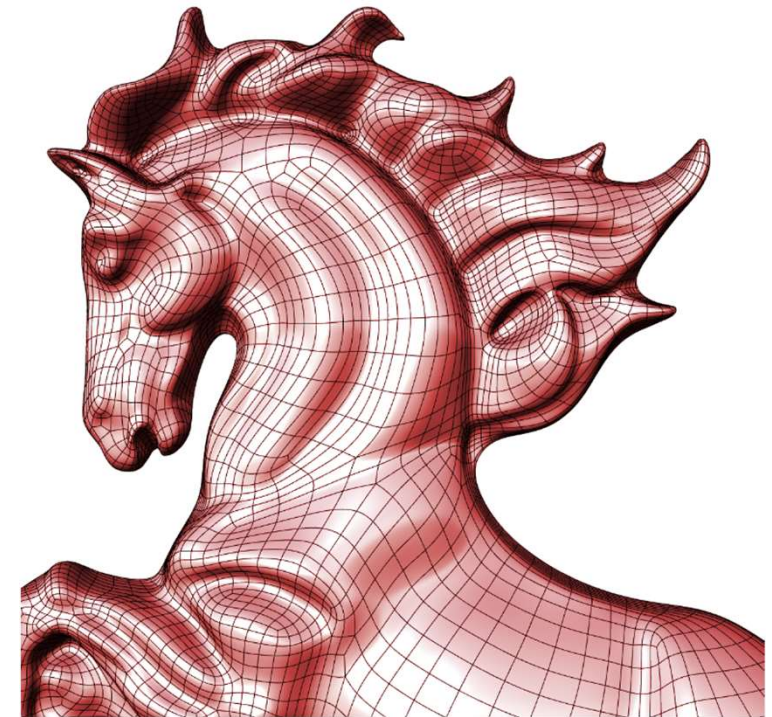
1- Introduction to DG & Horses3d

2- Multiphysics

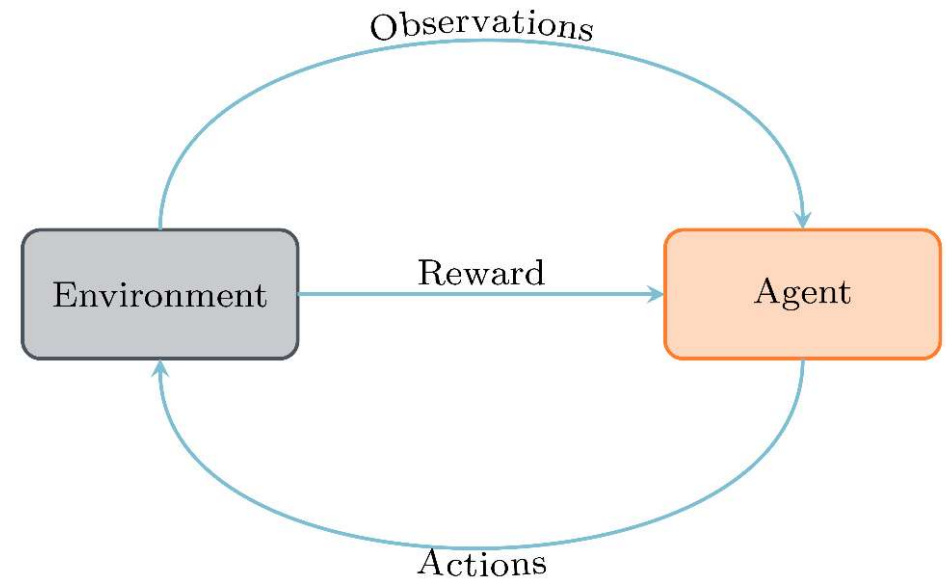
- Wind turbines
- Turbulence

3. Machine Learning + CFD

- Mesh adaption
- NN acceleration
- RL for automation



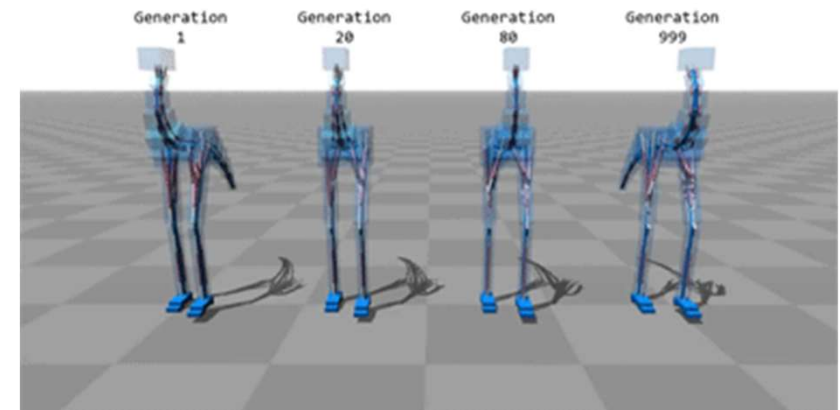
Machine Learning and Reinforcement Learning



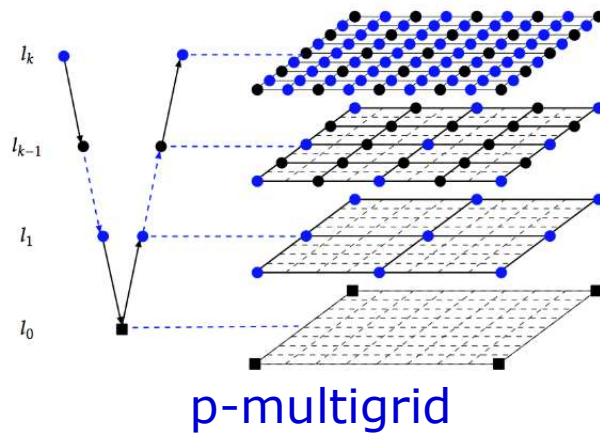
Go game



Chess game



Reinforcement learning for p-multigrid



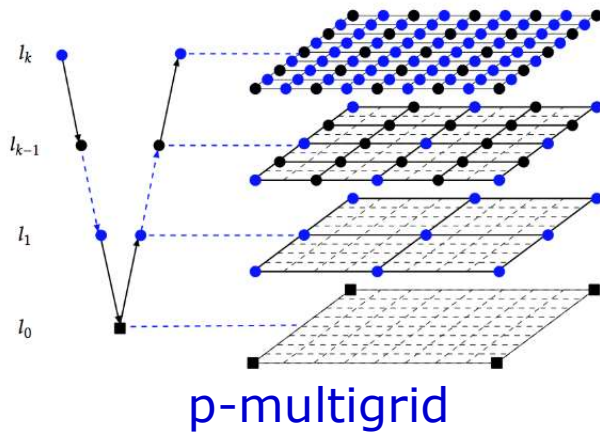
Cases				0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
Advection-diffusion	P3	L0	IC: Sine7		135	126	121	121	123	124	1186	U	U	
			IC: sine				121				1186	U	U	
			IC: exp				121				1186	U	U	
	P5	L0	IC: Sine7				471					U	U	U
			IC: sine				471					U	U	U
			IC: exp				471					U	U	U
	P7	L0	IC: Sine7				1207					1205	U	U
			IC: sine				1207					1205	U	U
			IC: exp				1207					1205	U	U
	P9	L0	IC: Sine7				2466					U	U	U
			IC: sine				2466					U	U	U
			IC: exp				2466					U	U	U
P3	R10	IC: Sine7	229	146	144	142	140	138	136	1188	U	U	U	

$$u_t + au_x - \nu u_{xx} = S$$

Optimal parameters in p-multigrid multigrid?

- Sweeps
- Relaxation between levels

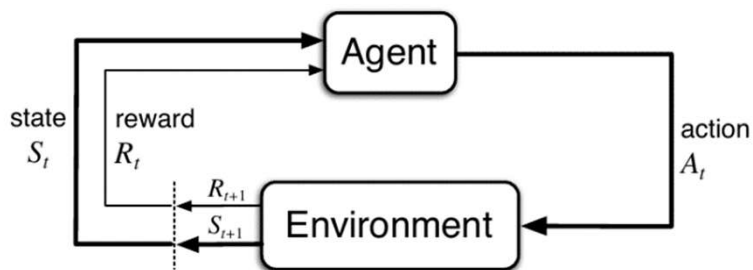
Reinforcement learning for p-multigrid



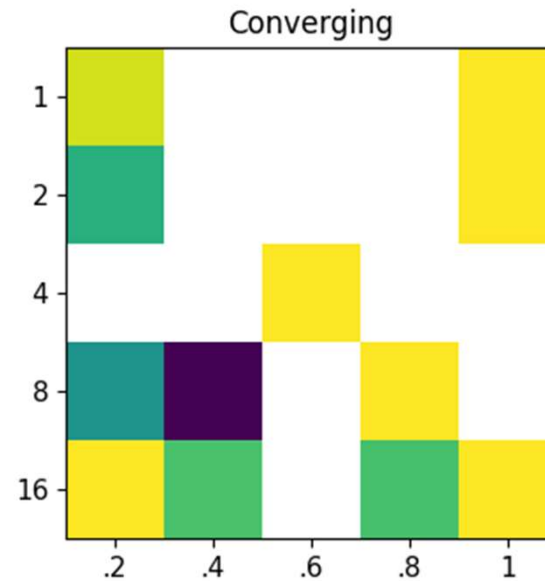
Cases		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
Advection-diffusion	P3	IC: Sine7										
		IC: sine	135	126	121	121	123	124	1186	U	U	
		IC: exp								1186	U	U
	P5	IC: Sine7				471				U	U	U
		IC: sine				471				U	U	U
		IC: exp				471				U	U	U
	P7	IC: Sine7				1207				1205	U	U
		IC: sine				1207				1205	U	U
		IC: exp				1207				1205	U	U
	P9	IC: Sine7				2466				U	U	U
IC: sine					2466				U	U	U	
IC: exp					2466				U	U	U	
P3	R10	IC: Sine7	229	146	144	142	140	138	136	1188	U	U

$$u_t + au_x - \nu u_{xx} = S$$

Reward: $f(\text{Relative drop in residual, time taken})$



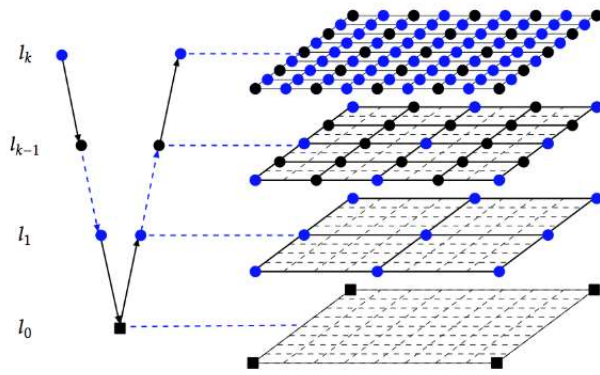
sweeps



Yellow \rightarrow action taken
Blue do not take it

Relax. between levels

Reinforcement learning for p-multigrid



p-multigrid

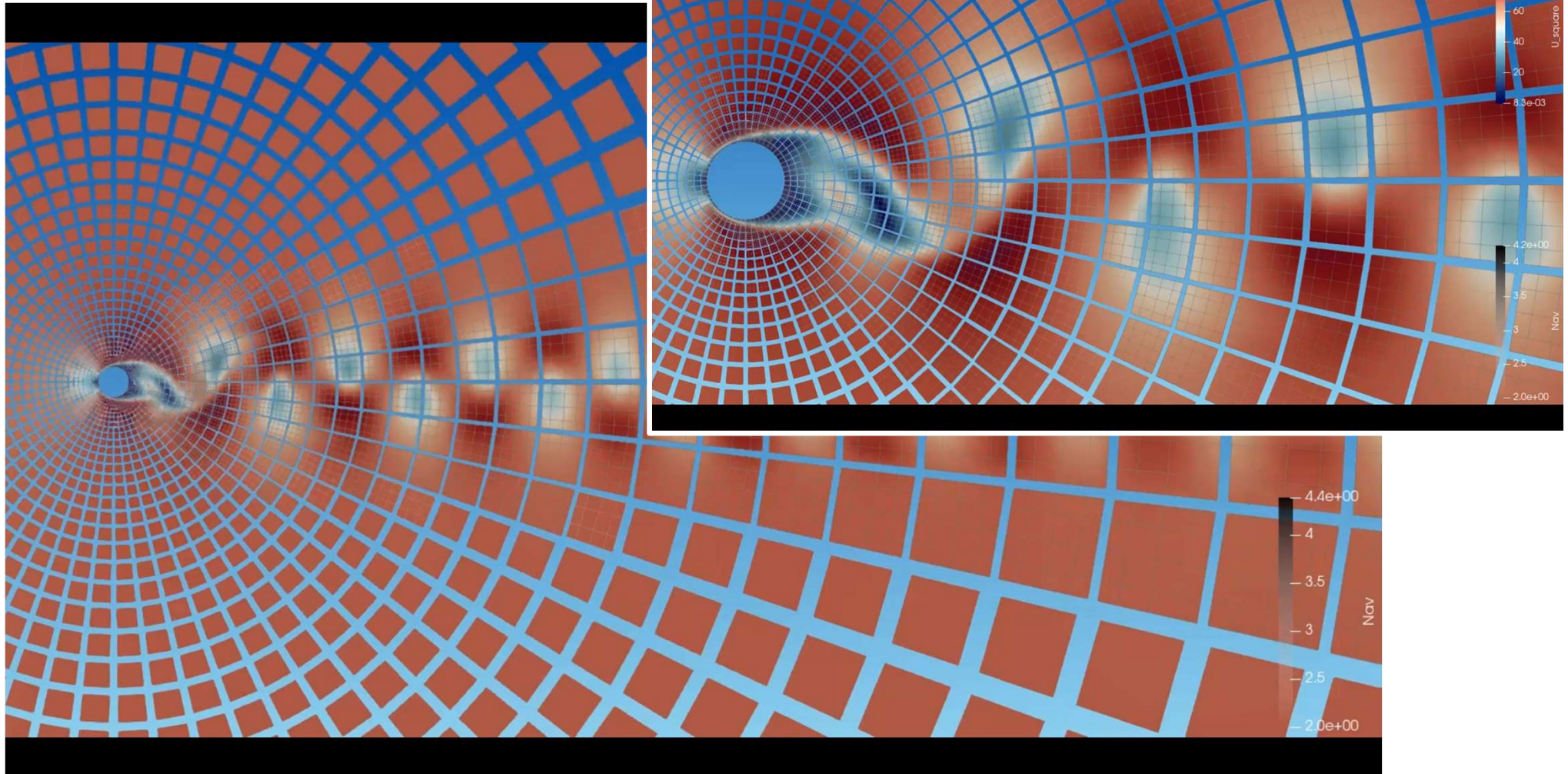
Cases		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
Advection-diffusion	P3	IC: Sine7		135	126	121	121	123	124	1186	U	U
		IC: sine				121				1186	U	U
	P5	IC: exp				121				1186	U	U
		IC: Sine7				471				U	U	U
	P7	IC: sine				471				U	U	U
		IC: exp				471				U	U	U
	P9	IC: Sine7				1207				1205	U	U
		IC: sine				1207				1205	U	U
	P3	IC: exp				1207				1205	U	U
		IC: Sine7				2466				U	U	U
R10	IC: Sine7	229	146	144	142	140	138	136	1188	U	U	

$$u_t + au_x - \nu u_{xx} = S$$

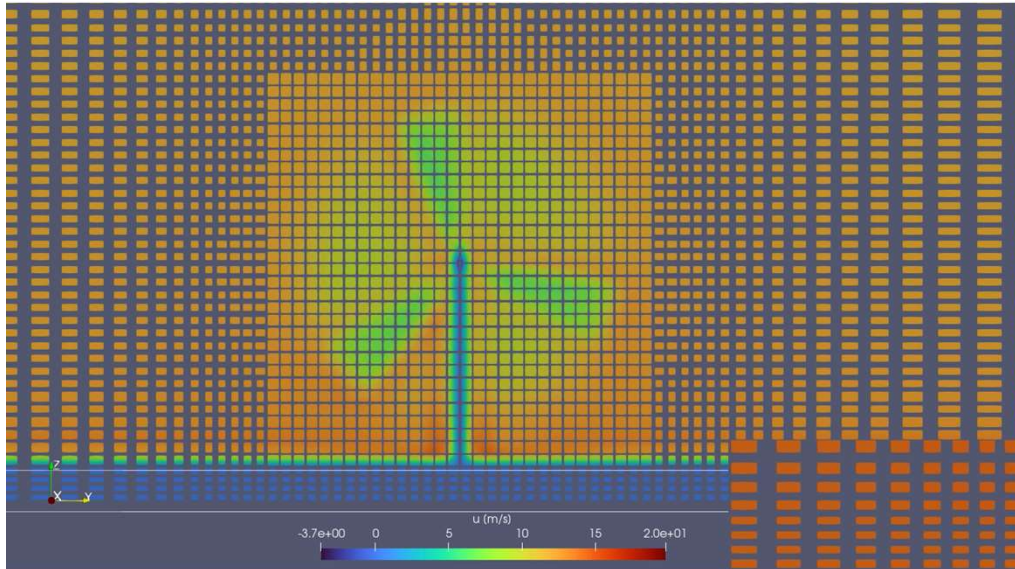
"Arbitrary"		MC		PPO		
runtime	iter	runtime	iter	runtime	iter	res
AD - Order 2						
a = 1., v = 0.01						
69.7168839	197	49.38292694	197	31.02763486	626	9.67E-09
a = 0.5., v = 0.01						
80.01094651	207	51.54315066	207	31.81400156	651	8.33E-09
a = 0.5., v = 0.5						
808.6031666	2178	480.8234568	2178	33.21327591	652	9.31E-09
a = 0.4, v = 0.6						
634.2691302	3166	582.4802358	3166	31.52360582	654	9.29E-09
a = 0.2, v = 0.8						
1476.47674	8063	1278.344407	7163	31.47797155	648	9.98E-09

Reinforcement learning for p-adaptation

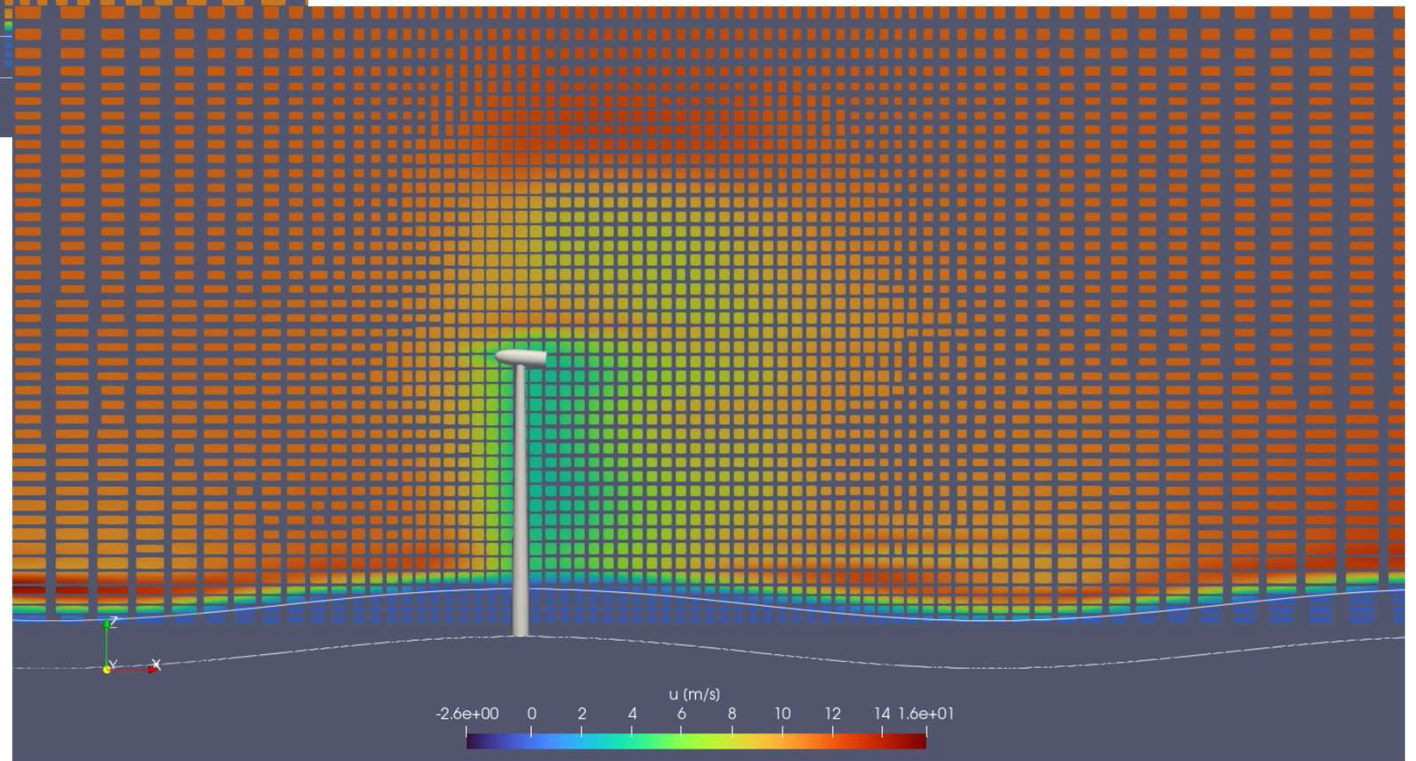
Cylinder $Re=100$



Reinforcement learning for p-adaptation

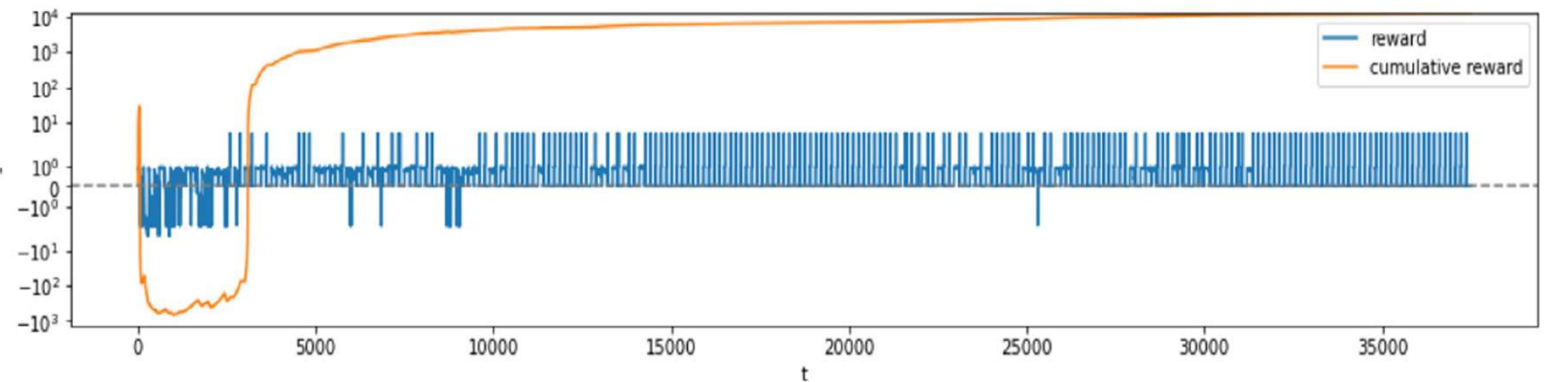
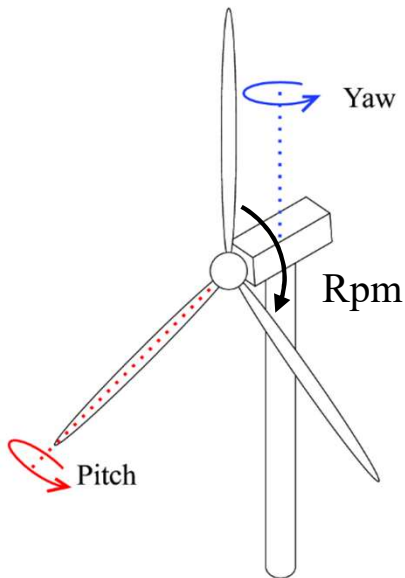


DTU 10MW offshore wind turbine



Reinforcement learning for wind turbine control

Training with simple winds

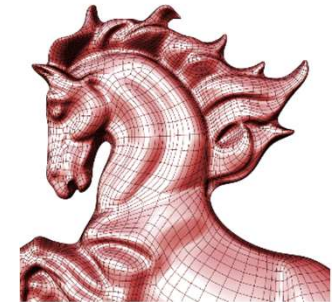


Validation with turbulent real winds

Metric	DDQN1	PID	Uncontrolled
Control Capacity Factor (%)	91.31	57.60	12.77
Capacity Factor (%)	20.95	12.49	1.59
Yearly Production (MWh)	4162.95	2481.97	316.12

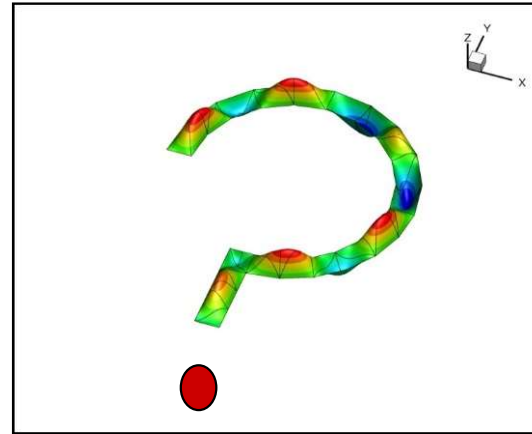
Conclusions

- **High order DG methods fairly well developed**
 - Incompressible flows & Compressible flows
- **Multiphysics:**
 - Wind turbines with various methods
 - Turbulence (iLES & explicit LES)
 - Aero-acoustics
 - Supersonic & Shocks
- **AI-based Solver**



Thank you very much

esteban.ferrer@upm.es



@FerrerCfd



Esteban Ferrer

<http://sites.google.com/site/eferrerdg/publications>

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