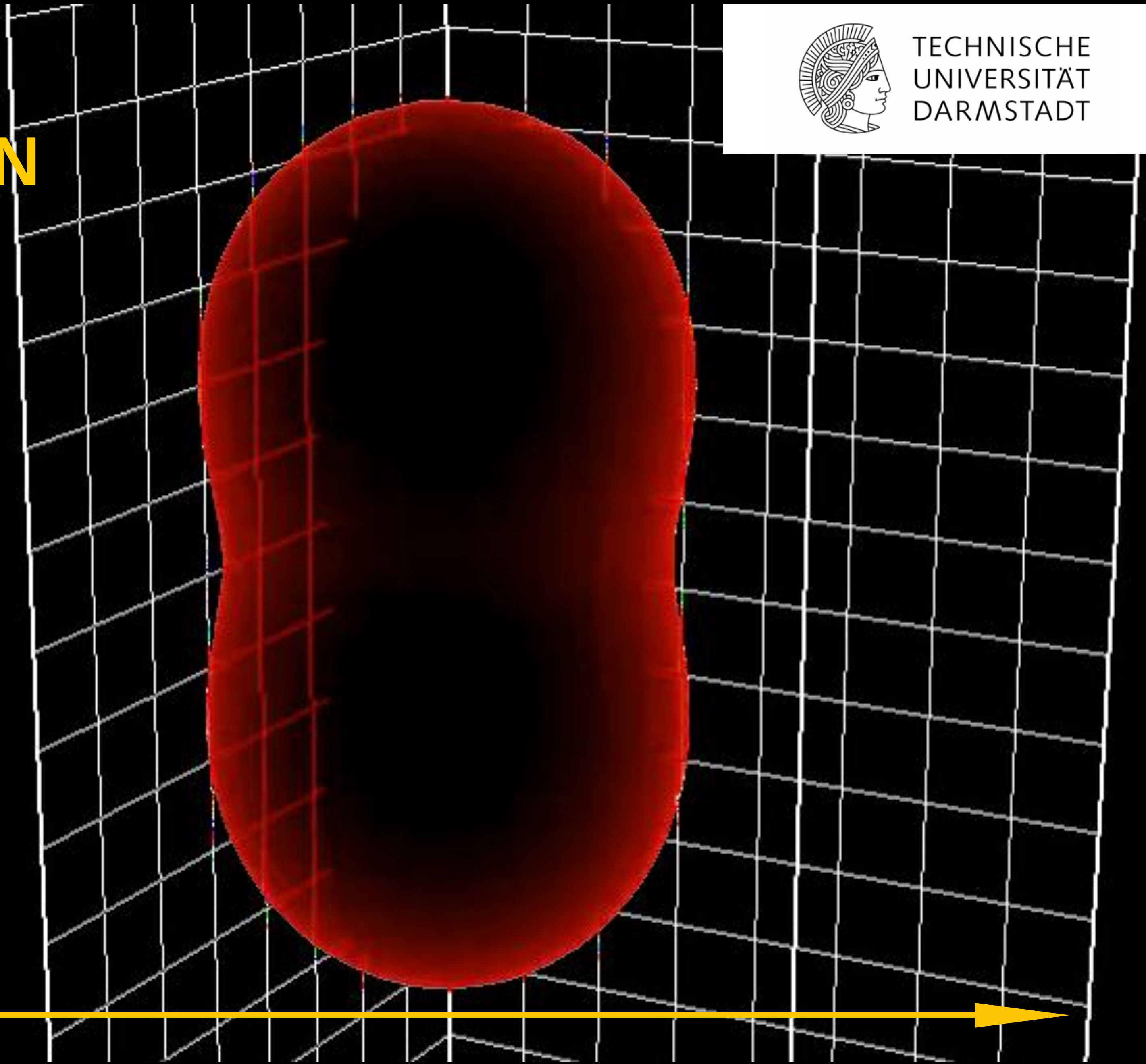
NUMERICAL SIMULATION OF FLOWS WITH DYNAMIC INTERFACES

Martin SMUDA
Florian KUMMER

ERCOFTAC Autumn Festival 2025
Darmstadt, 9th Oct., 2025



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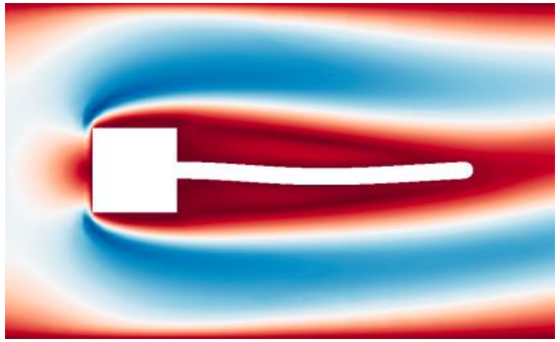
MASCHINENBAU
We engineer future
FDY

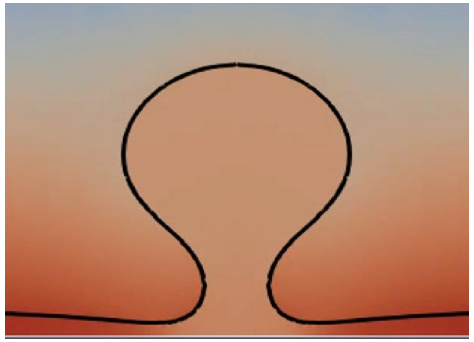
GOALS

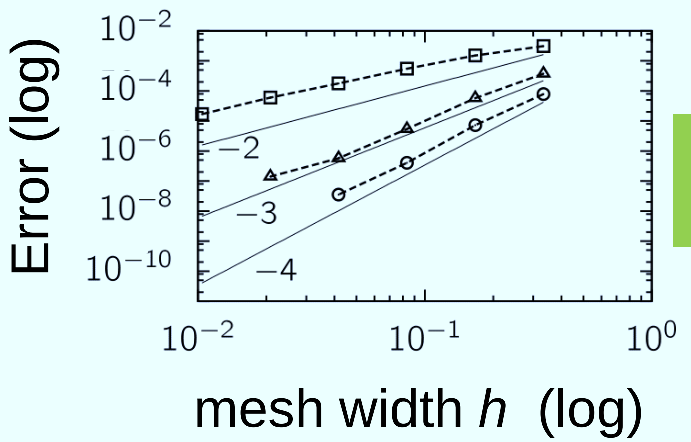
Simulation Methods for flows with **DYNAMIC INTERFACES**

- Moving Geometry
- Fluid Interfaces

- (Static Geometry: eliminate need for meshing)





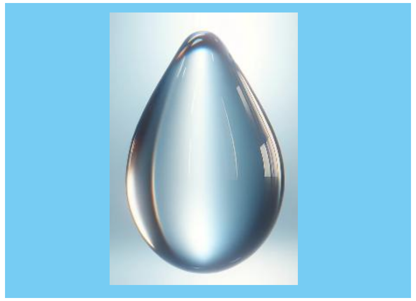


$$\mathrm{Err} \sim h^k$$

- using **HIGH ACCURACY, HIGH ORDER METHODS**
  - Either:
    - better accuracy on same mesh
    - coarser mesh for same accuracy

  - high order methods: better suitability for modern hardware (GPUs)

DYNAMIC INTERFACES



Multiphase flows

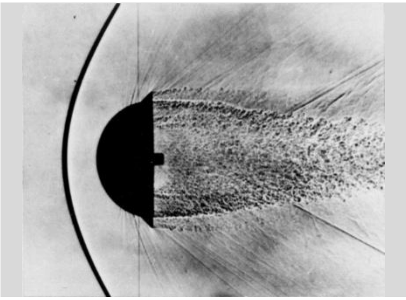
Heat transport & phase transition

Fluid-Structure interaction

Particle laden flows

Three-phase contact line

Reactive flows



Supersonic flows & shock waves

Moving technical geometries

DYNAMIC INTERFACES

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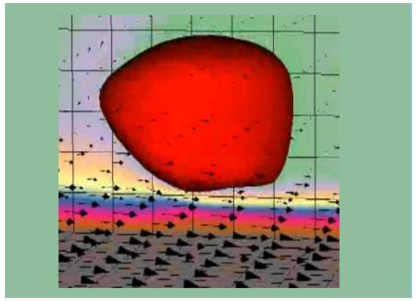
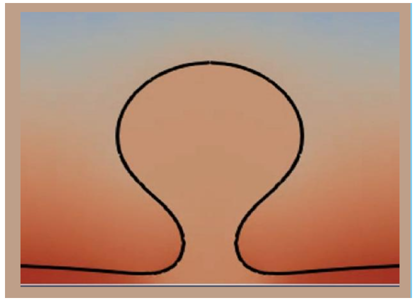
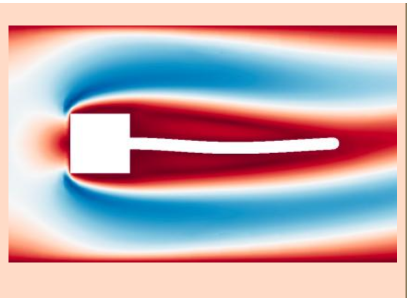


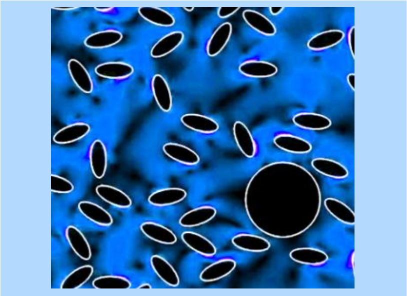
Multiphase flows



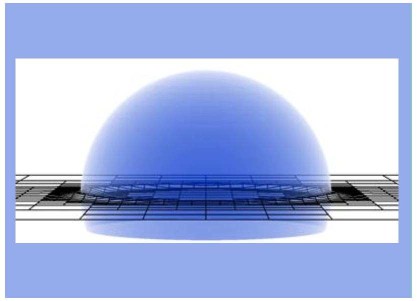
Heat transport & phase transition



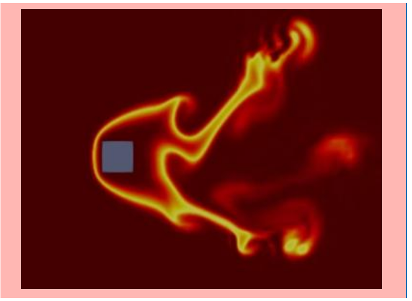
Fluid-Structure interaction



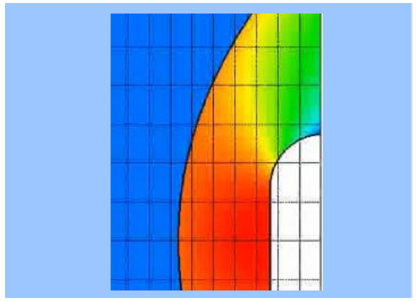
Particle laden flows



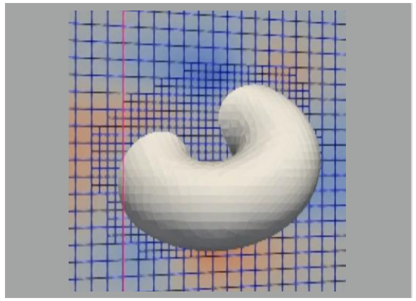
Three-phase contact line



Reactive flows



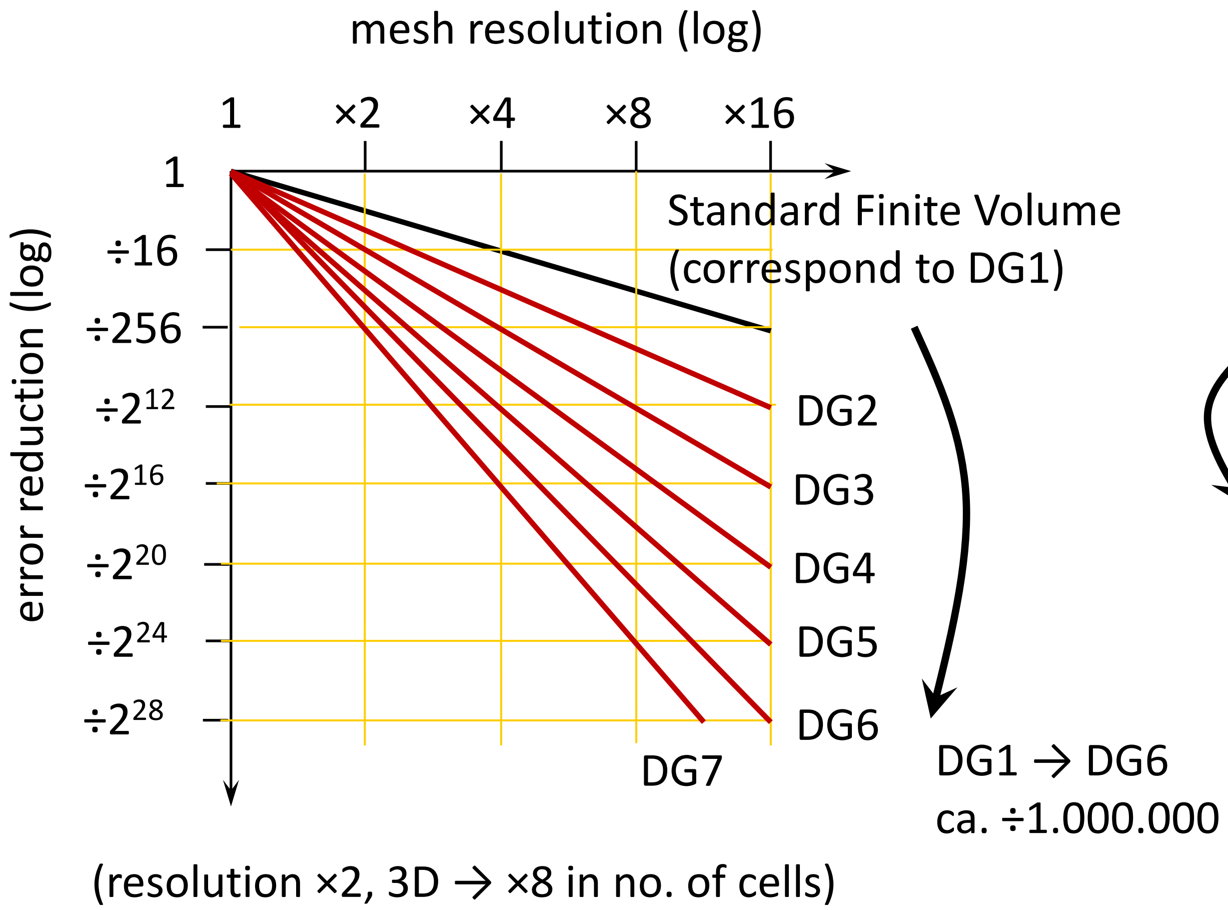
Supersonic flows & shock waves

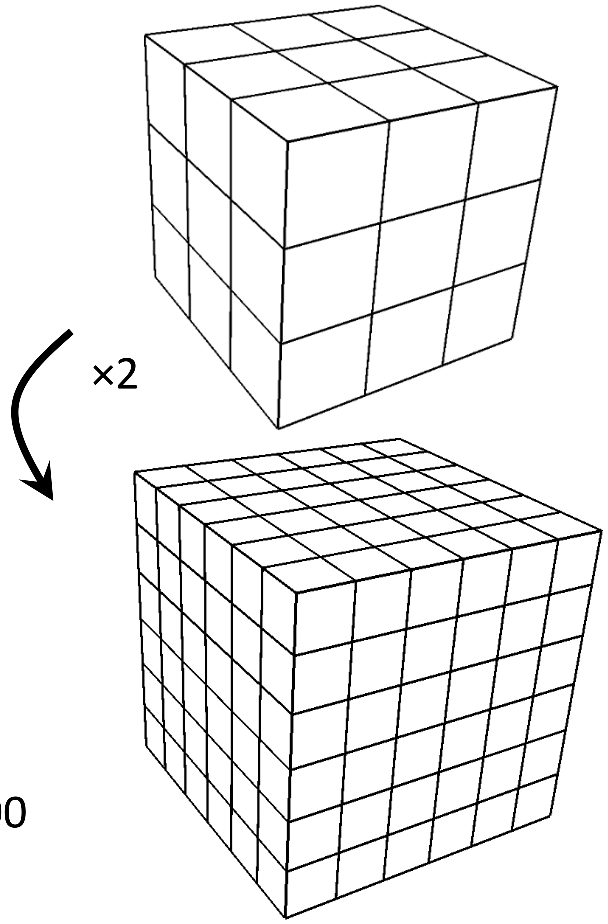


Moving technical geometries

XDG method / BoSSS code

HIGH ACCURACY / HIGH ORDER METHODS





OUTLINE

1 Introduction: Dynamic Interfaces and High Order

2 High Order/ High Accuracy:
Discontinuous **G**alerkin (DG)

3 DG for Dynamic Interfaces
e**X**tended **D**iscontinuous **G**alerkin (XDG)

4 Applications

5 Summary

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2 HIGH ORDER / HIGH ACCURACY: DISCONTINUOUS GALERKIN (DG)

- How do they work?
- When do they achieve high order? — $\mathrm{Err} \sim h^k$
- When do they fail? — $\mathrm{Err} \sim h^1$

DG IN ONE DIMENSION

$$u(x) := \cos\left(\frac{x\pi}{2}\right)(x+2)$$

$$\approx$$

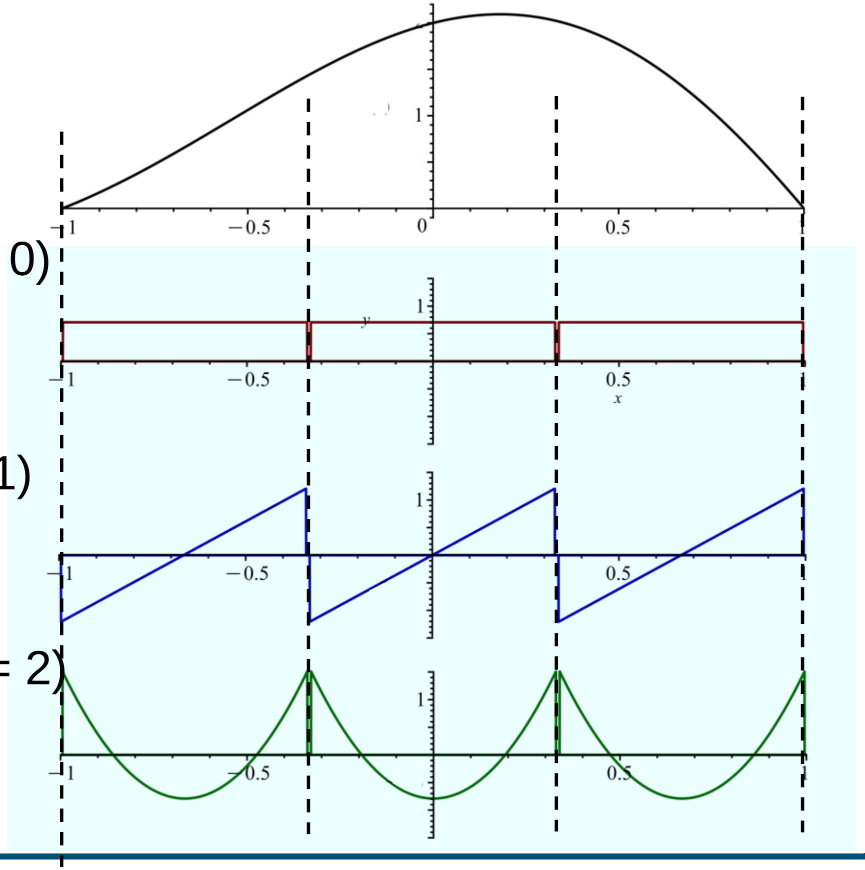
$$u_h(x) := \sum_{\text{cells } j} \sum_{\text{modes } n} \phi_{j,n}(x)\tilde{u}_{jn}$$

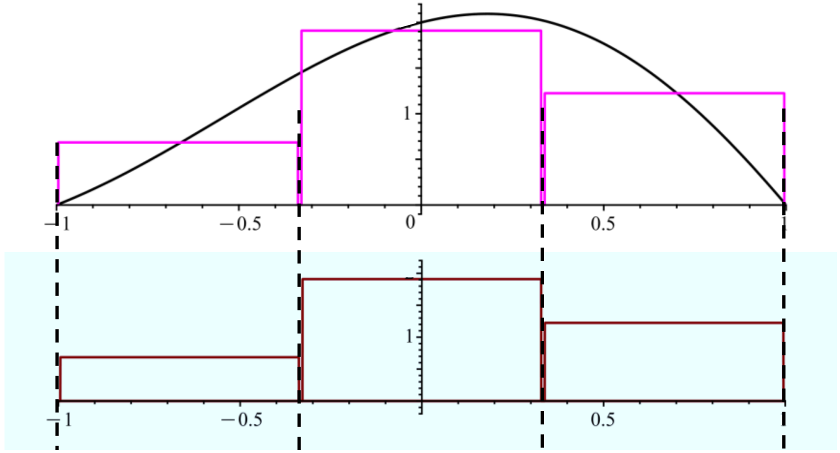
$$\int_{-1}^{1} (u(x) - u_h(x))^2 \, dx \to \min$$

const. (p = 0)

lin. (p = 1)

quad. (p = 2)





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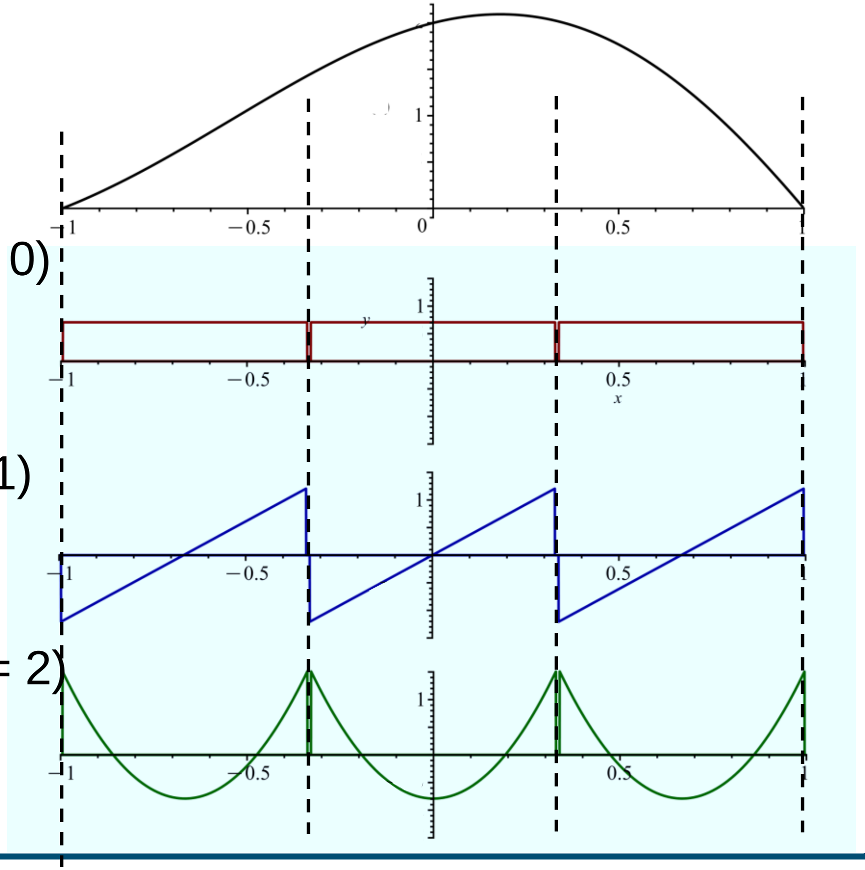

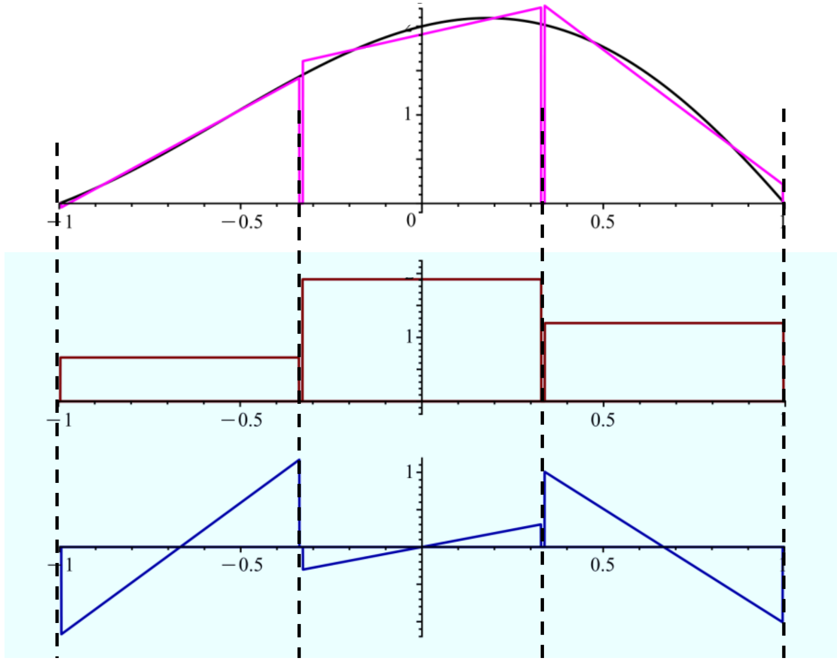

DG IN ONE DIMENSION

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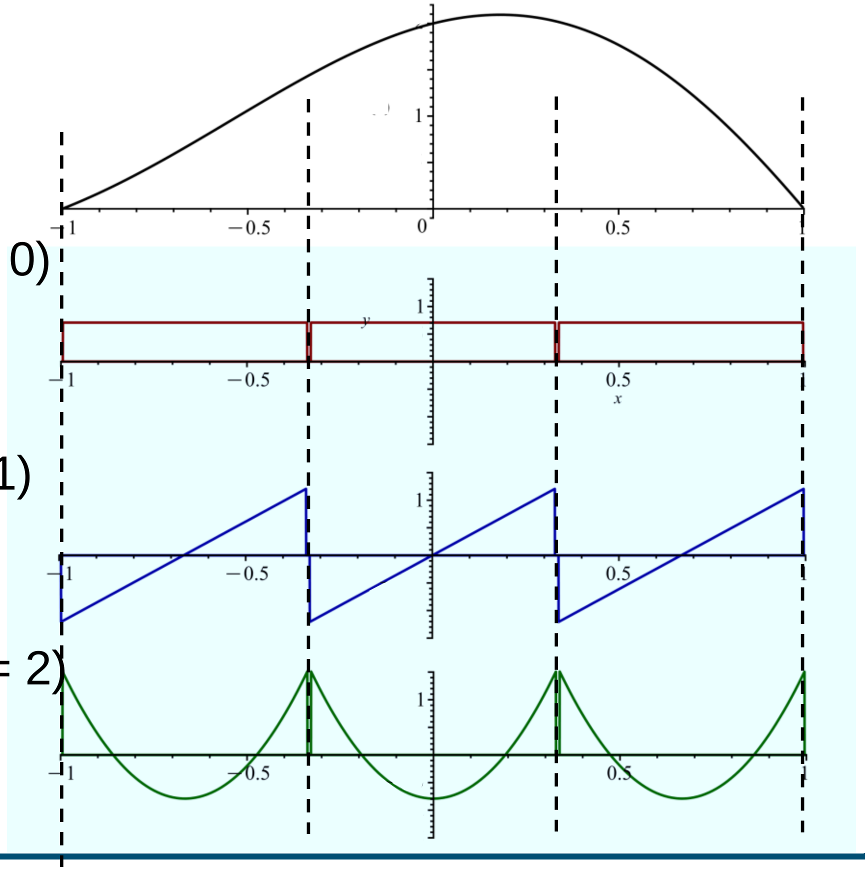

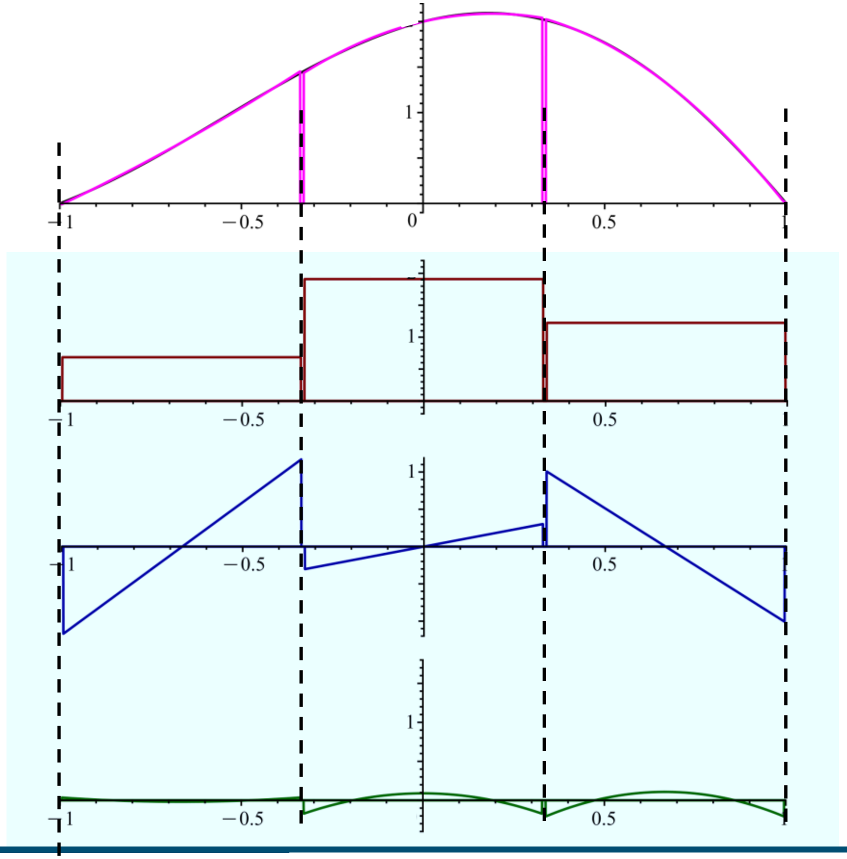

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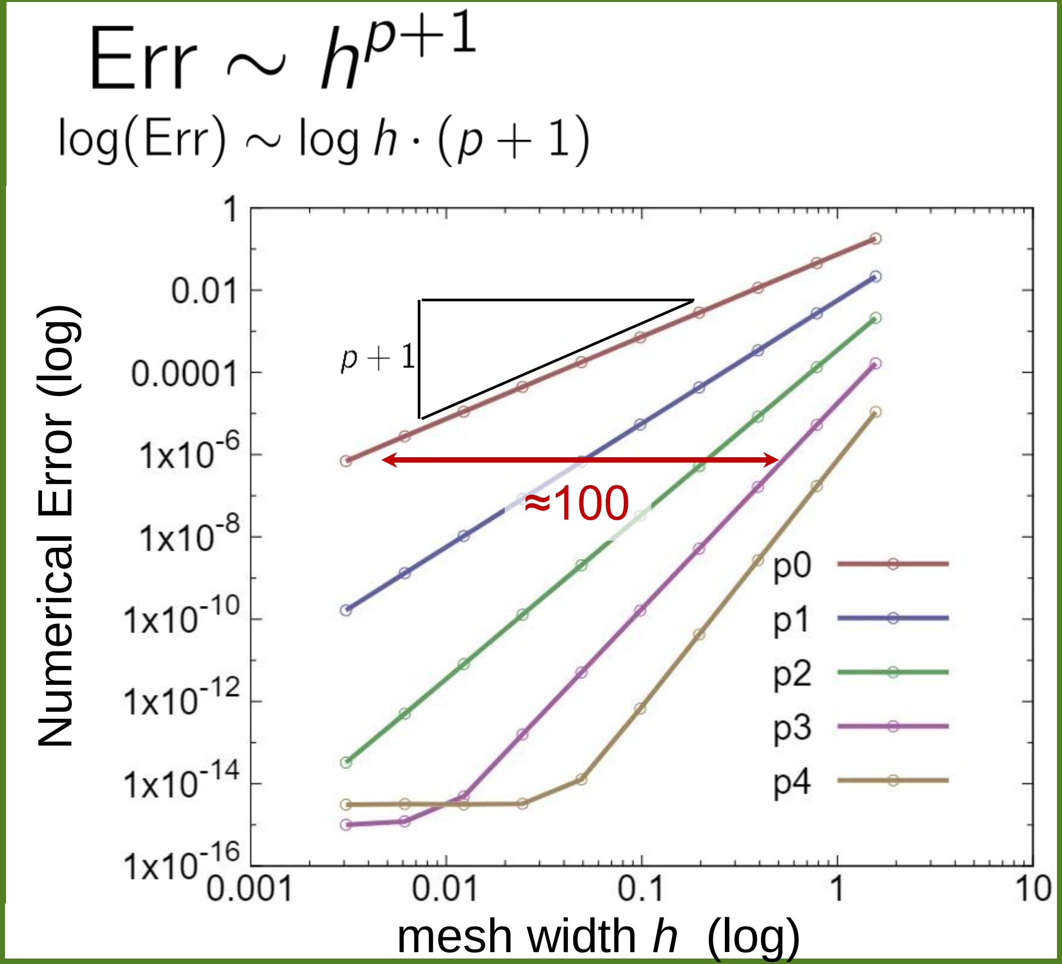

const. (p =
lin. (p =
quad. (p

$$\sum_{\text{cells } j} \sum_{\text{modes } n} \phi_{j,n}(x)\tilde{u}_{jn}$$

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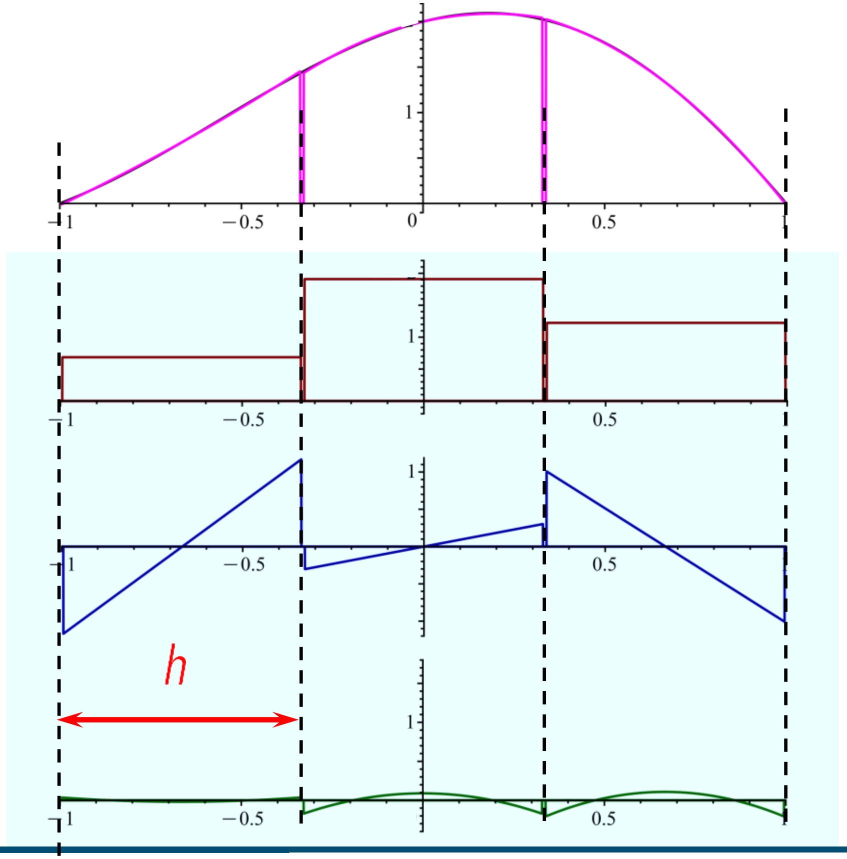

DG IN TWO DIMENSIONS

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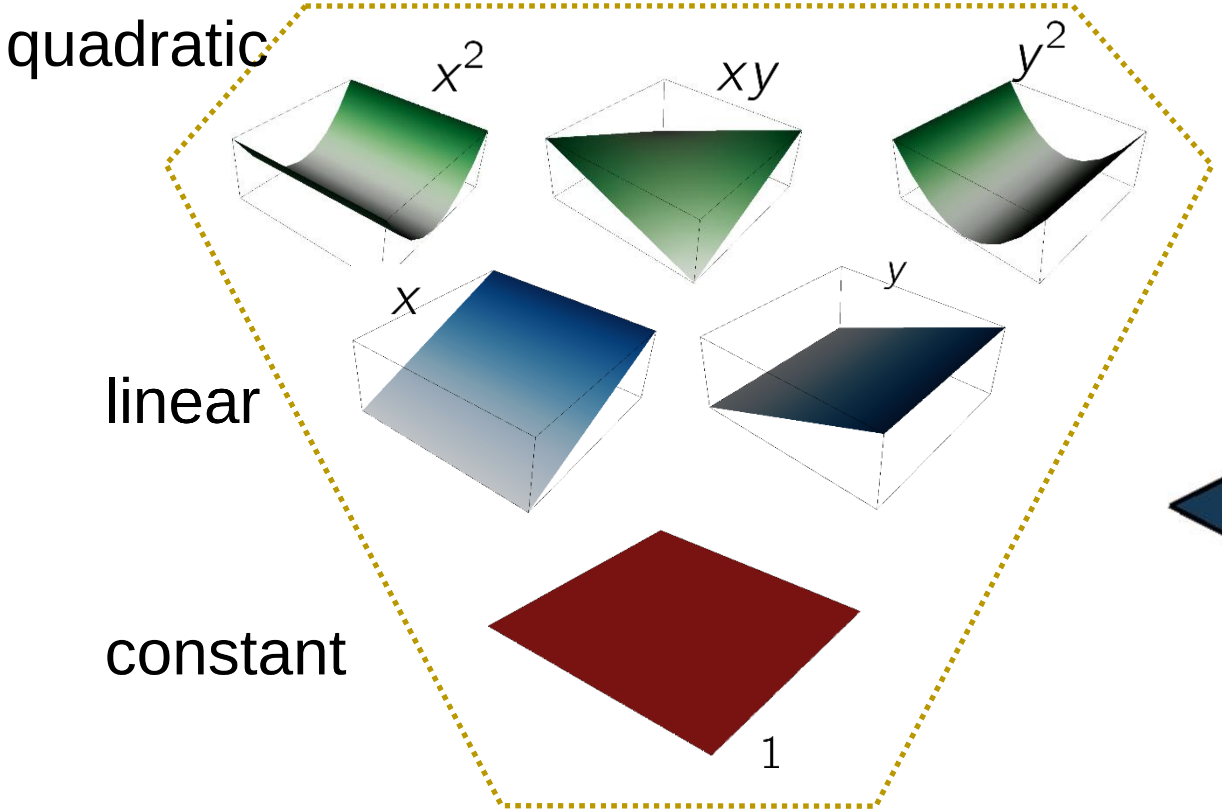
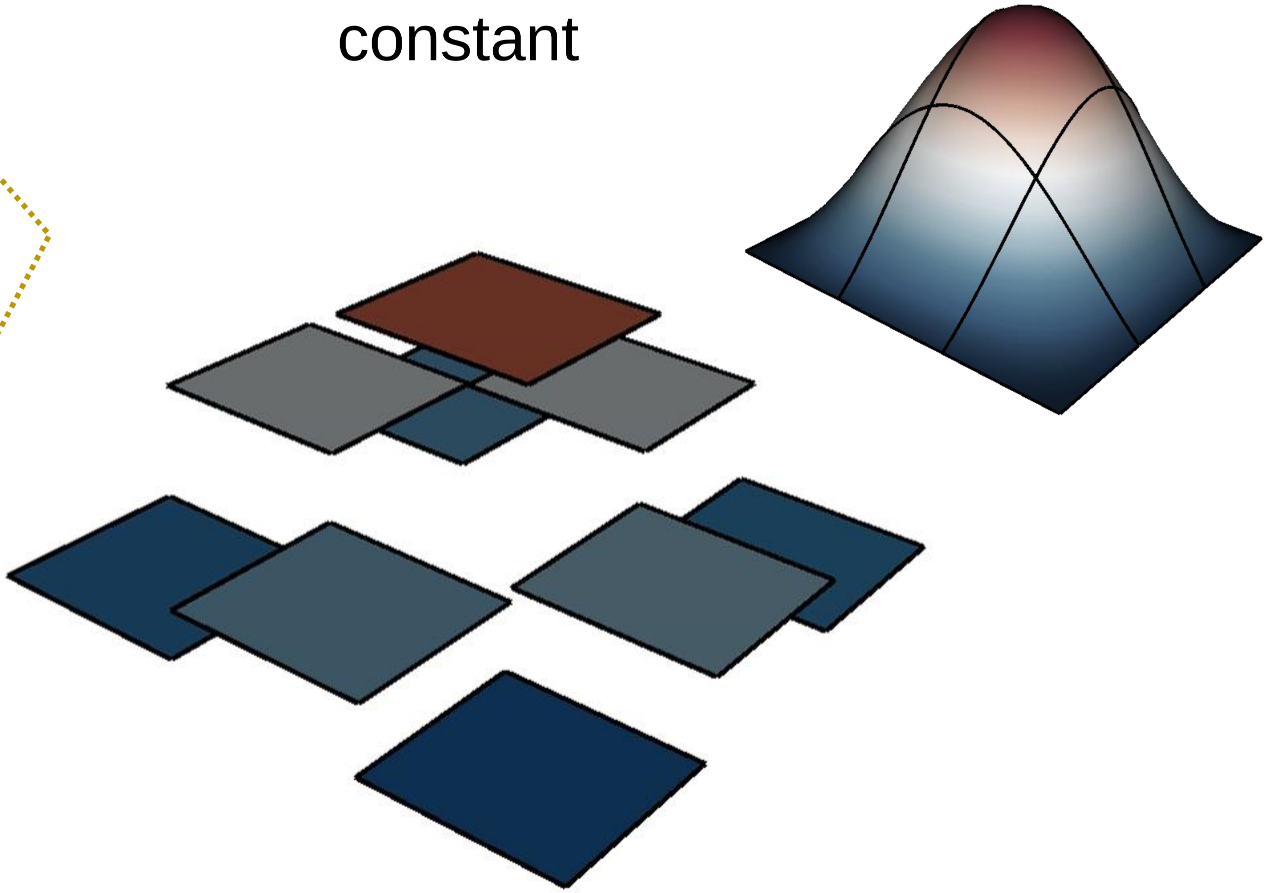
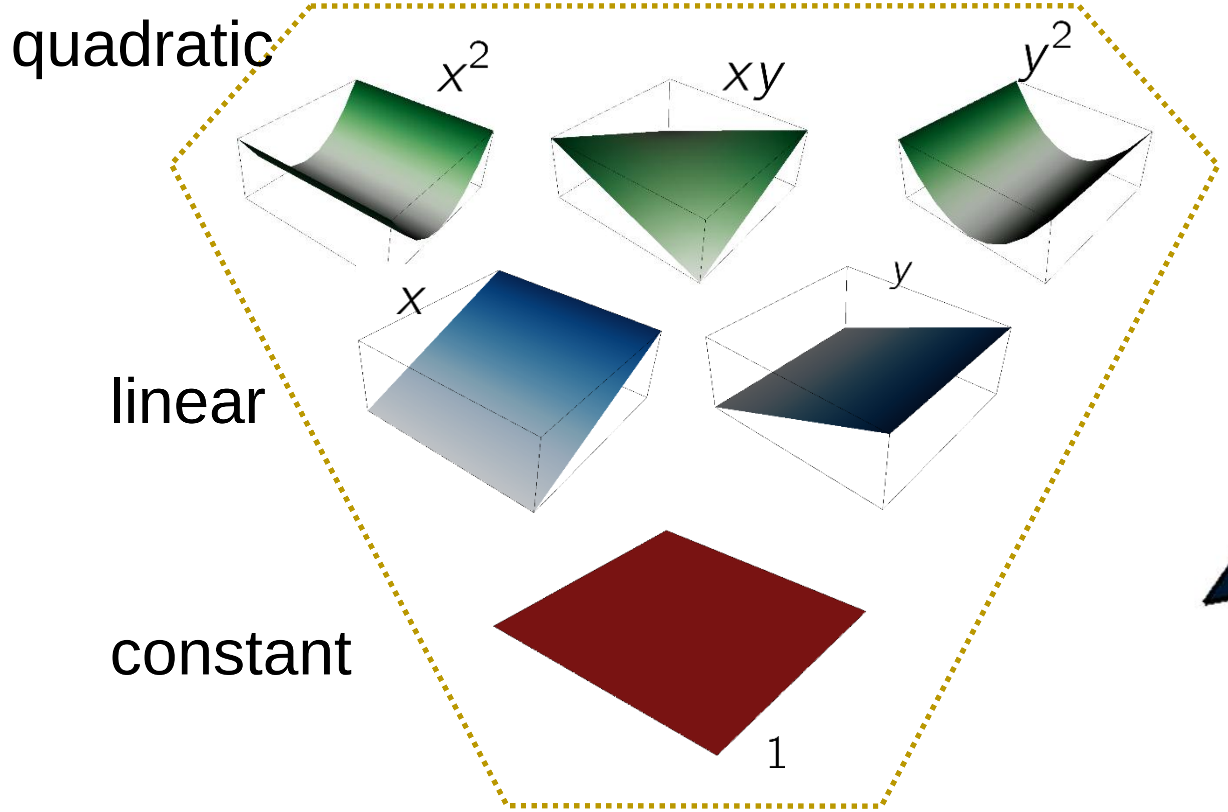
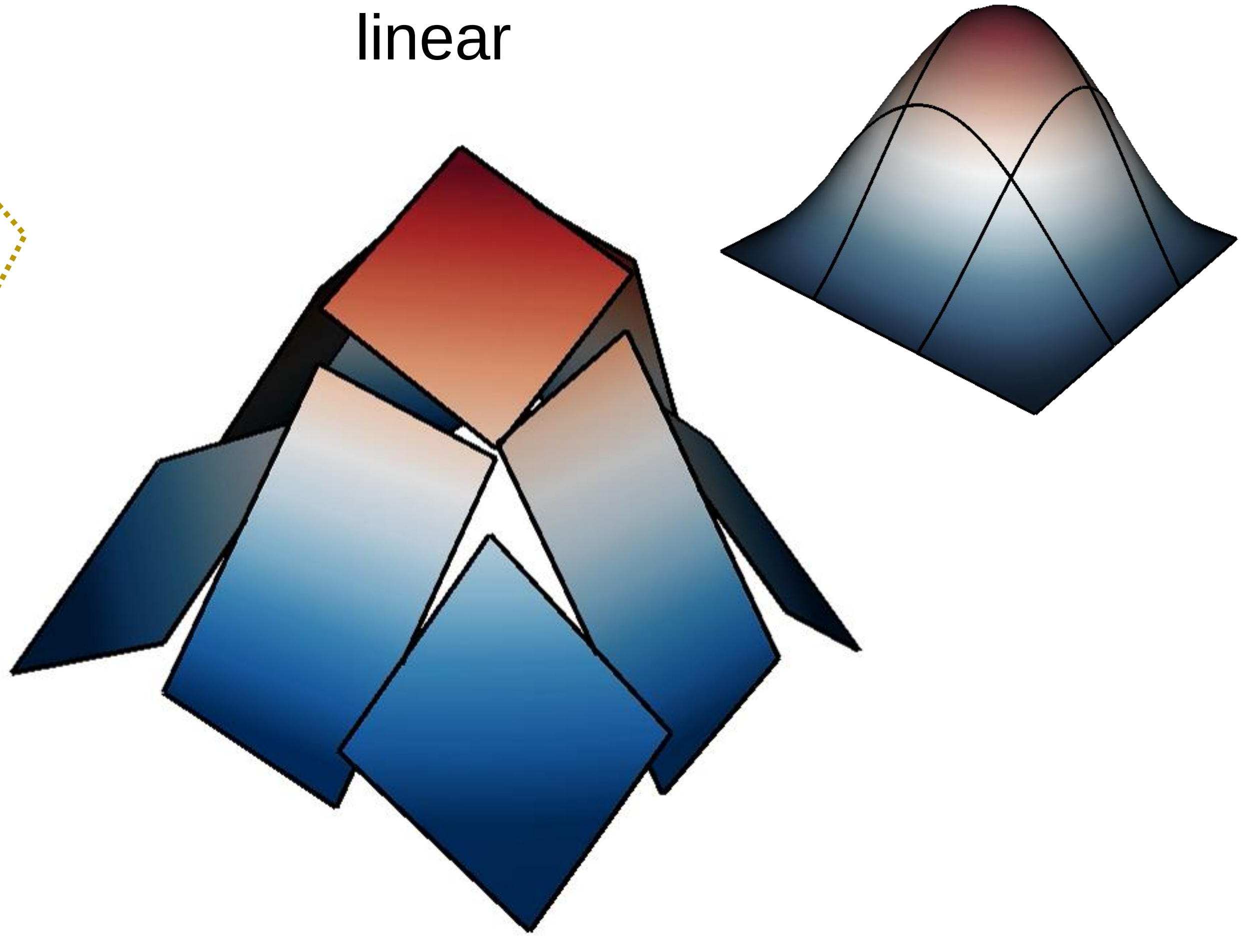




DG IN TWO DIMENSIONS





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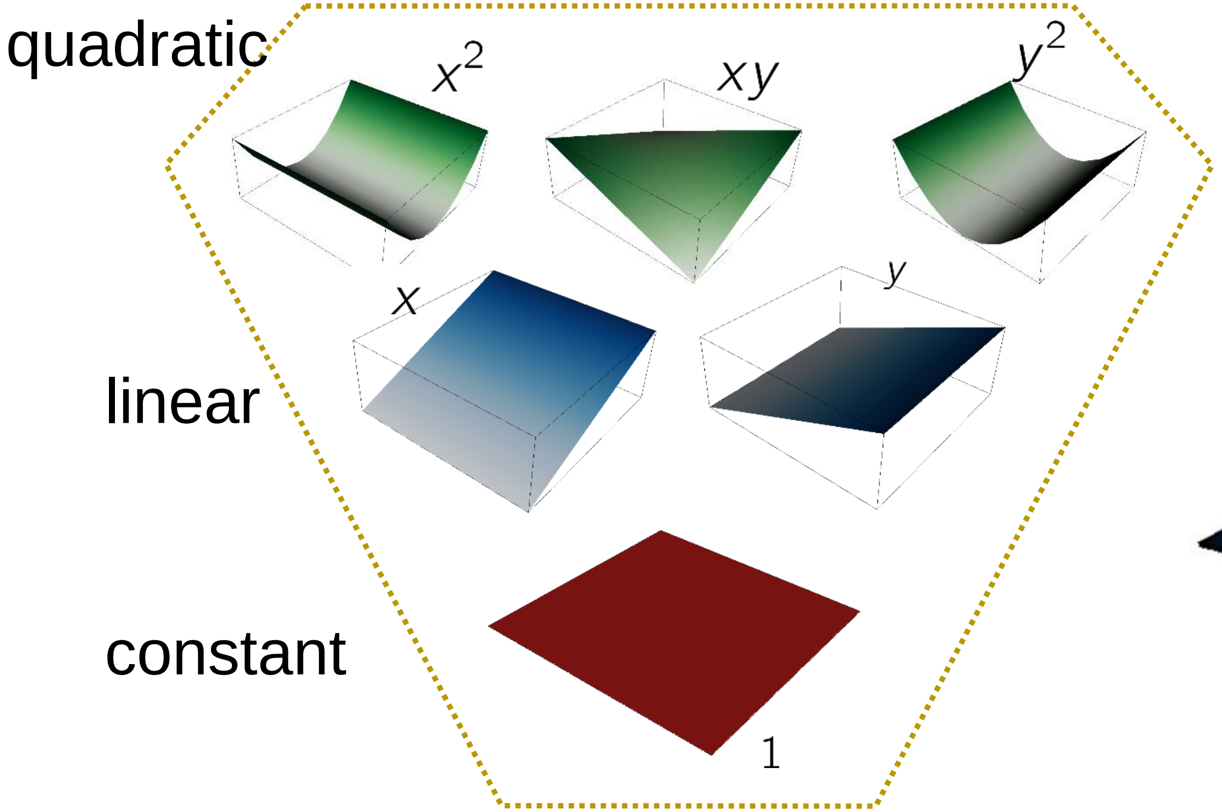
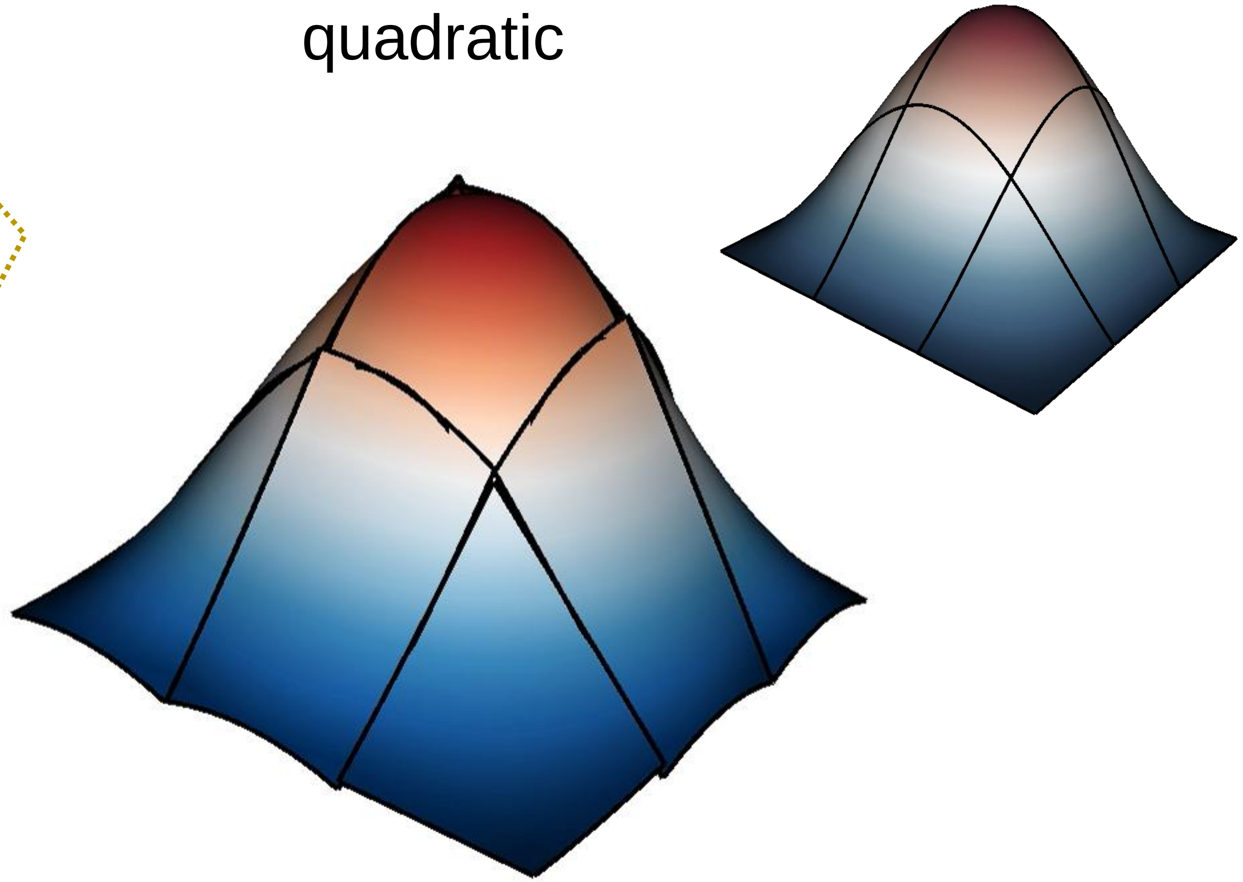


quadratic

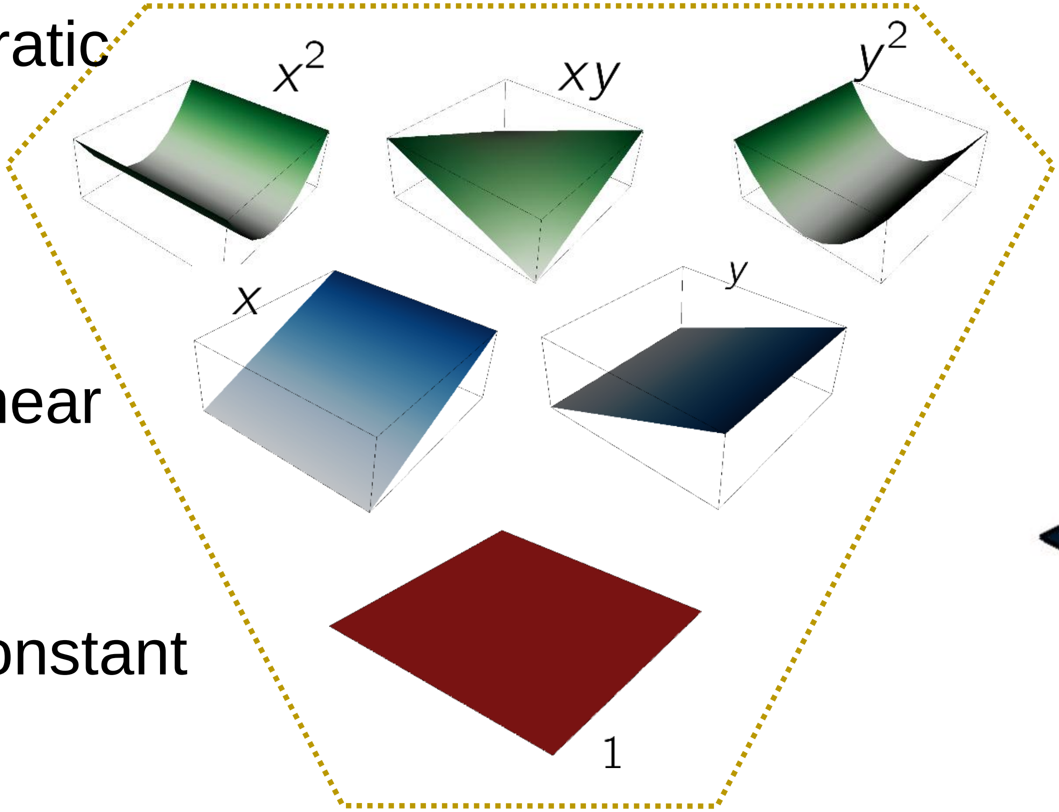


DG IN TWO DIMENSIONS

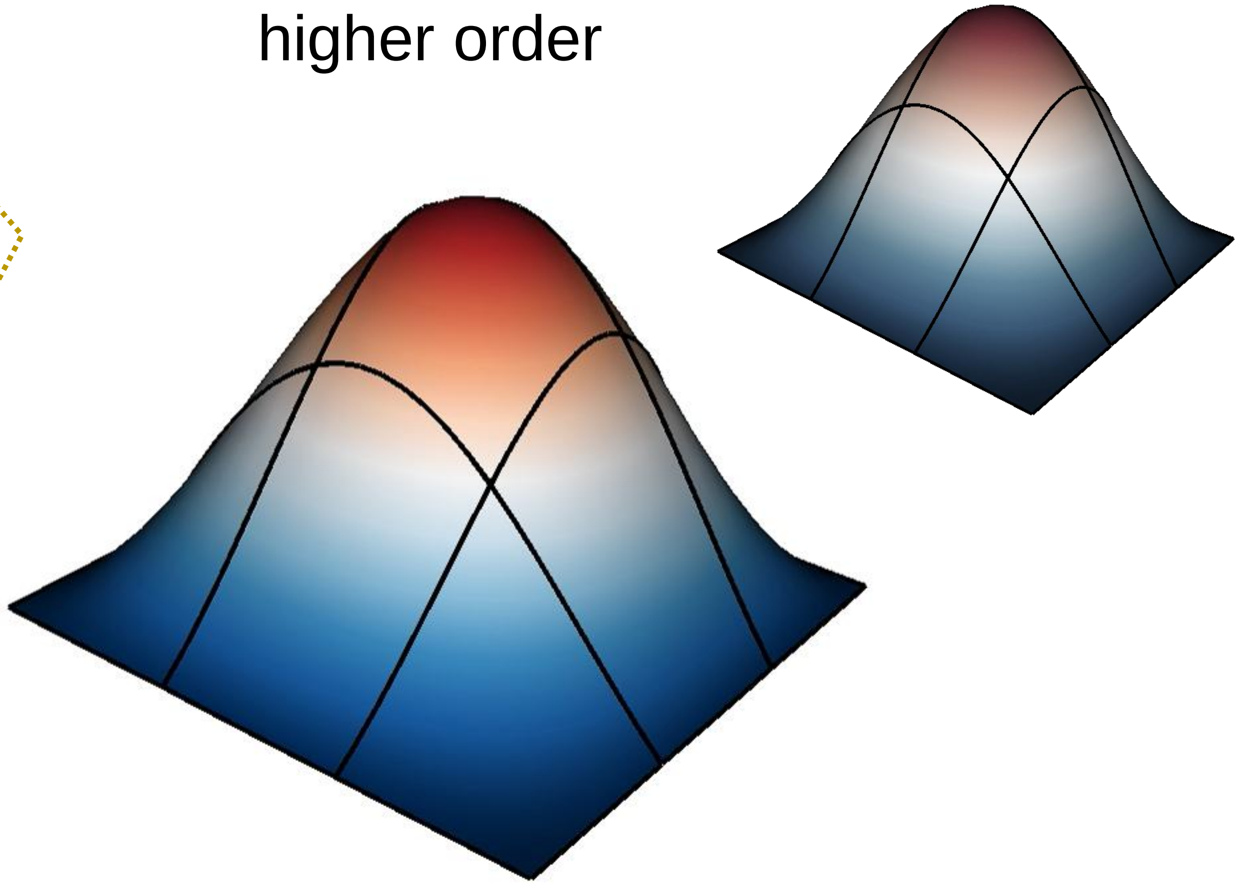
quadratic

linear

constant



higher order



DISCONTINUITIES AND INTERFACES

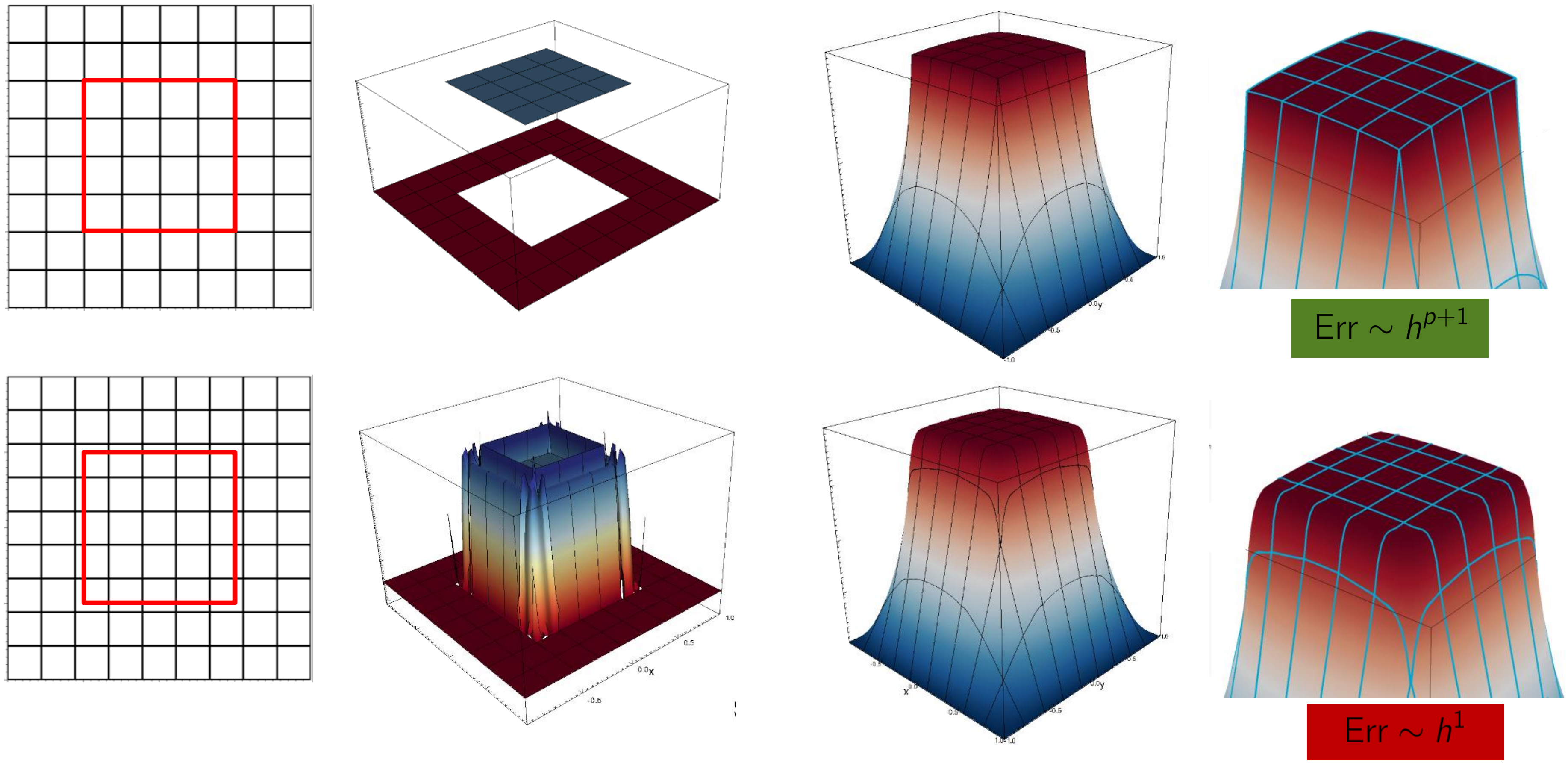
Mesh | RHS g | PDE solution u

$$-\nu \Delta u = g \text{ in } (-1,1)^2$$

$$u = 0 \text{ on bndy}$$

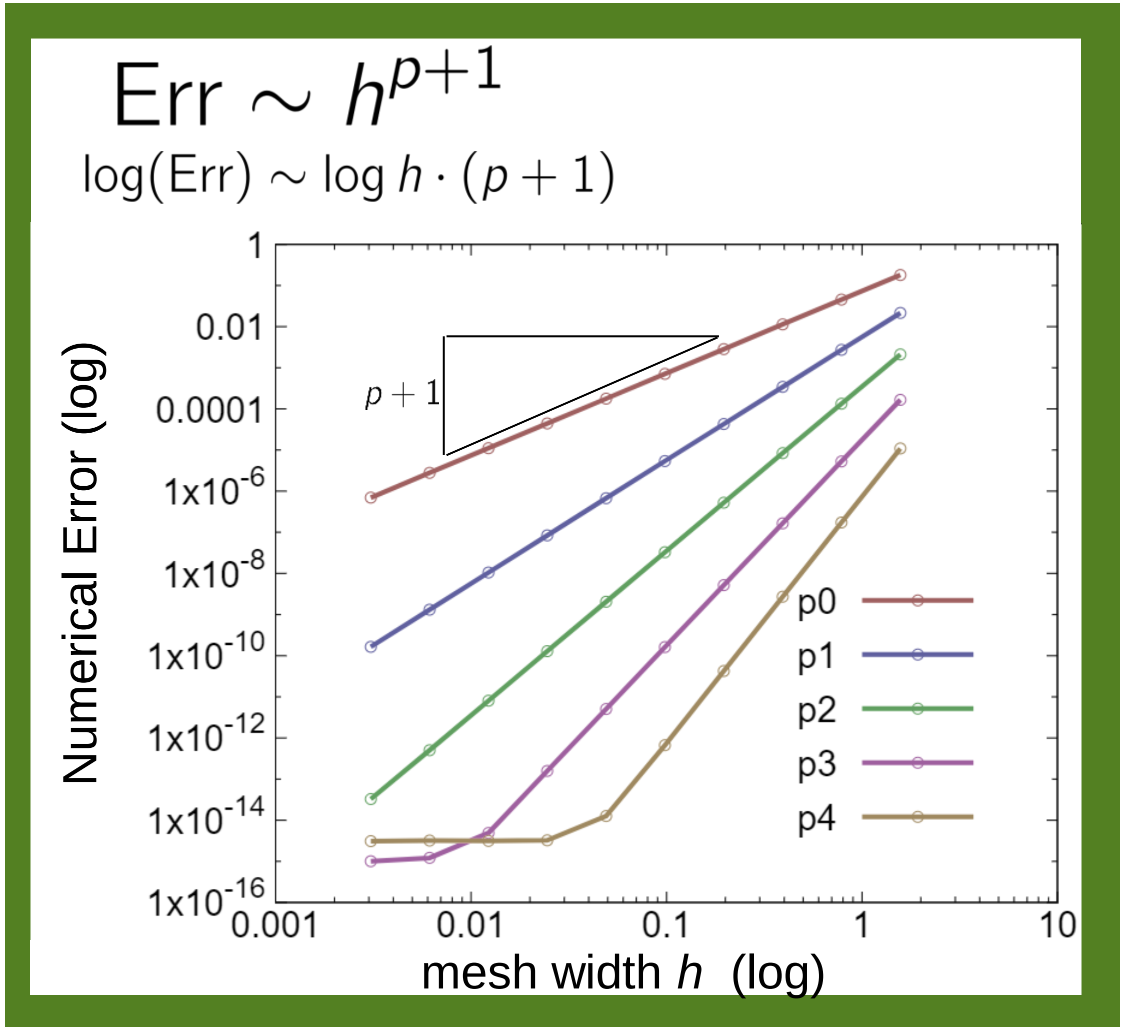
$$g = \begin{cases} 1 & -0.5 \leq x, y \leq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

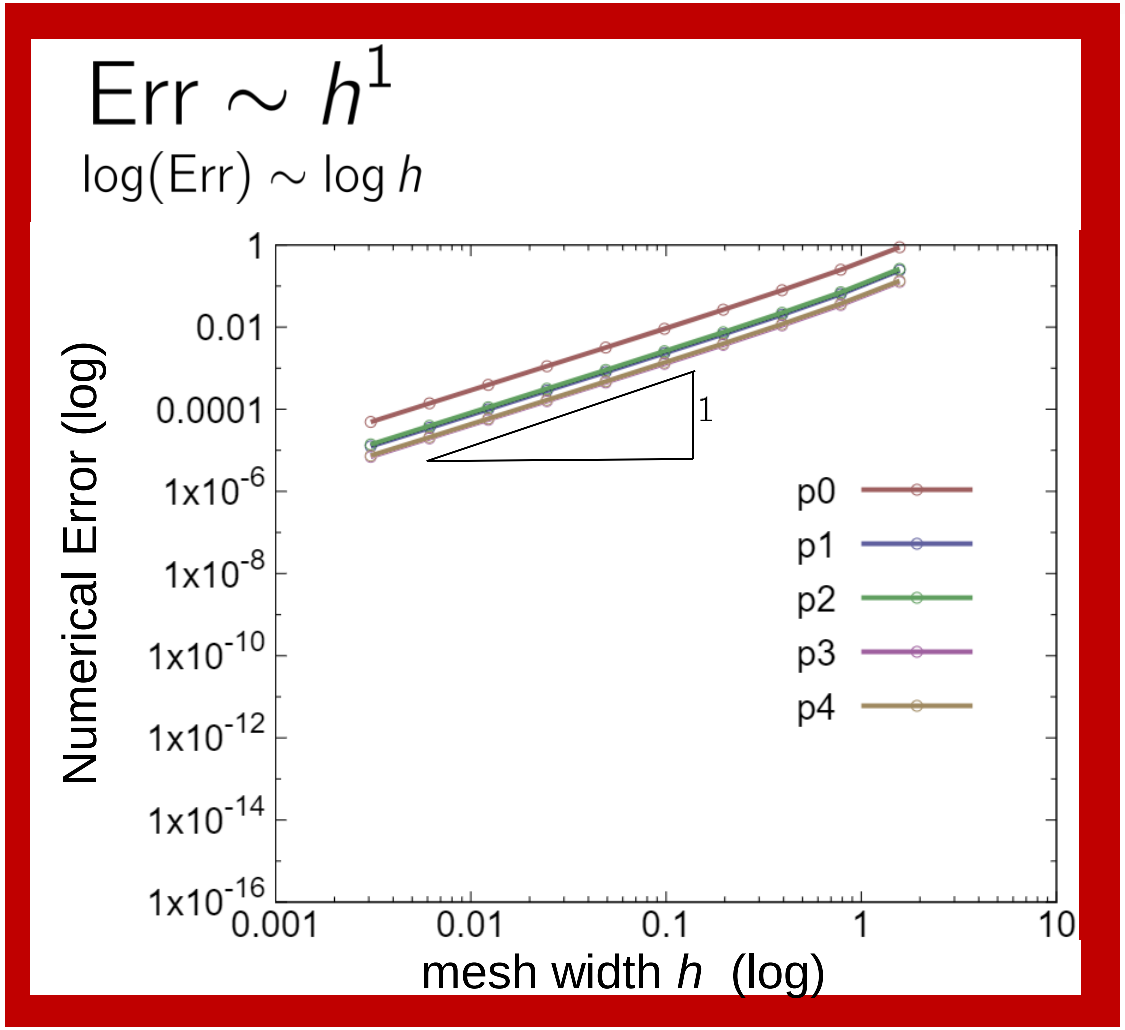
$$\nu = \begin{cases} 100 & -0.5 \leq x, y \leq 0.5 \\ 1 & \text{otherwise} \end{cases}$$



[I. Babuška, 1970: The Finite Element Method for Elliptic Equations with Discontinuous Coefficients]

DISCONTINUITIES AND INTERFACES – ERROR DEGENERATION

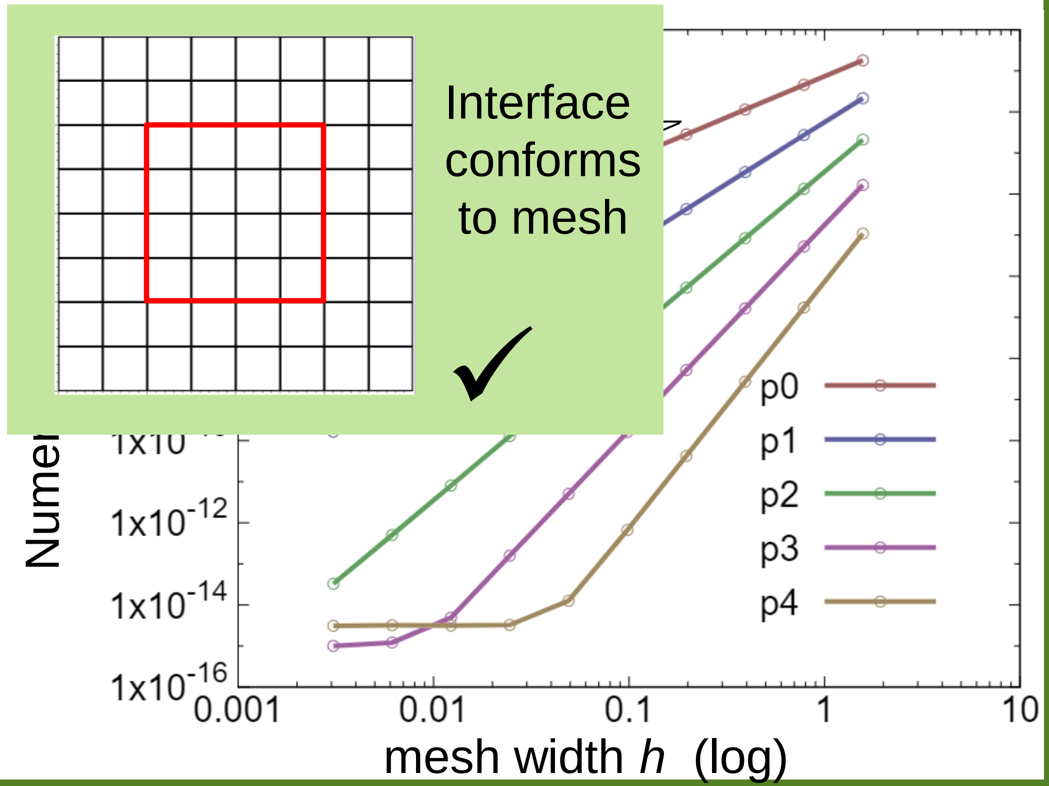




DISCONTINUITIES AND INTERFACES – ERROR DEGENERATION

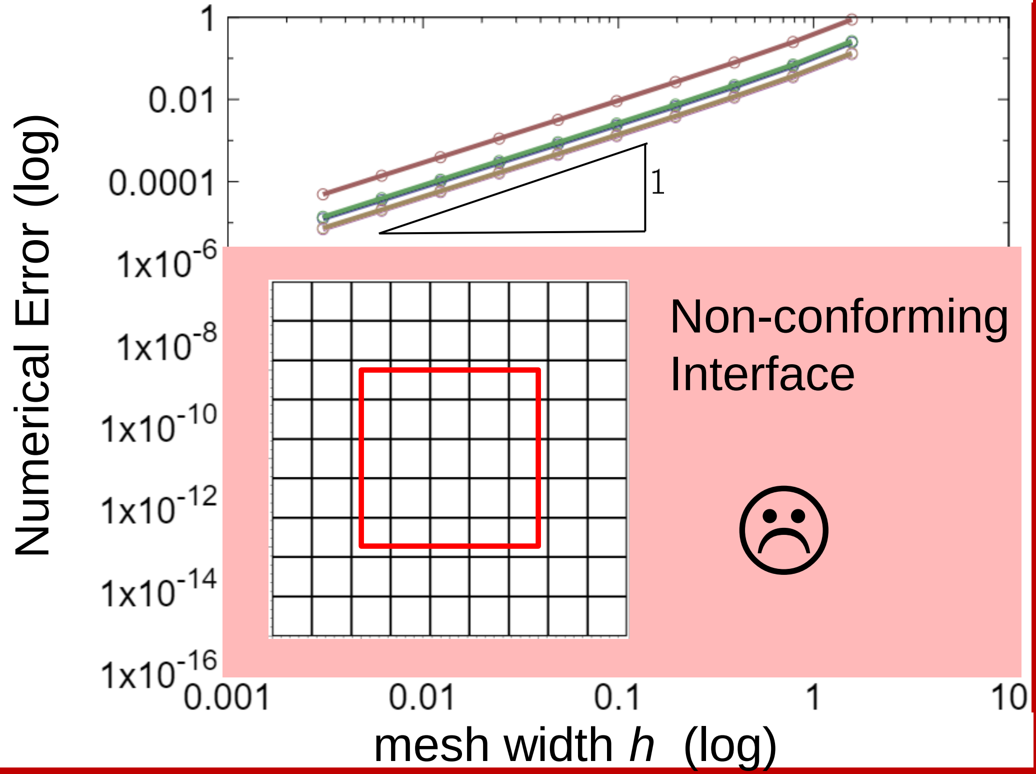
$$\mathrm{Err} \sim h^{p+1}$$

$$\log(\mathrm{Err}) \sim \log h \cdot (p+1)$$



$$\mathrm{Err} \sim h^{1}$$

$$\log(\mathrm{Err}) \sim \log h$$



DYNAMIC INTERFACES AND DG

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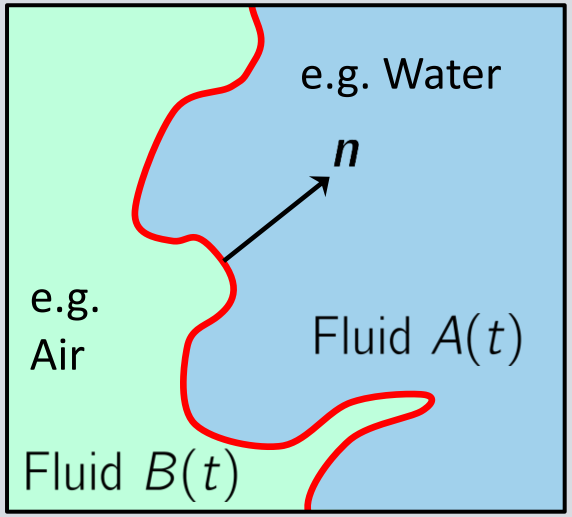
- **Challenges:**
  - interface location a priori unknown
  - temporal evolution
  - topology changes
- **(Nearly) impossible with interface-conforming meshing**

➢ **eXtended Discontinuous Galerkin**

1. tracking of interface position
2. representation of discontinuities by
   conforming basis functions

Multiphase flows

- Phases/sub-domains $A(t)$, $B(t)$



3

DG FOR DYNAMIC INTERFACES EXTENDED DG - XDG

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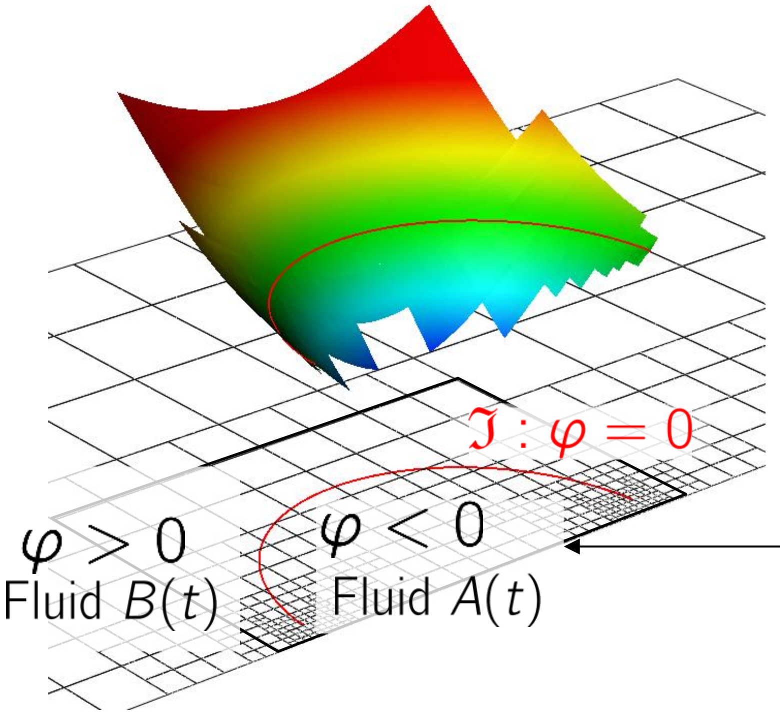
- Interface Description: Level-Set
- Interface-conforming XDG basis

➢ High order / high accuracy

LEVEL-SET-METHOD: INTERFACE EVOLUTION

Level-Set field $\varphi(t, \boldsymbol{x})$

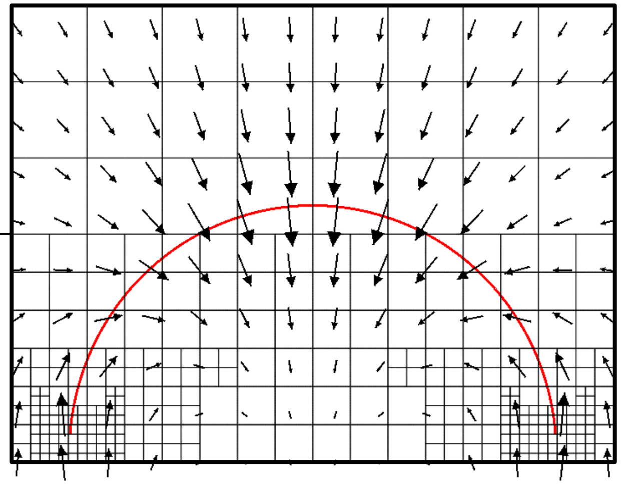


Ansatz: $\mathfrak{I} : \quad \varphi(t, \boldsymbol{x}(t)) = 0 \quad \Big| \frac{\mathrm{d}}{\mathrm{d}t}$

$$\partial_t \varphi + \nabla \varphi \cdot \dot{\boldsymbol{x}} = 0$$

Level-set advection: $\partial_t \varphi + \nabla \varphi \cdot \boldsymbol{u} = 0$

Level-set velocity $\boldsymbol{u}$

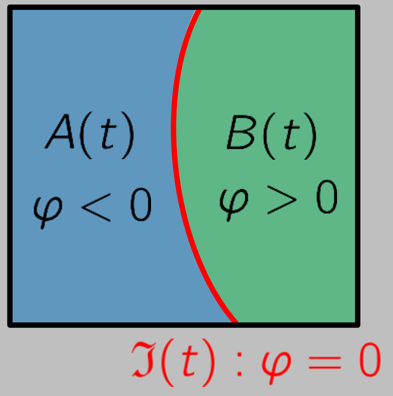


INTERFACE-CONFORMING XDG BASIS

Cut Cells

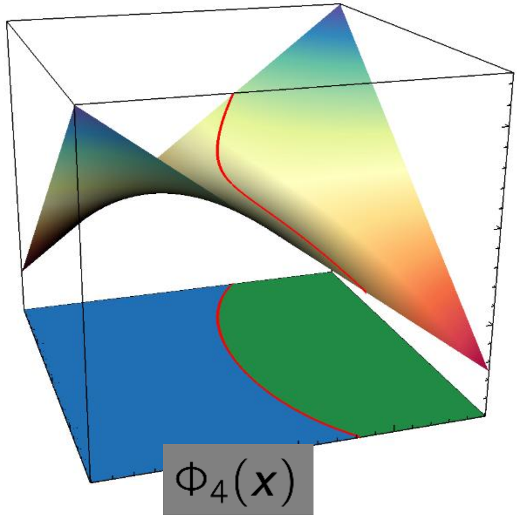
Level-Set

$\varphi(t, \vec{x})$



DG Approximation

$$u(\vec{x}, t) = \sum_{n=1}^{N} \tilde{u}_{j,n} \Phi_{j,n}(\vec{x})$$



INTERFACE-CONFORMING XDG BASIS

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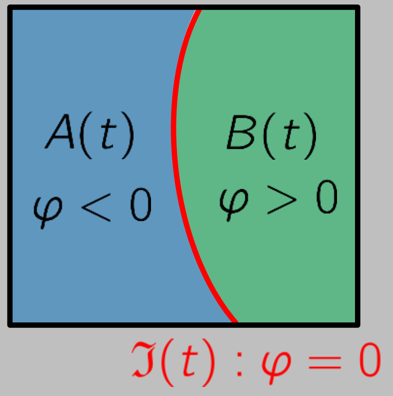
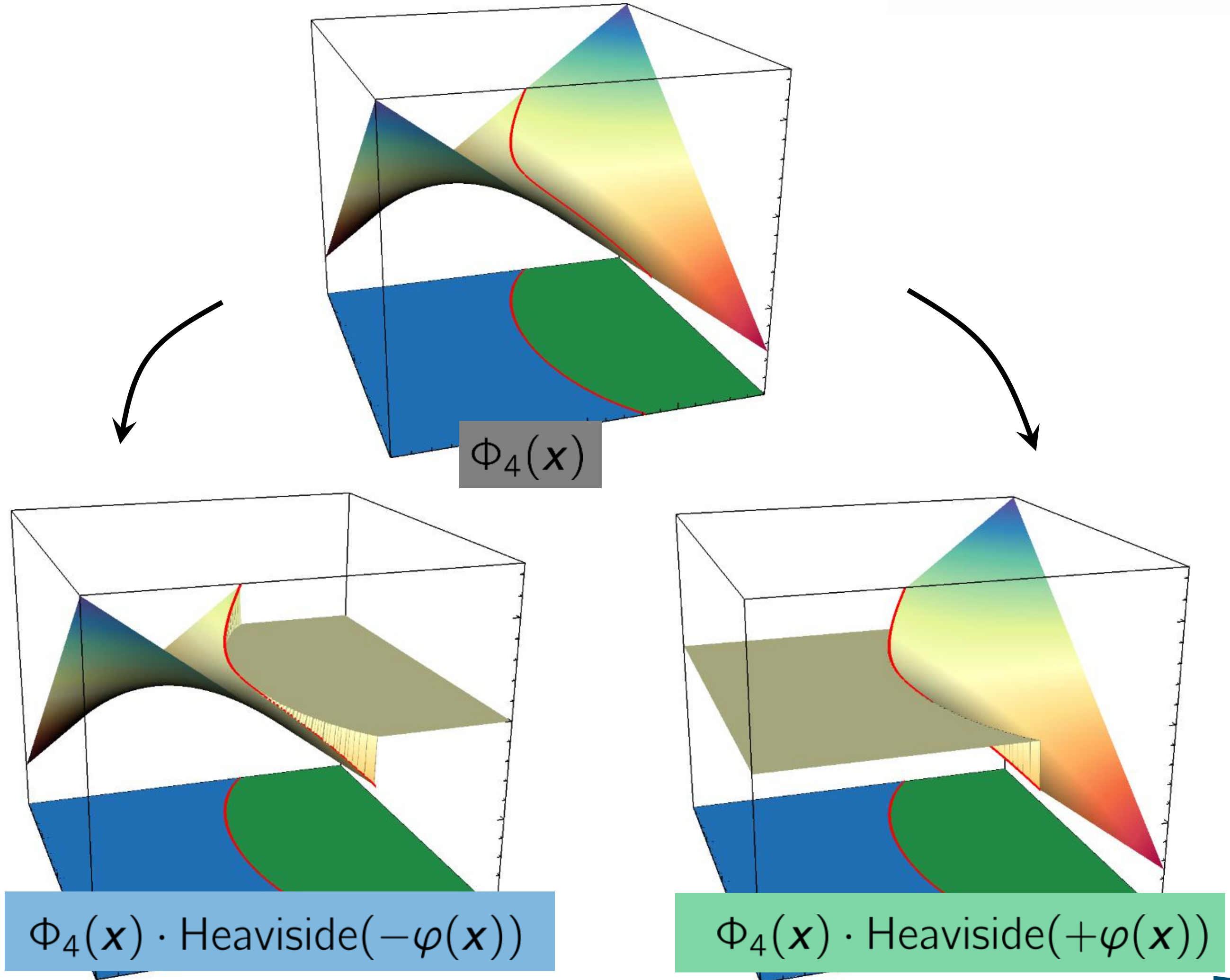
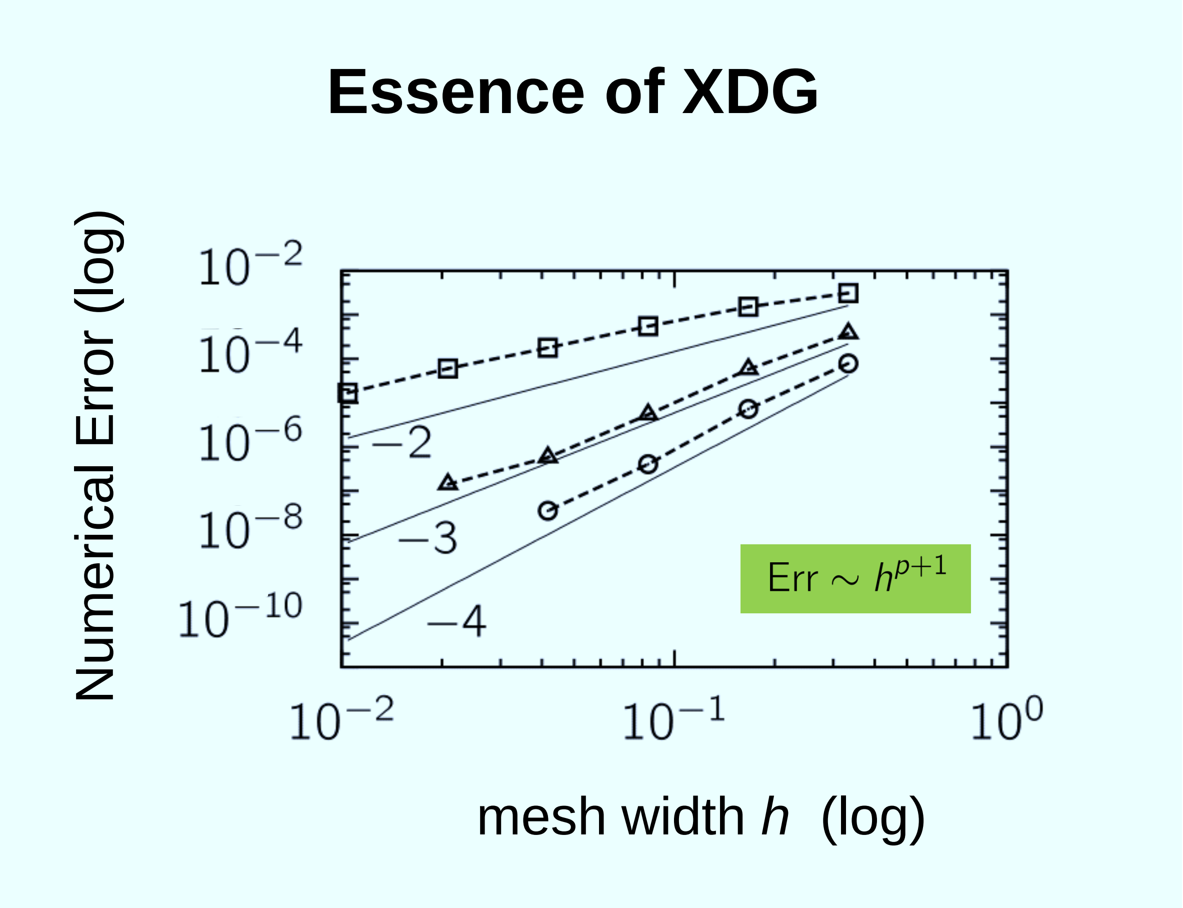


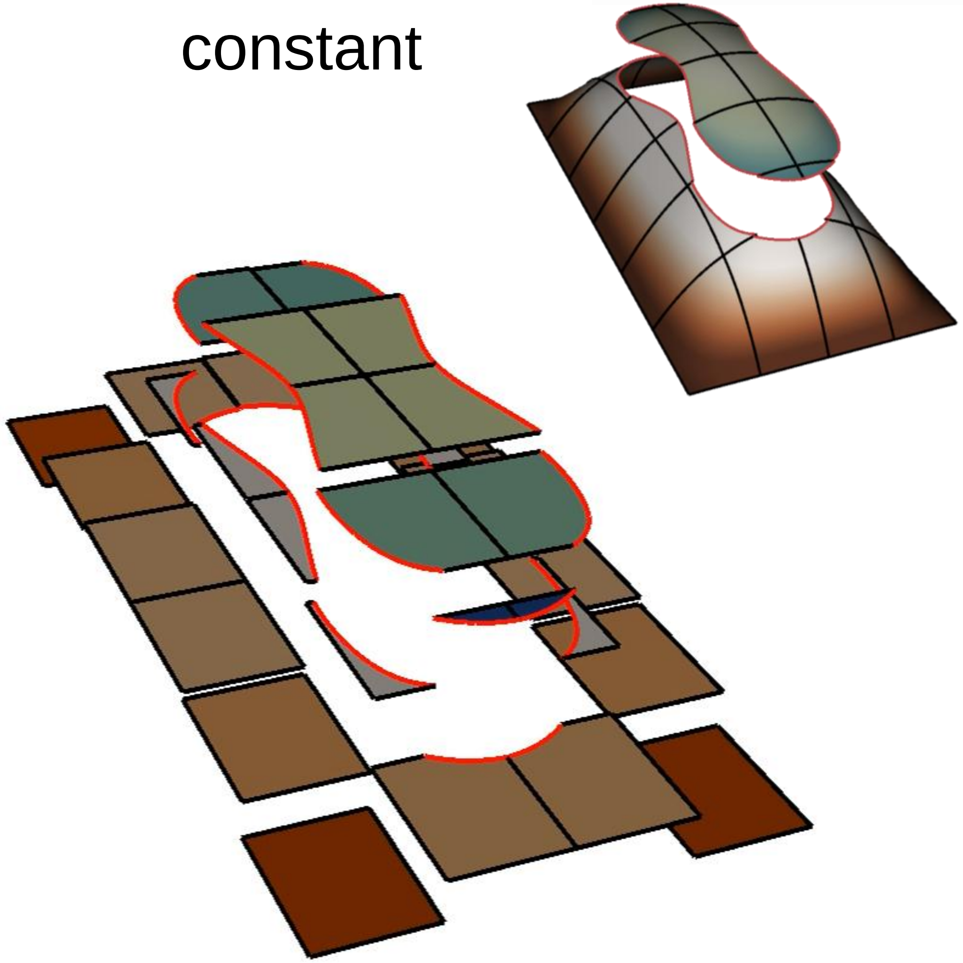
XDG Approximation

$$u(\vec{x}, t) = \sum_{n=1}^{N} [\tilde{u}_{A,n} \Phi_n(\vec{x}) \text{Heaviside}(-\varphi) + \\ + \tilde{u}_{B,n} \Phi_n(\vec{x}) \text{Heaviside}(+\varphi)]$$



SHARP REPRESENTATION OF INTERFACES: EXTENDED DISCONTINUOUS GALERKIN





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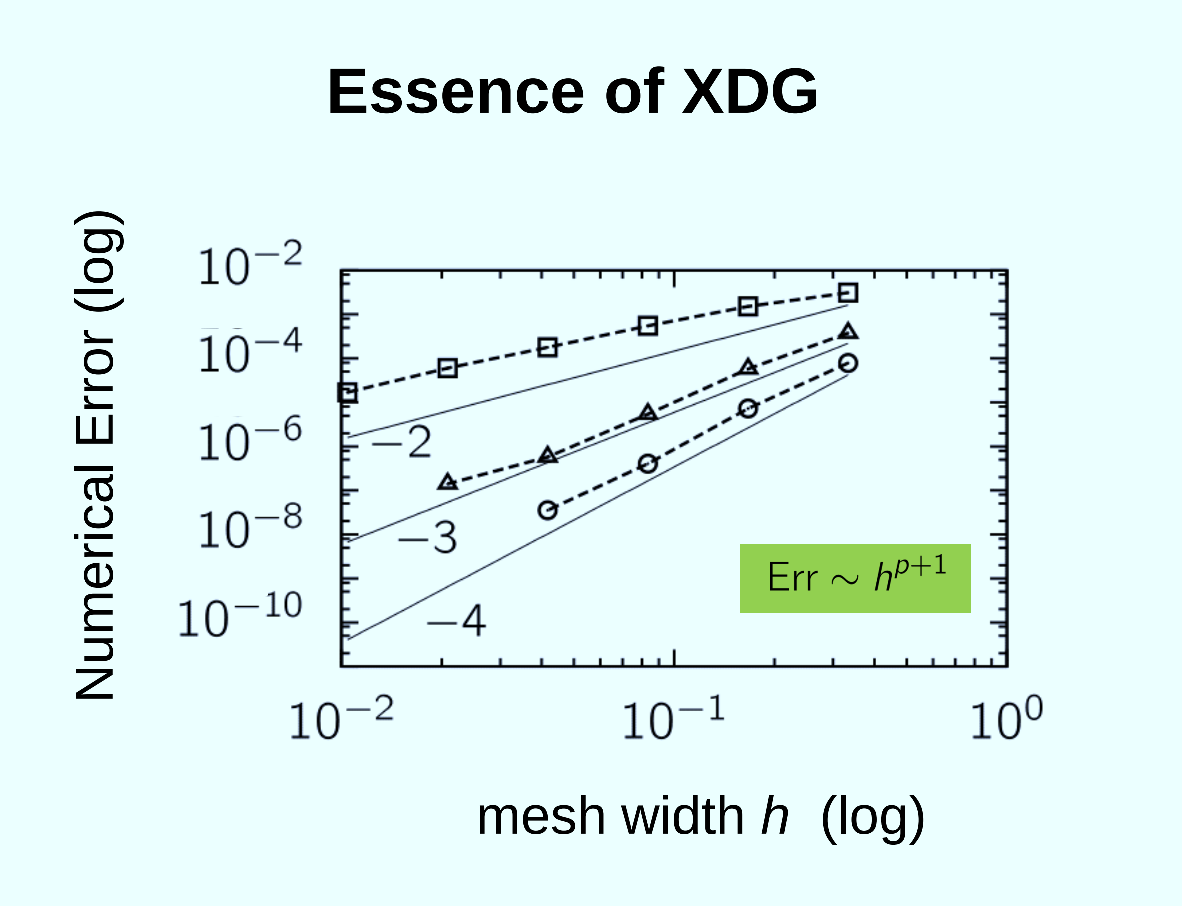
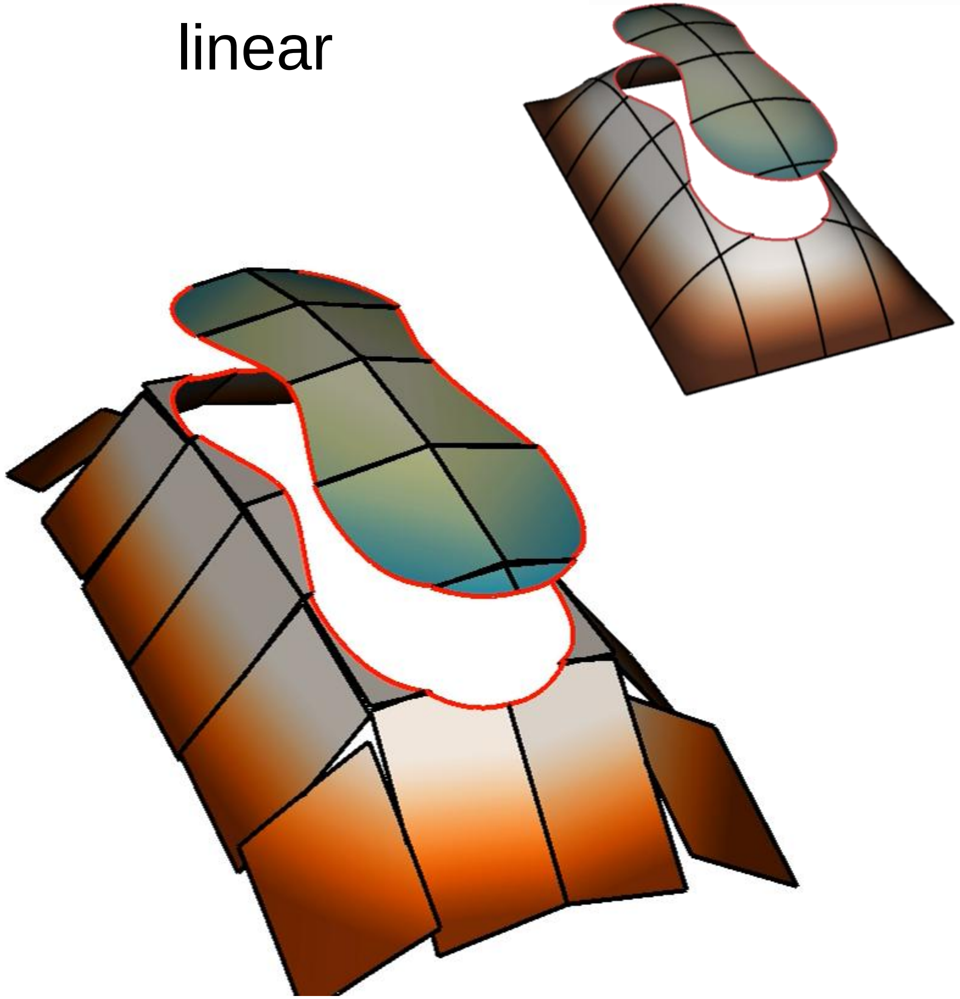
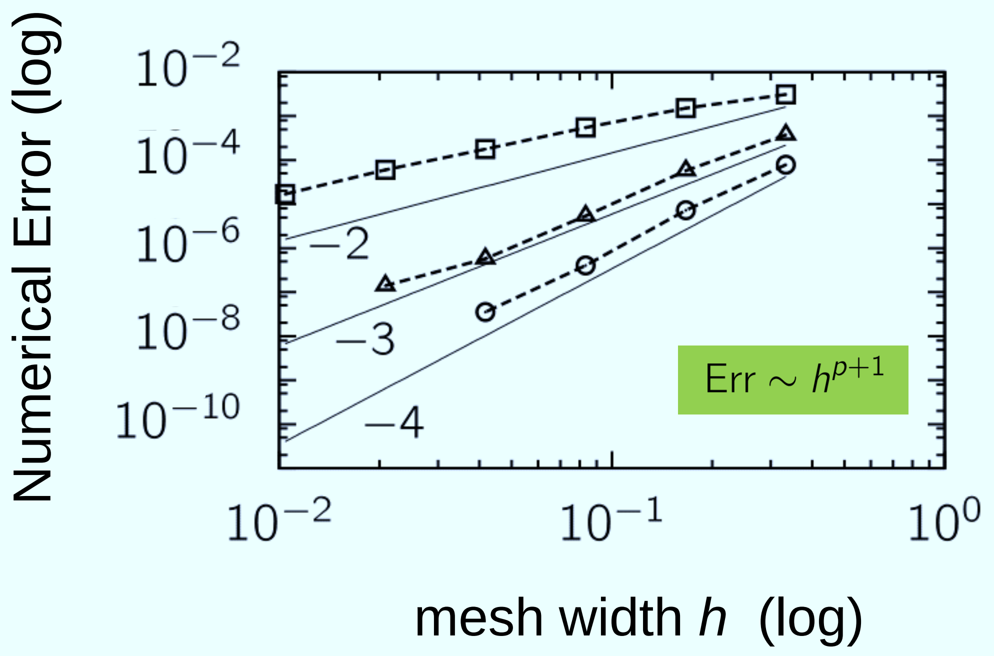


linear

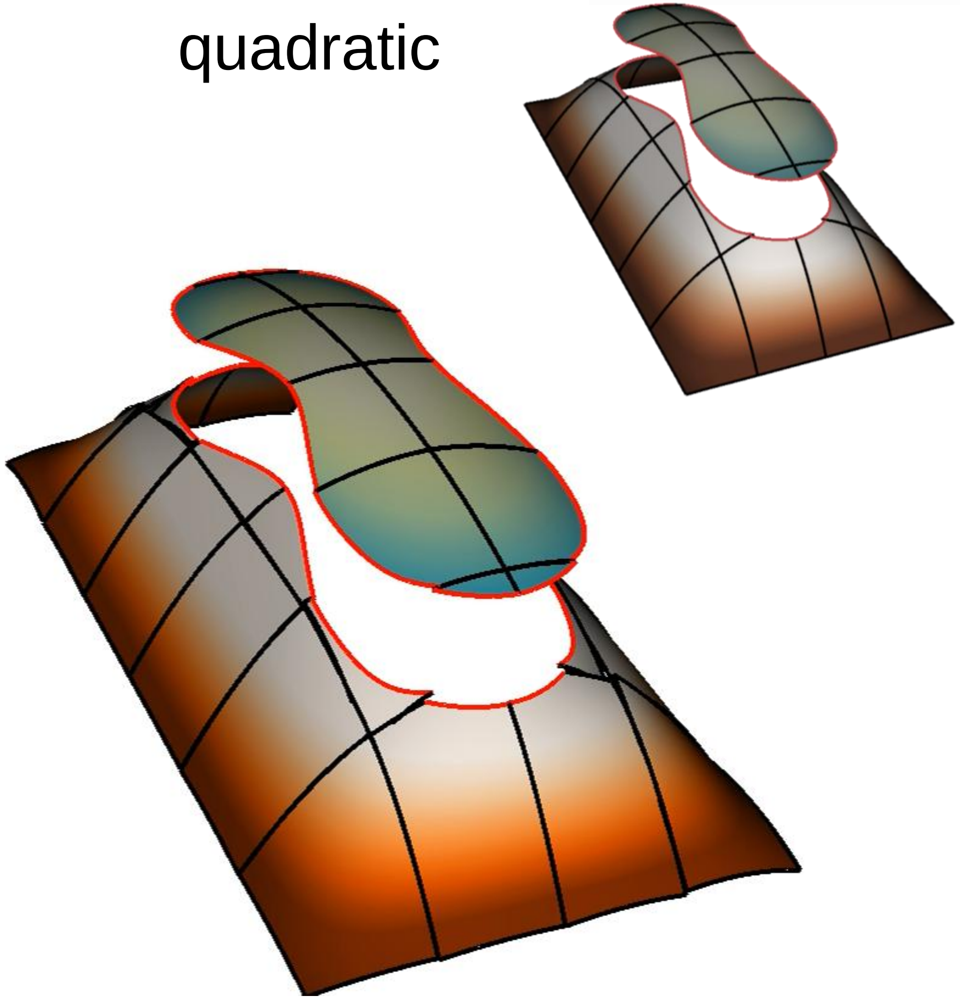


SHARP REPRESENTATION OF INTERFACES: EXTENDED DISCONTINUOUS GALERKIN

Essence of XDG



quadratic



SHARP REPRESENTATION OF INTERFACES: EXTENDED DISCONTINUOUS GALERKIN

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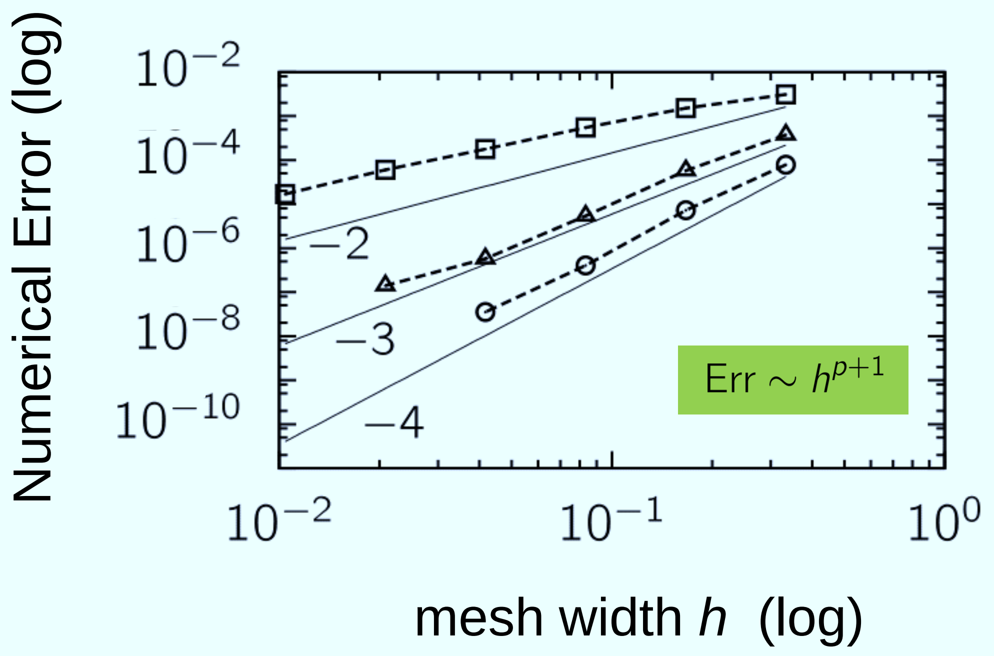
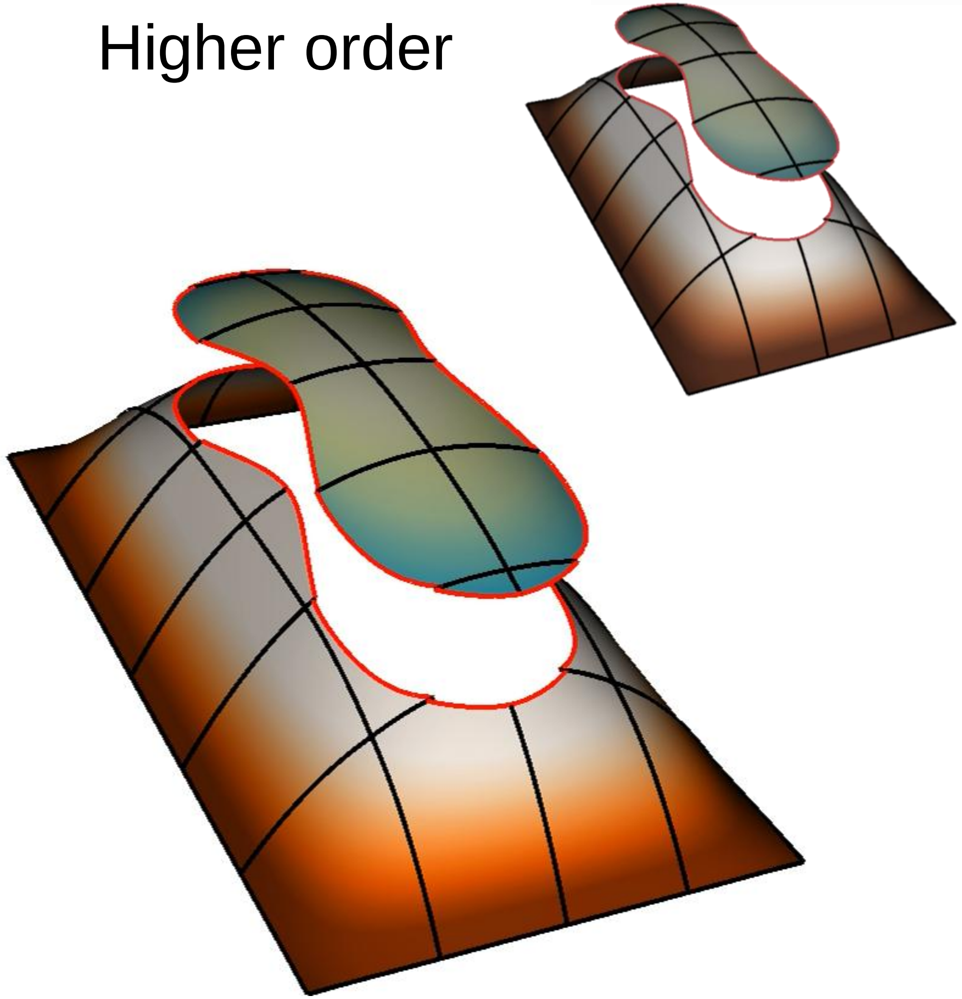


Higher order

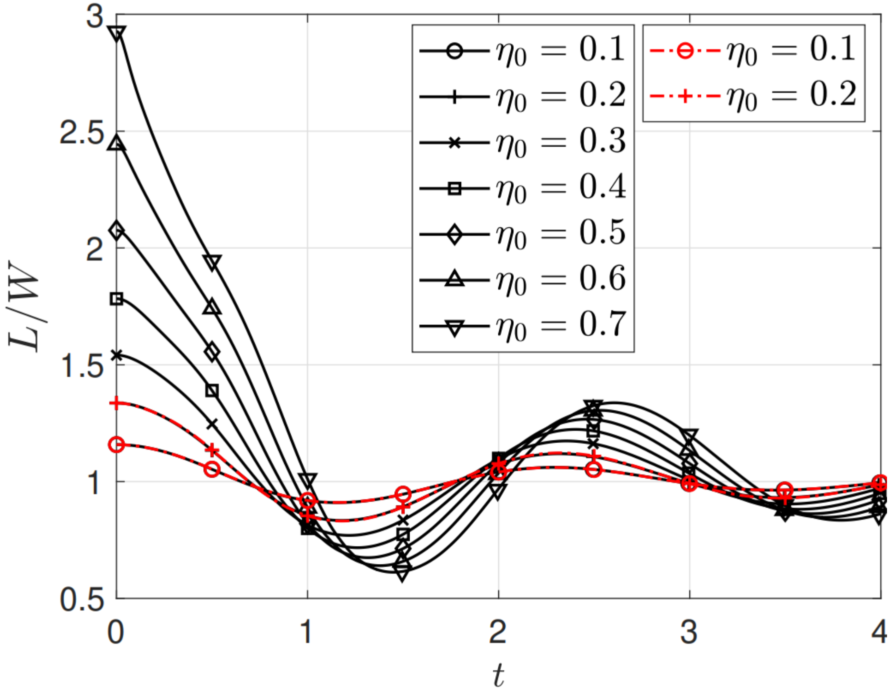


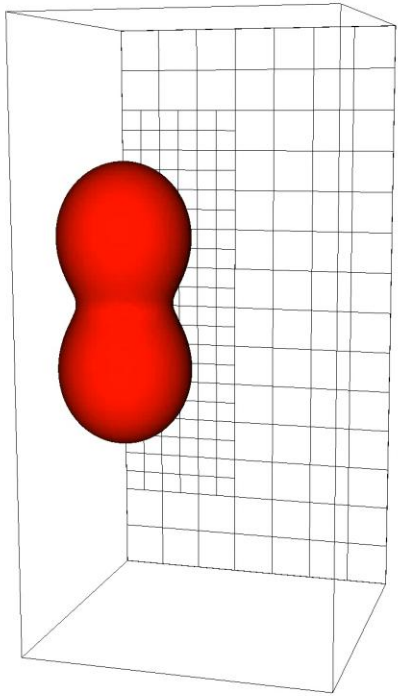
4 APPLICATIONS

- Oscillating Droplet
- Leidenfrost on Soft Substrate
- Shock Tracking in high-speed supersonic flows

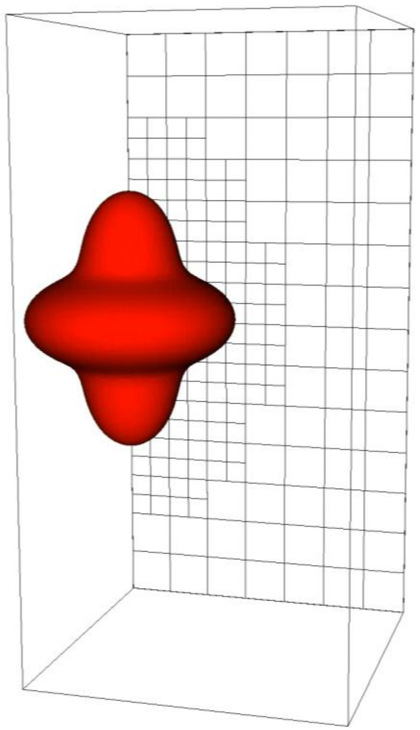
OSCILLATING DROPLET

Validation against WNLT





$m = 2, \eta_0 = 0.4$



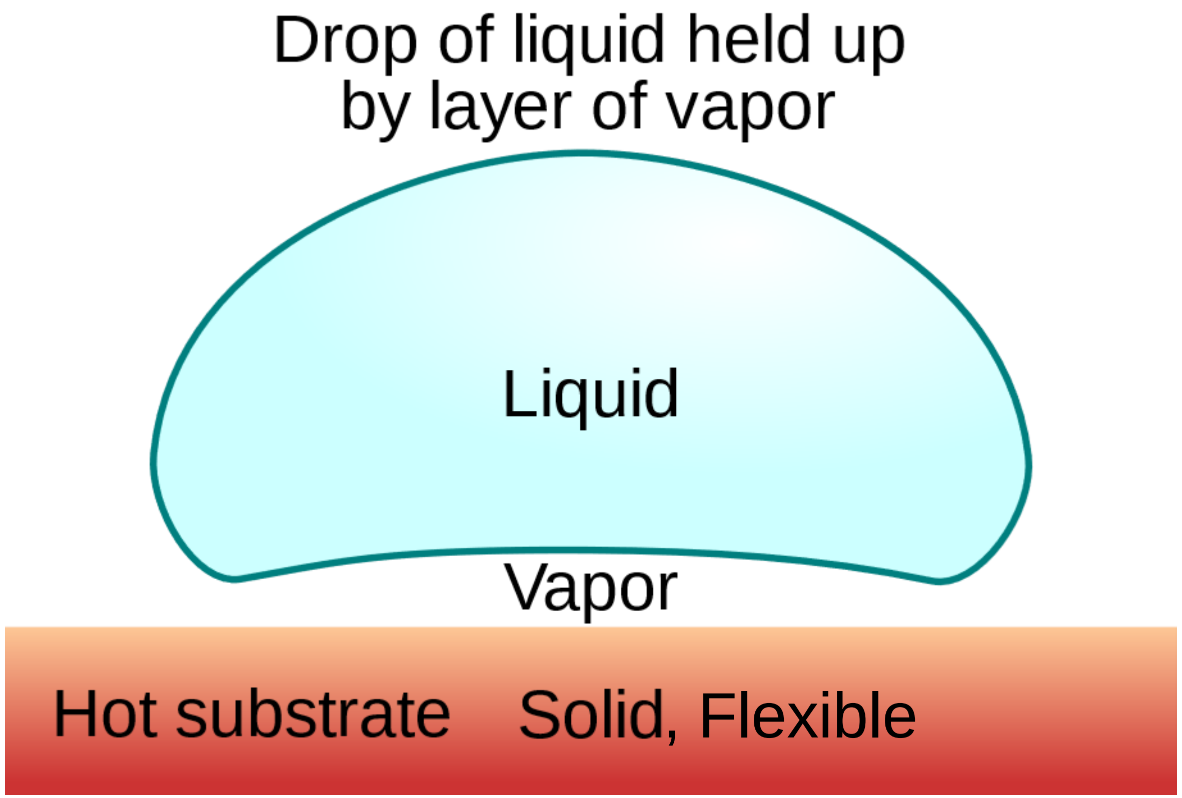
$m = 4, \eta_0 = 0.4$

- **Methodological Challenges:**
  - stable coupling between Level-Set and flow solver
  - adaptive mesh refinement and load balancing
- **Achievement:**
  - high accuracy on coarse meshes
  - verify against theory

LEIDENFROST ON SOFT SUBSTRATES

- **Application:**
  - Droplet levitating on own vapor on soft substrate
  - comparison of hard and soft substrates

- **Challenges**
  - Multi-Physics:
    - Multiphase
    - Heat Transfer and Evaporation
    - Fluid-Structure-Interaction

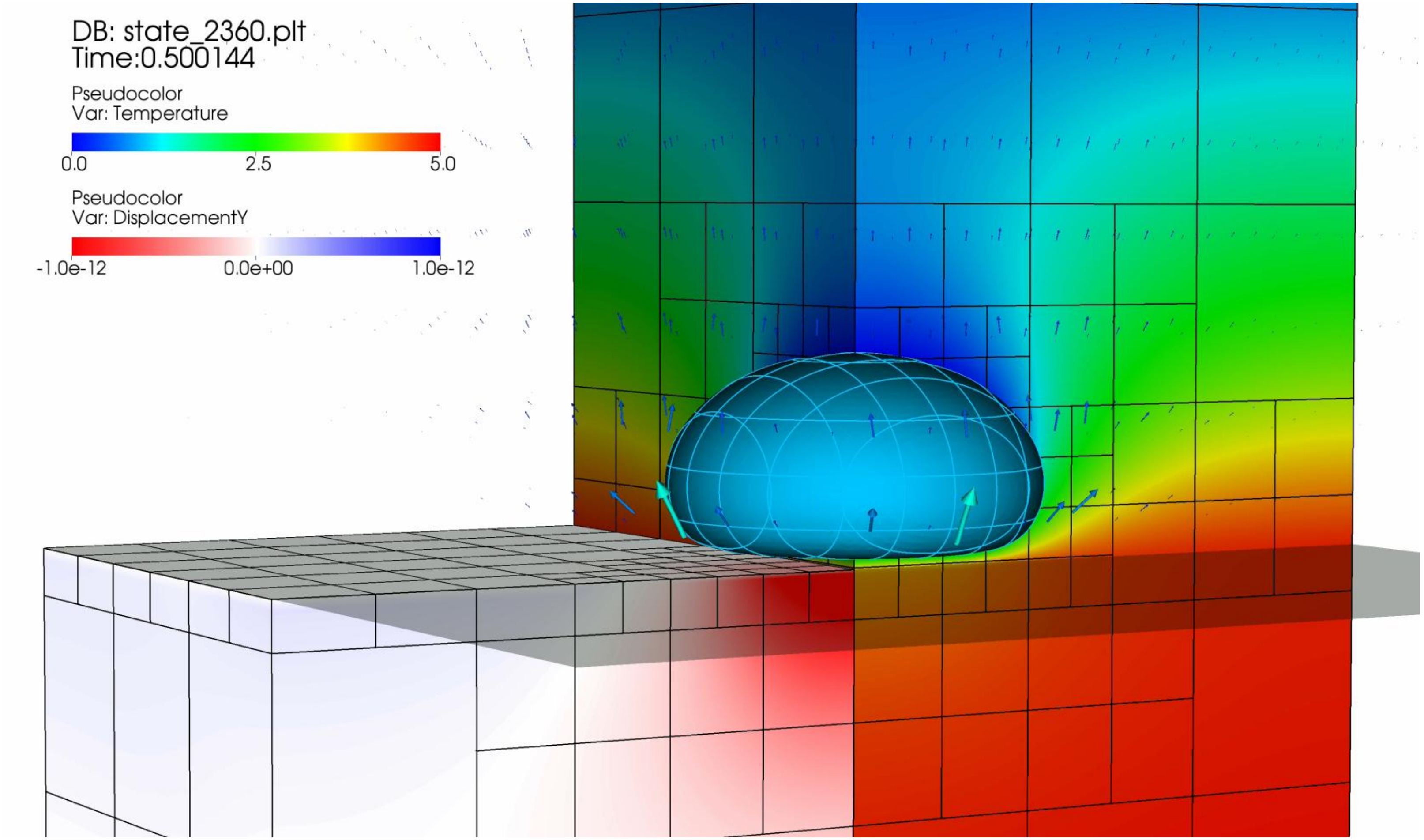


LEIDENFROST ON SOFT SUBSTRATES

Highlights:

- **Fully coupled, monolithic solver**
  - fluid-structure interaction,
  - heat transfer
  - phase transition

- **Two Level Sets!**
  - fluid-fluid boundary
  - fluid-solid boundary



SHOCK TRACKING IN HIGH-SPEED SUPERSONIC FLOWS

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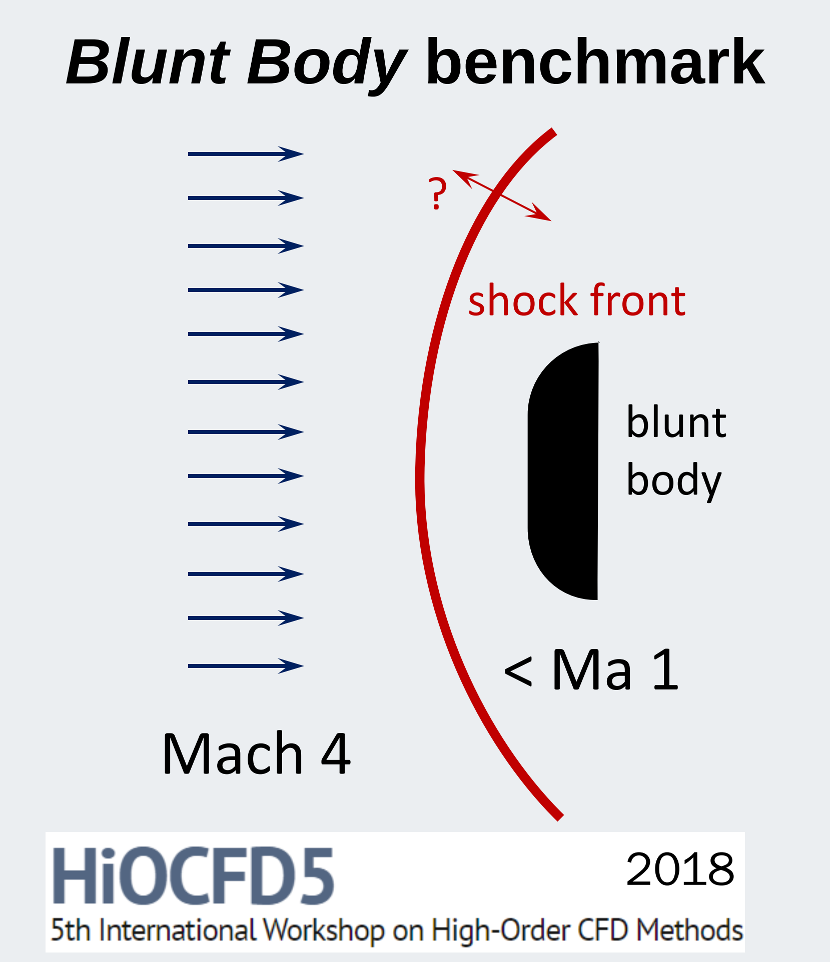
- **Application**
  - XDG to supersonic flows
  - Benchmark: blunt body

- **Methodological Challenge:**
  - convectional methods loose accuracy
  - **Nonlinear optimization problem:** shock location and flow solution



SHOCK TRACKING IN HIGH-SPEED SUPERSONIC FLOWS

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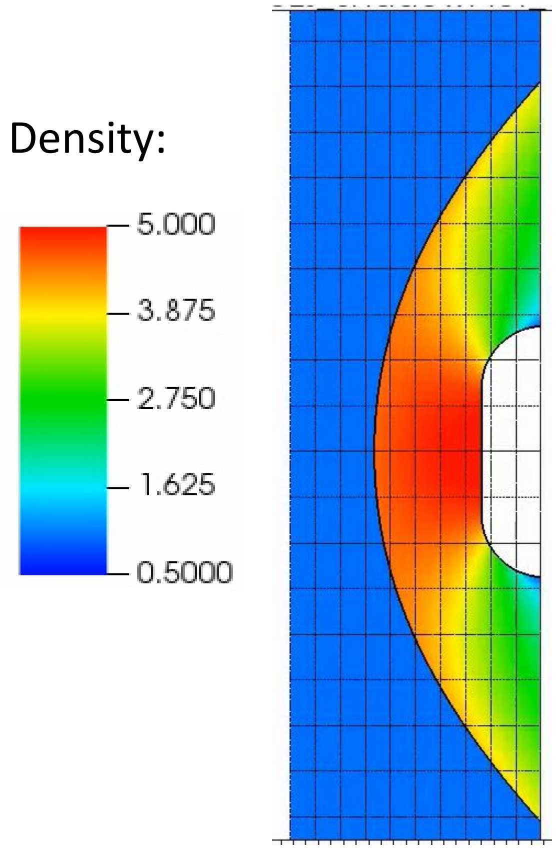
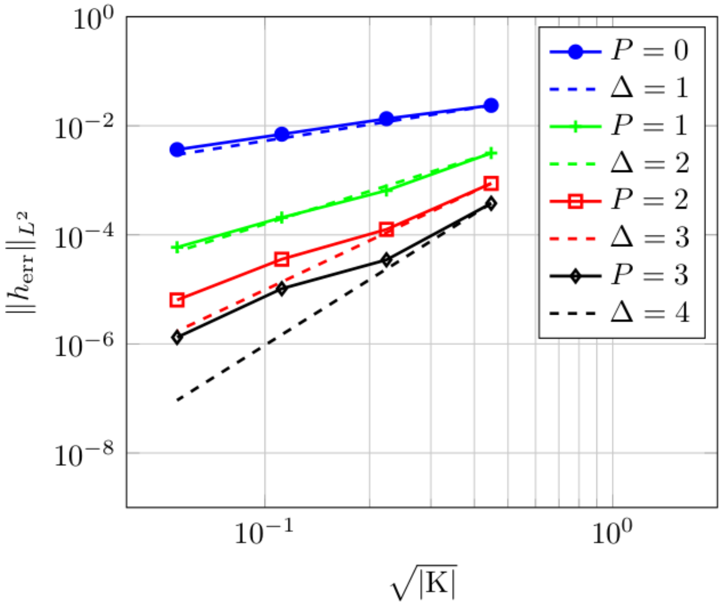
- h^p **accuracy for shock waves**
  - XDG is useful beyond Multi-Phase

- **Nonlinear optimization: shock location and flow solution**

Density:



Enthalpy Error Convergence:



Vandergrift *et.al.*, 2023

5 SUMMARY

SUMMARY

- DG methods are superior to Finite Volume
  - Much higher accuracy
  - (Not in this talk: Much better suited for modern computer architectures)
- Standard DG **fails** for simulations with interfaces
  - Core Problem: mesh/basis is not conformal
- XDG fixes this!
  - Conformal Polynomial Basis

➢ XDG makes DG much more accessible for general simulations
  - Moving Interfaces and Discontinuities
  - Static Geometry: replace expensive meshing with Level-Set representation

BOSSS IS OPEN SOURCE

get your copy @ github.com/FDYdarmstadt/BoSSS

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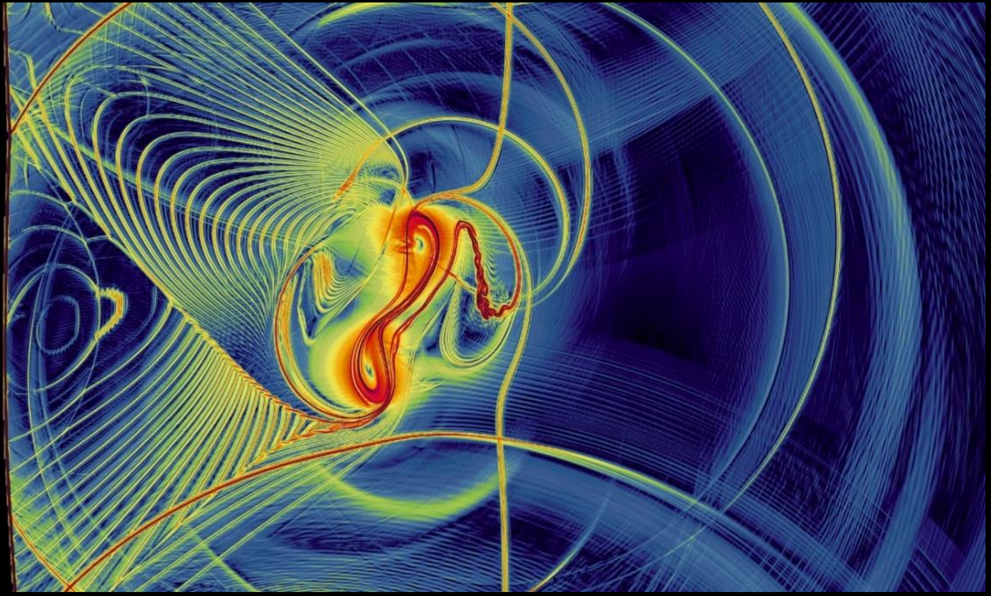
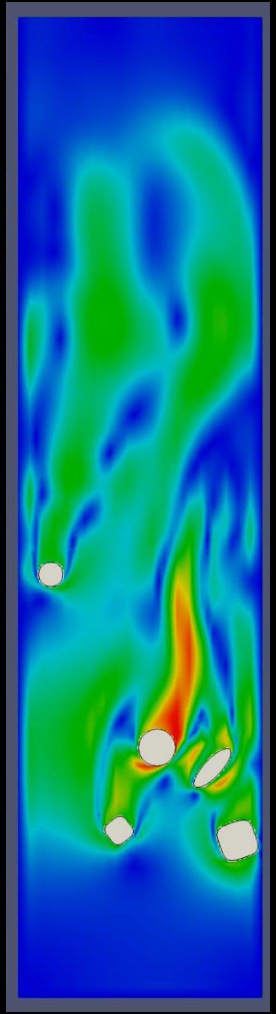
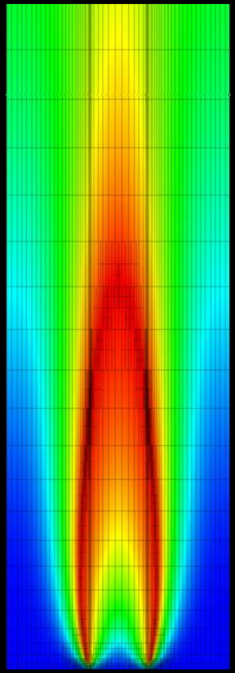
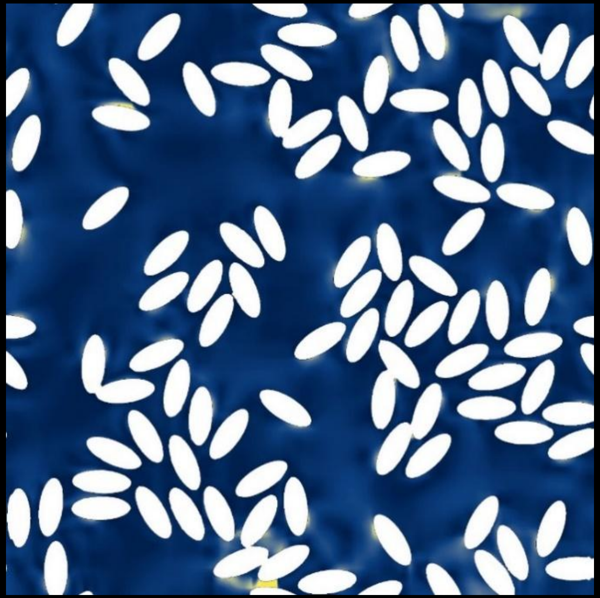






◁ combustion



k-ω ▽ compressible △ IBM ▷

▽ Voronoi △ multi-body