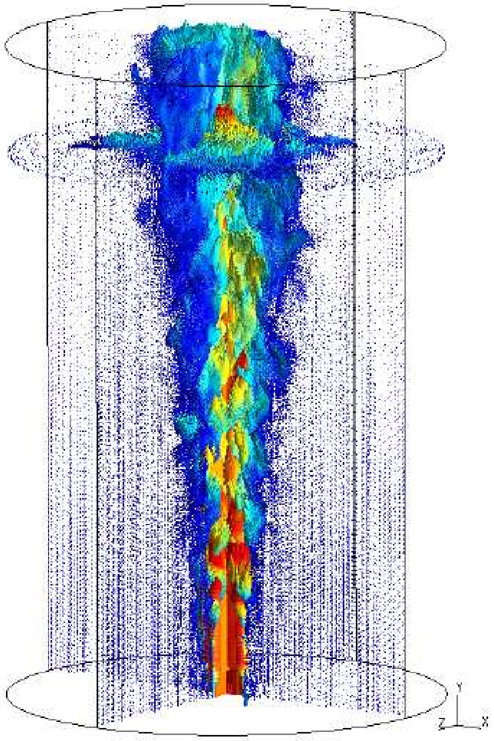

Discontinuous Galerkin Method
Implementation and application to various turbulent flows

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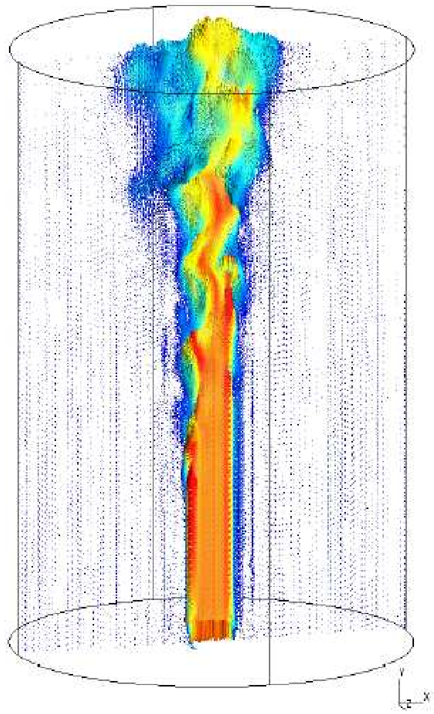
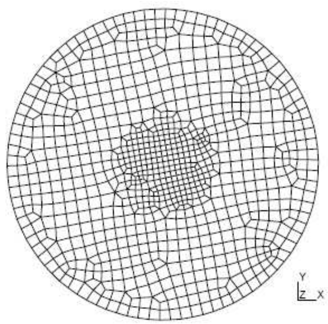
Presentation plan

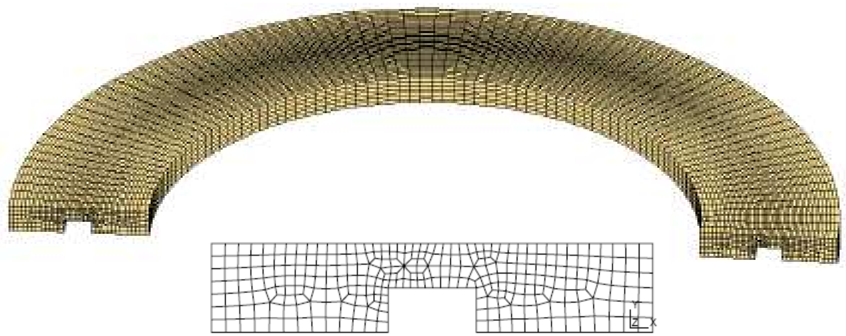
- Introduction
- Discontinuous Galerkin method for Navier-Stokes equation
- Test cases (cavity flow, shear layer)
- Sample simulations of turbulent flows
  - Free, round jet
  - Flow between rotating disks
    - Smooth disks
    - Disks with a step
- Summary



Introduction

- Experience with high-order methods on structured grids (SAILOR code)
  - Spectral, pseudospectral, compact finite difference
  - LES (Large Eddy simulation) of jet flows, shear flows, combustion, wall bounded flows in simple (academic) geometries
- Development of DioGenes code (Discontinuous Galerkin for Navier-Stokes equation)
  - Incompressible viscous flow (projection method)
  - Unstructured grids (2D and 3D) – quadrilateral and hexahedral elements

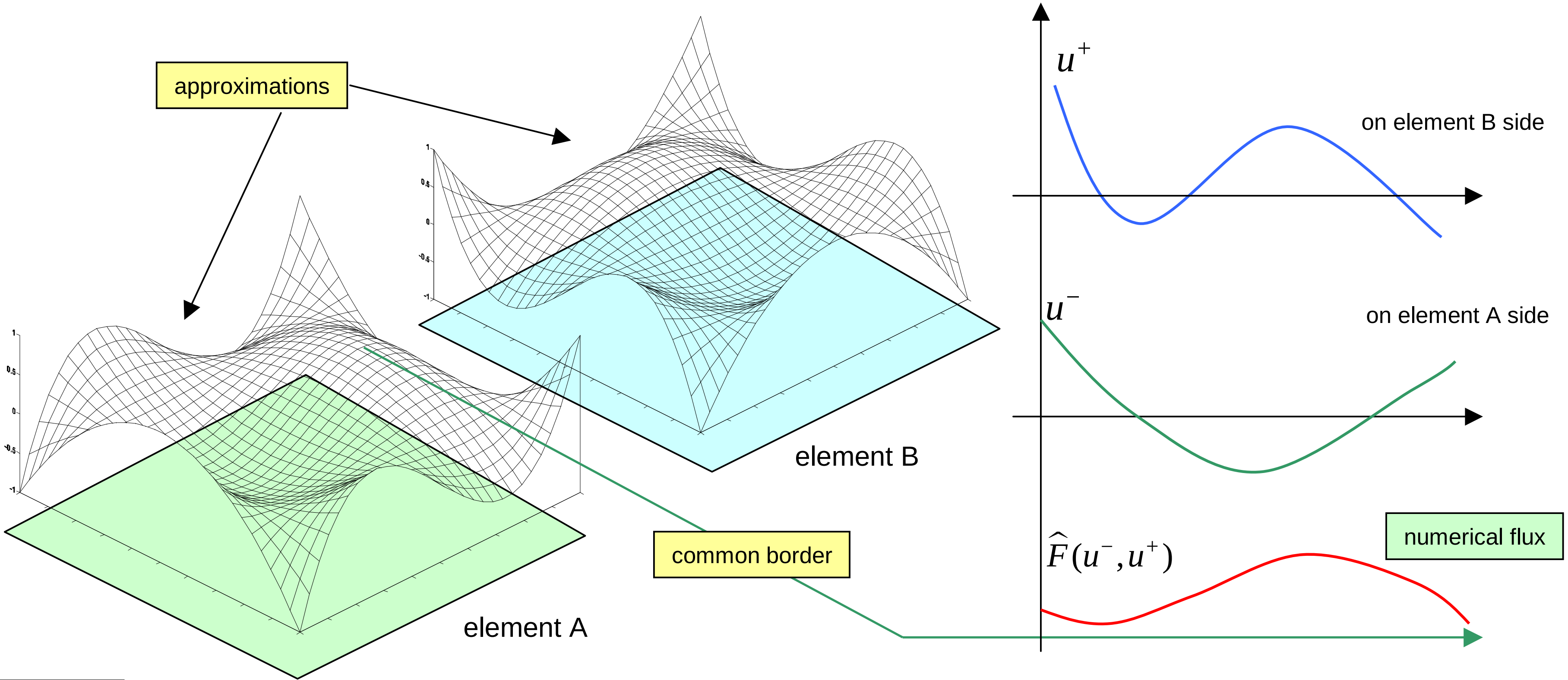


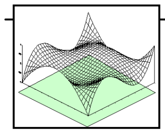


  - Explicit in time (Runge-Kutta DG, 1-3 order)
  - Parallelised (MPI)
    - Seperate module – partitioner – developed for organization of communication between processors

Discontinuous Galerkin method (DGM)

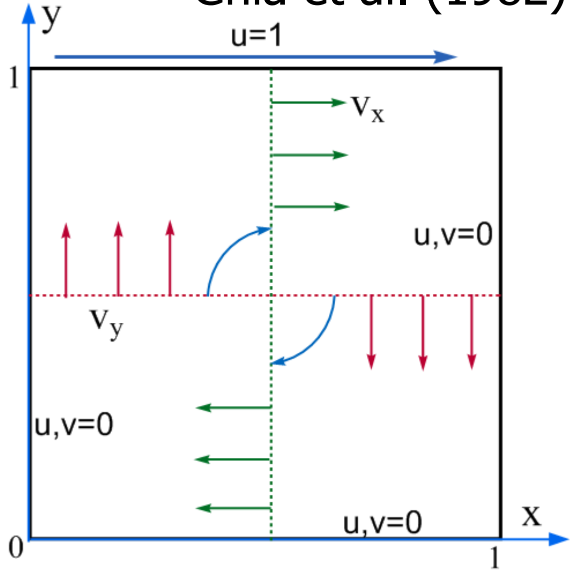
- Belongs to the family of FEM (Finite Element) methods, but share some properties with FVM (Finite Volume)
- Features:
  - Discontinuous basis functions (discontinuity on the element border)
  - Numerical flux for coupling of the elements (FVM concept)

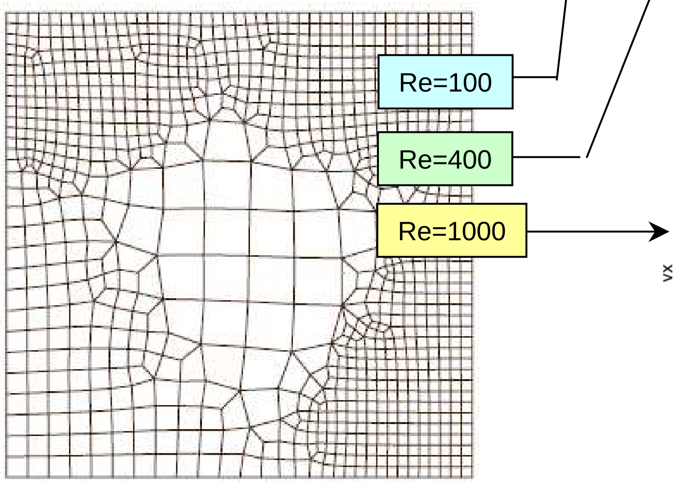


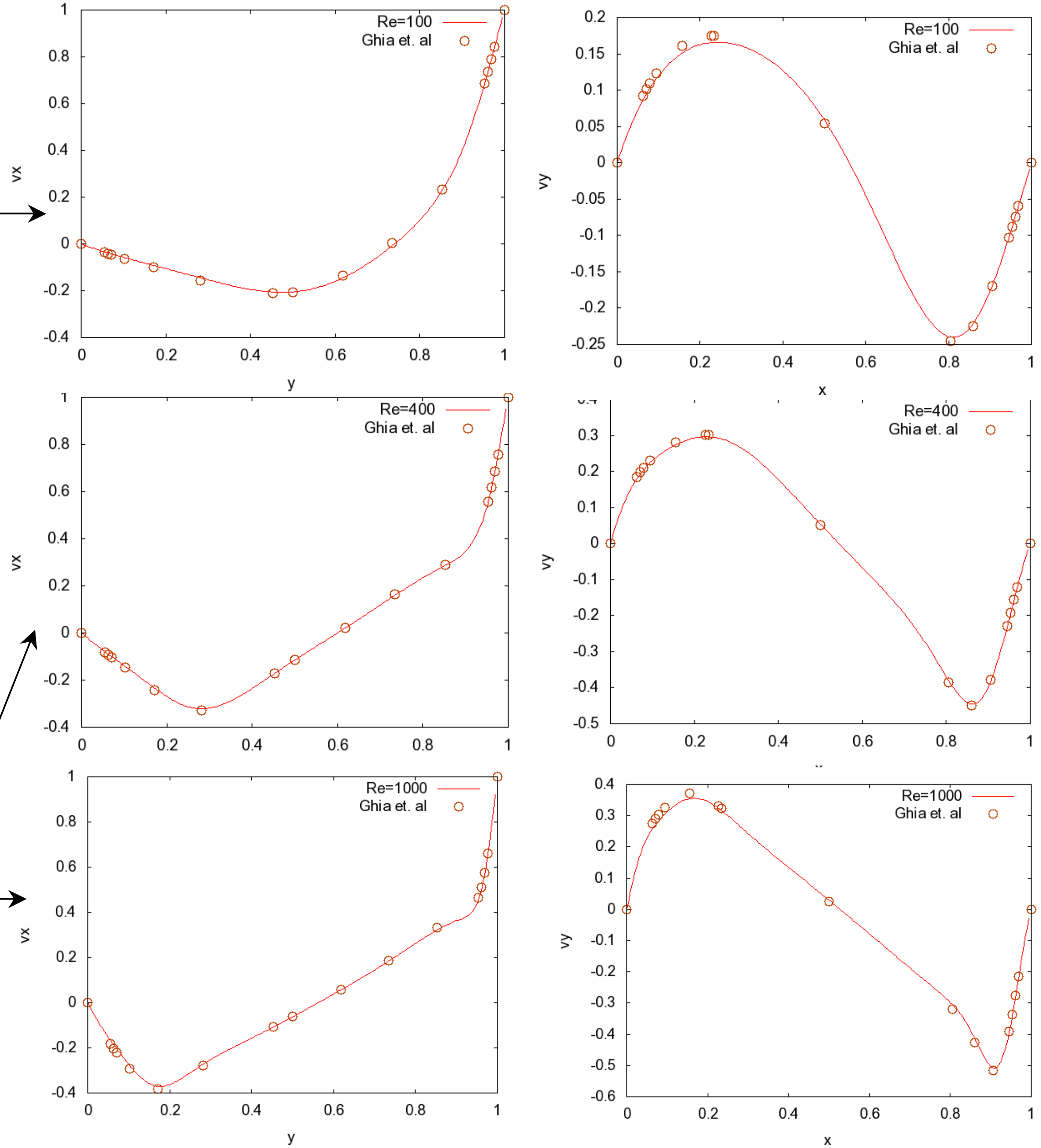


Cavity flow

- Verification of the code
  - Ghia et al. (1982)

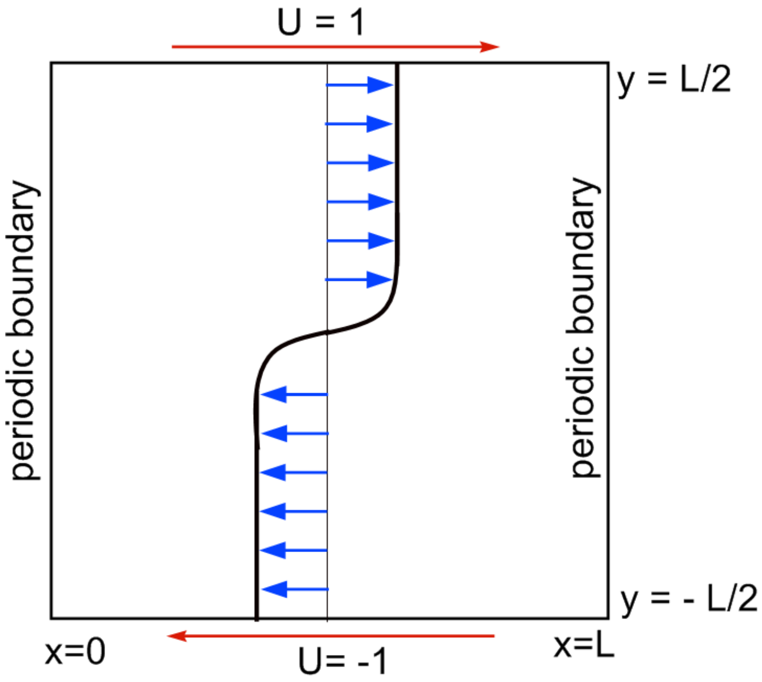






Temporal shear layer

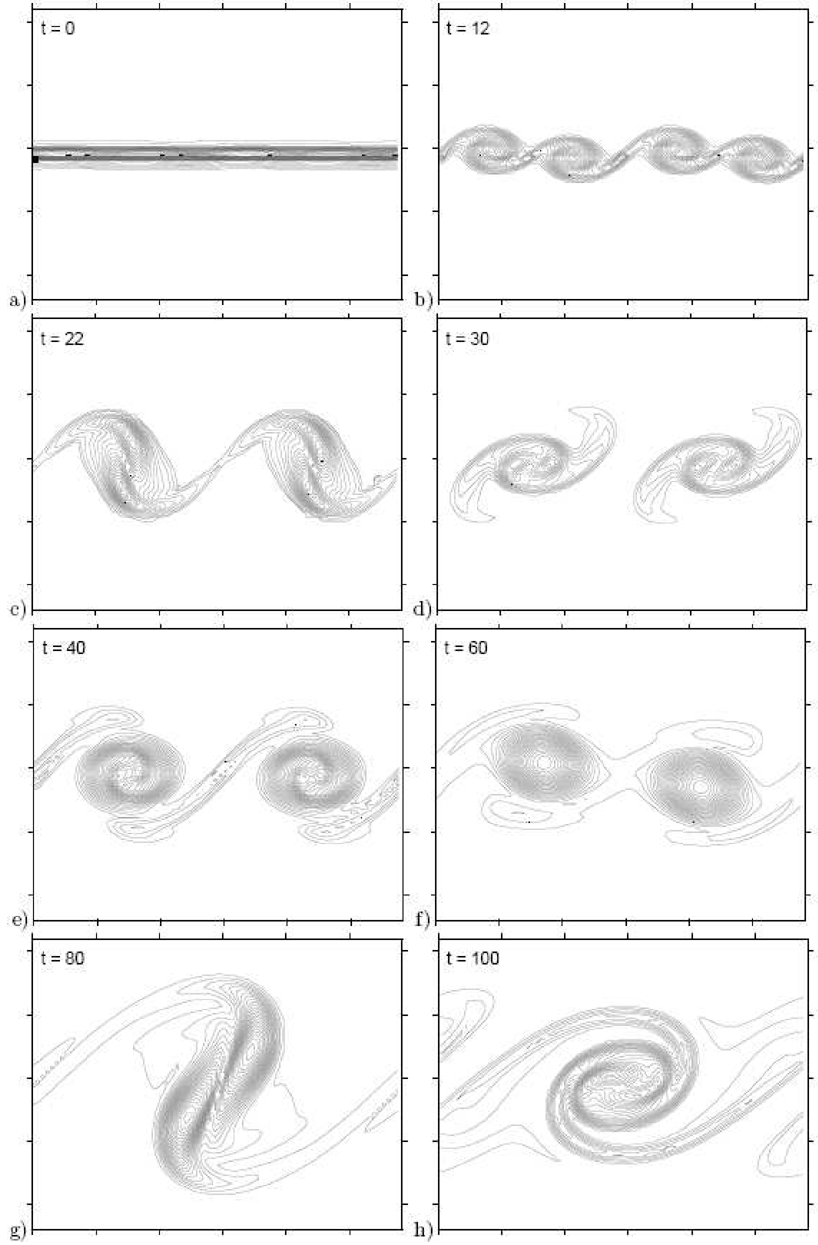
- Simple turbulent flow



Amplitudes of unstable modes

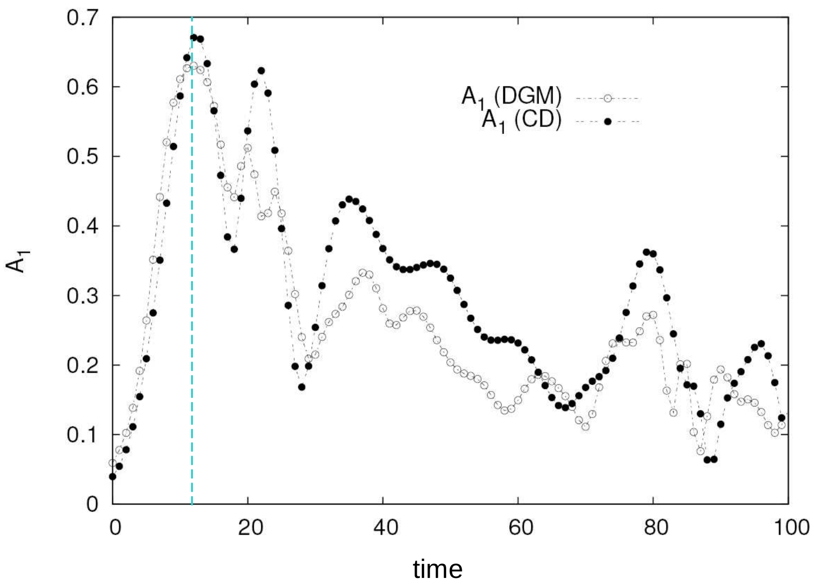
$$A_\alpha = \left[\int_{-L/2}^{+L/2} 2\hat{u}_x(\alpha)\hat{u}_x^*(\alpha) dy \right]^{\frac{1}{2}}$$

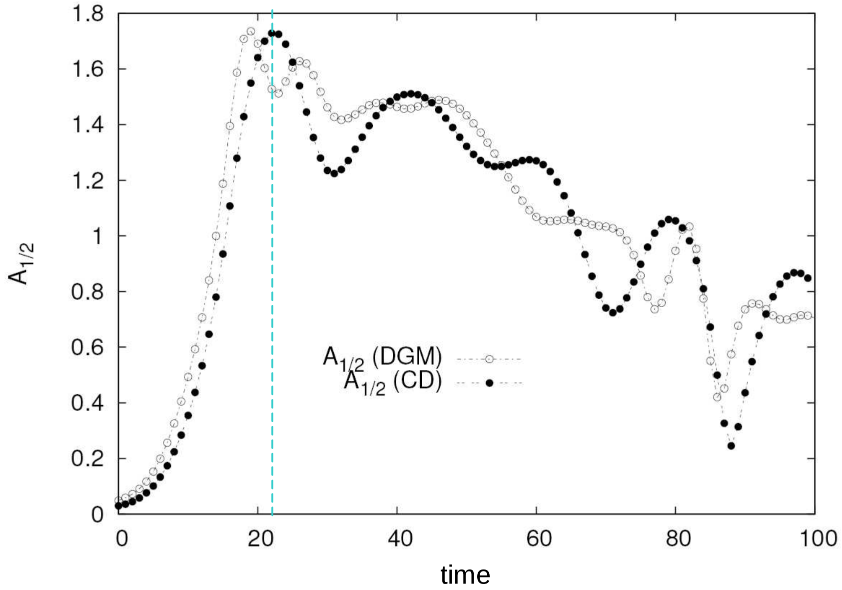
Vorticity isocontours - DGM (80x80, P=1)

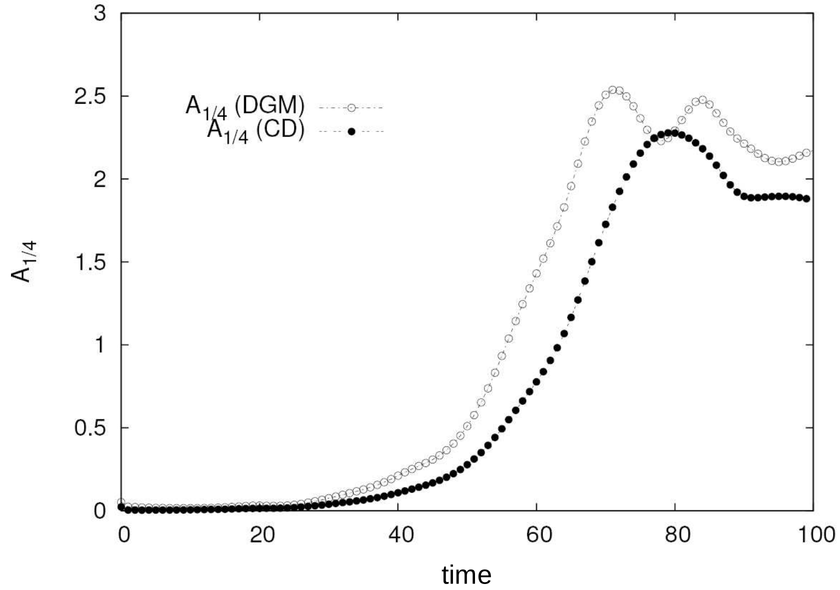


Temporal shear layer

- Evolution of unstable modes







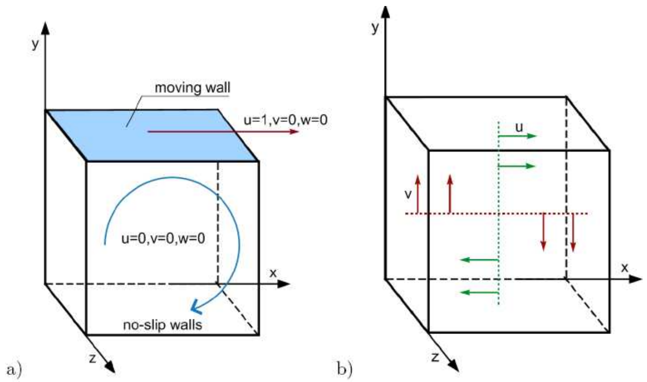
Times of max. ampl. (Moser, Rogers, 1993)

$t_1 = 11.9$	$t_{1/2} = 22.5$

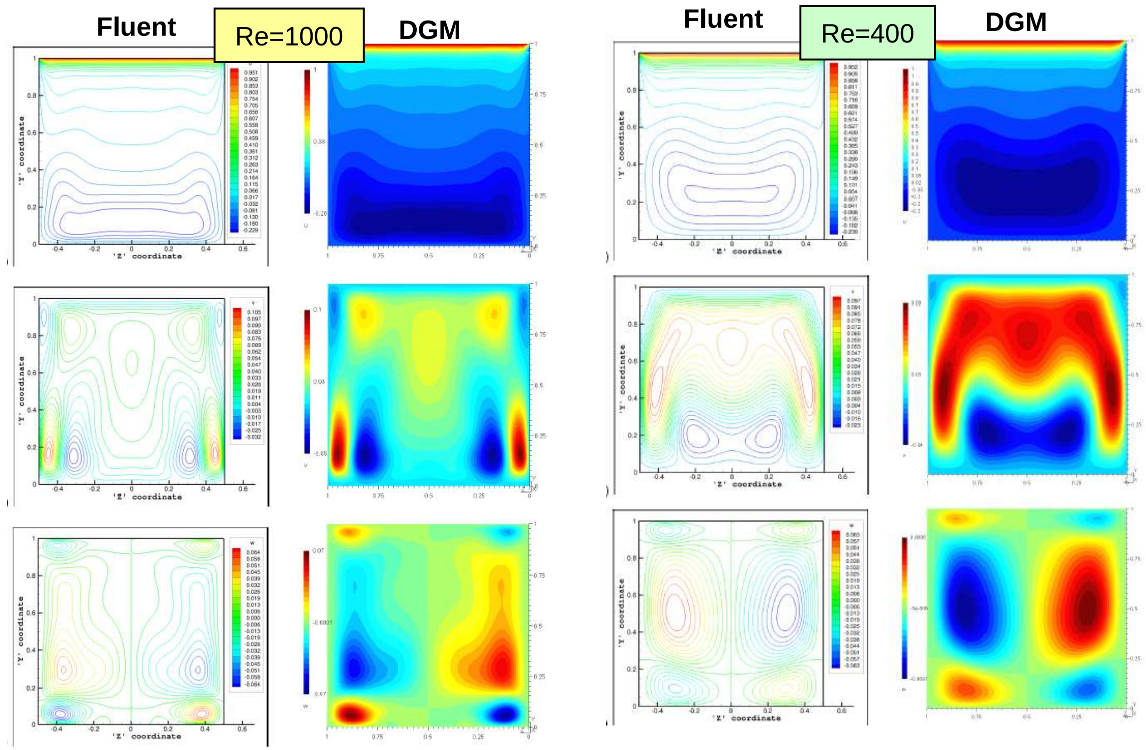
CD – 128x128, Re=250
DGM – 80x80, P=1

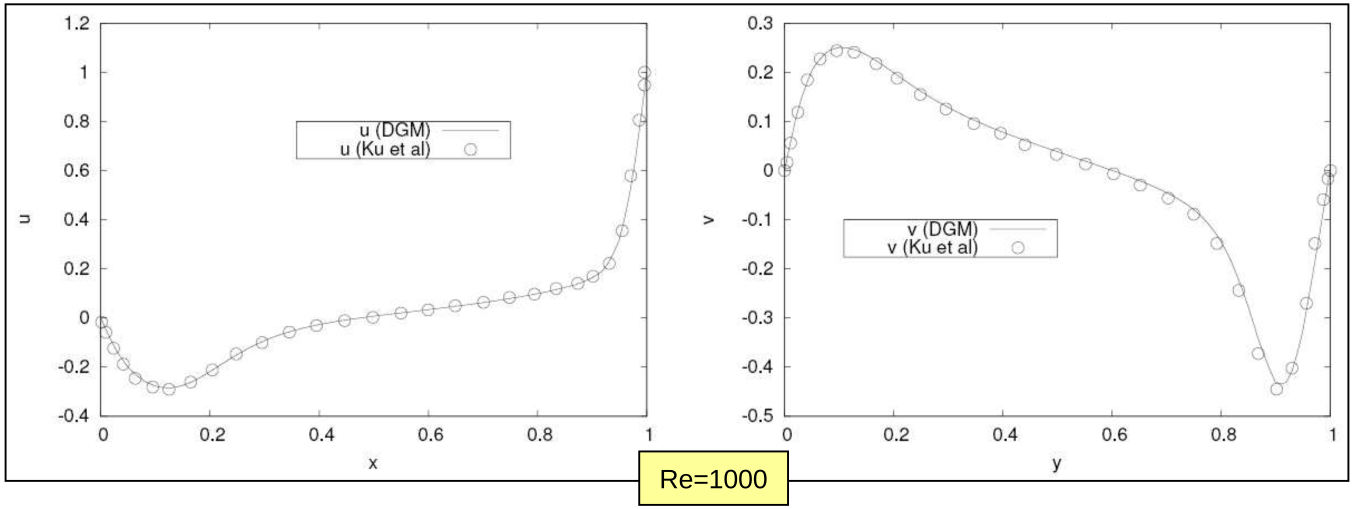
Cavity flow (3D)

- Verification of the code
  - Ku et al. (1987)



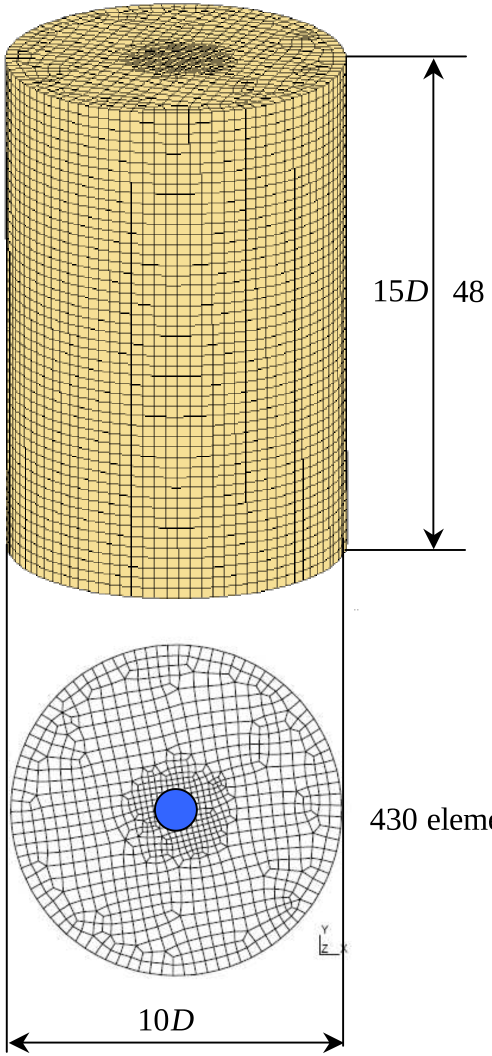
$20 \times 20 \times 20$ elements, $P = 2$





Simulation of a round jet

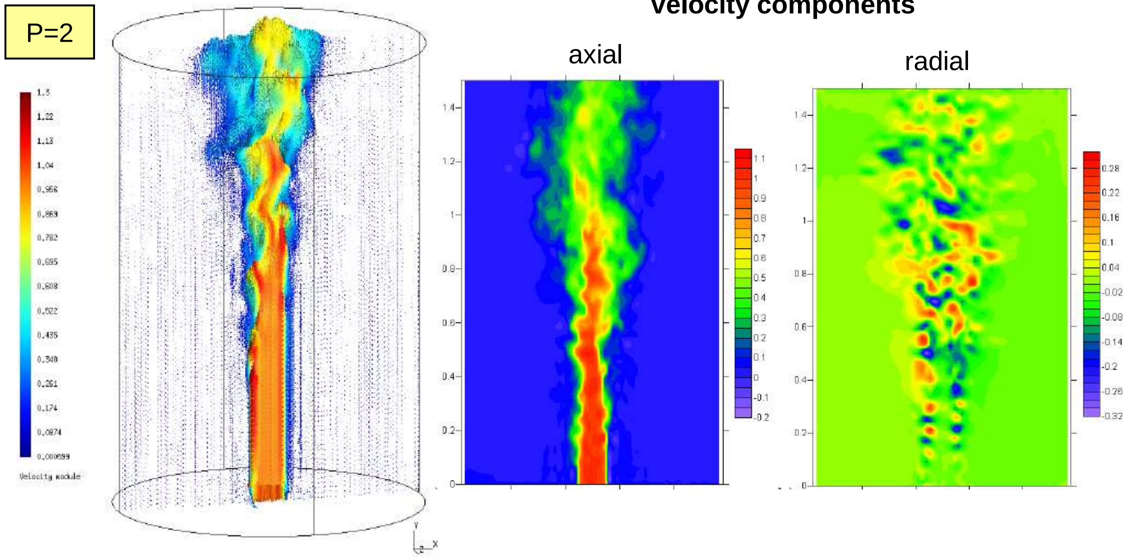
20640 el. (total)

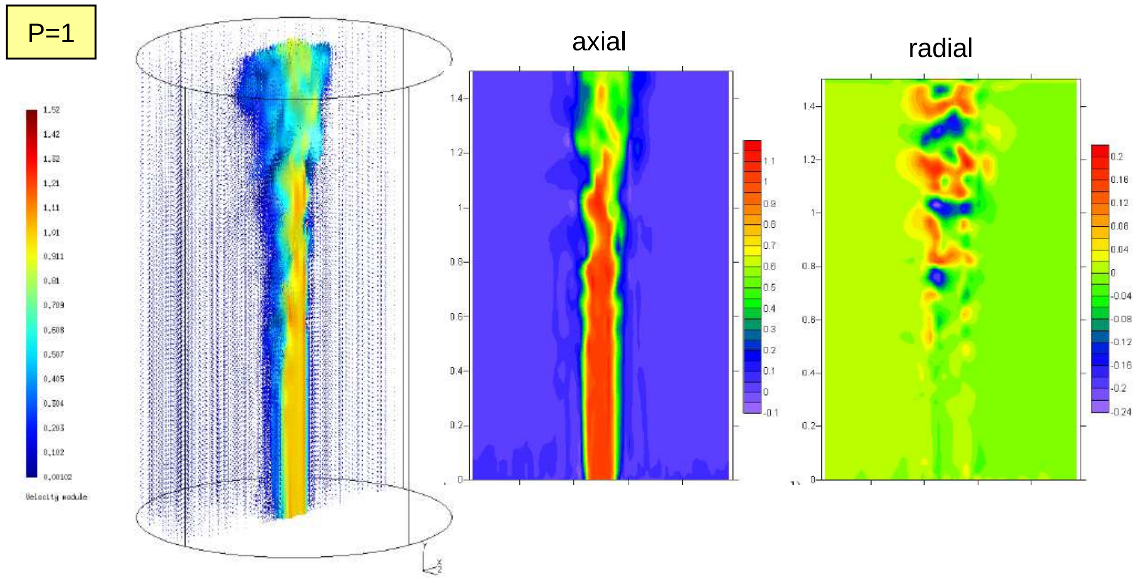


$$\mathrm{Re} = 20000$$

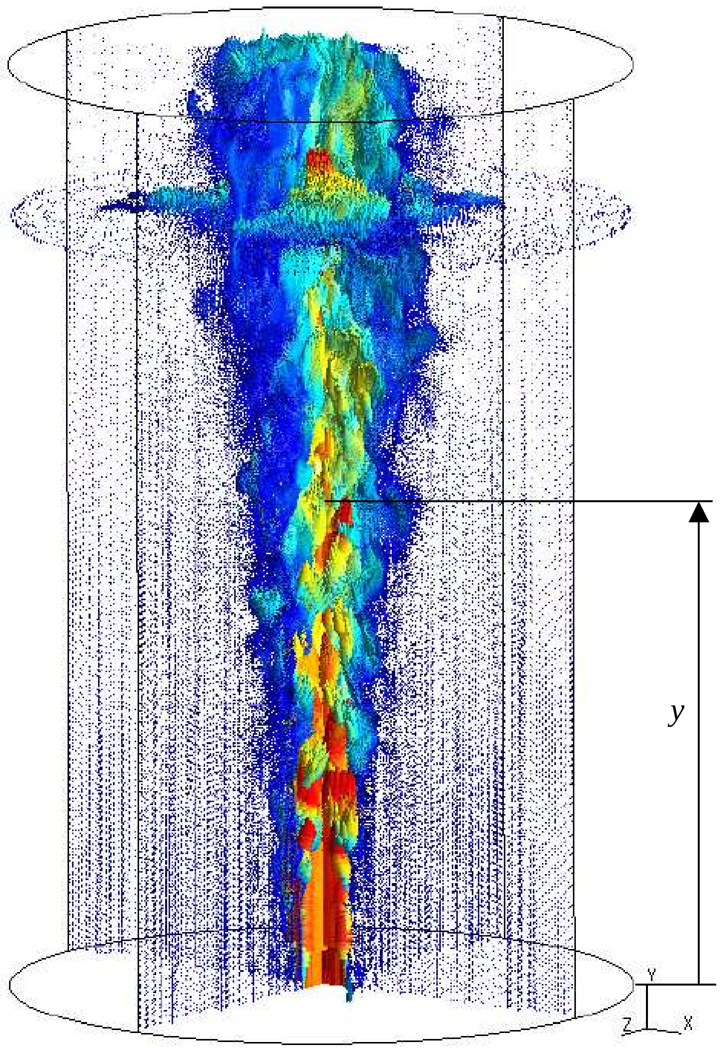
$$\frac{R}{\theta} = 20$$

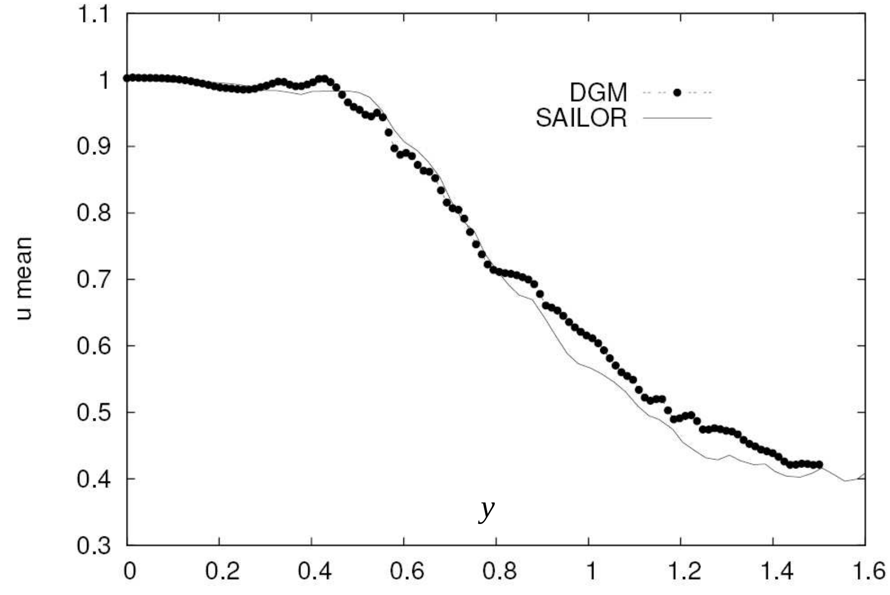
velocity components

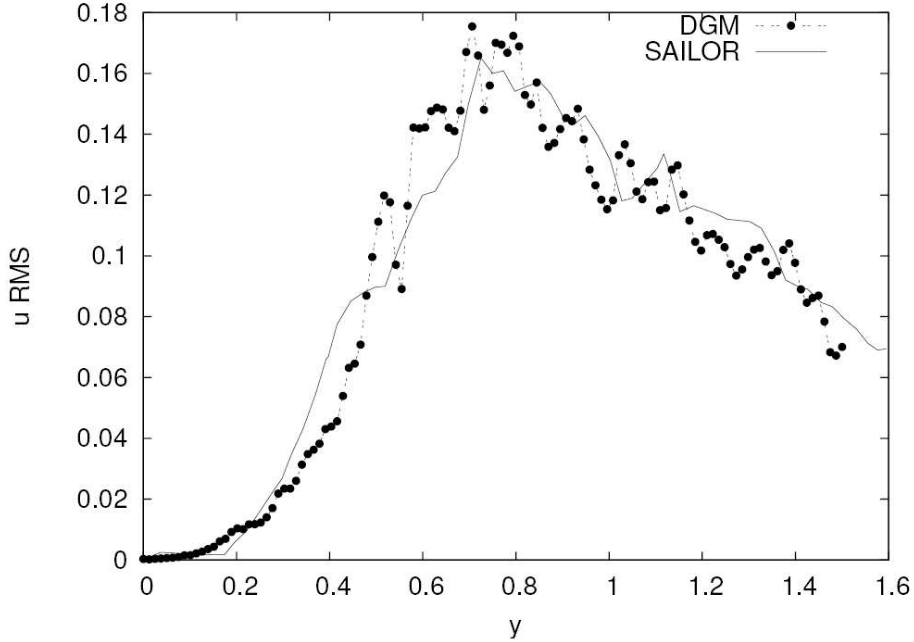




Simulation of a round jet

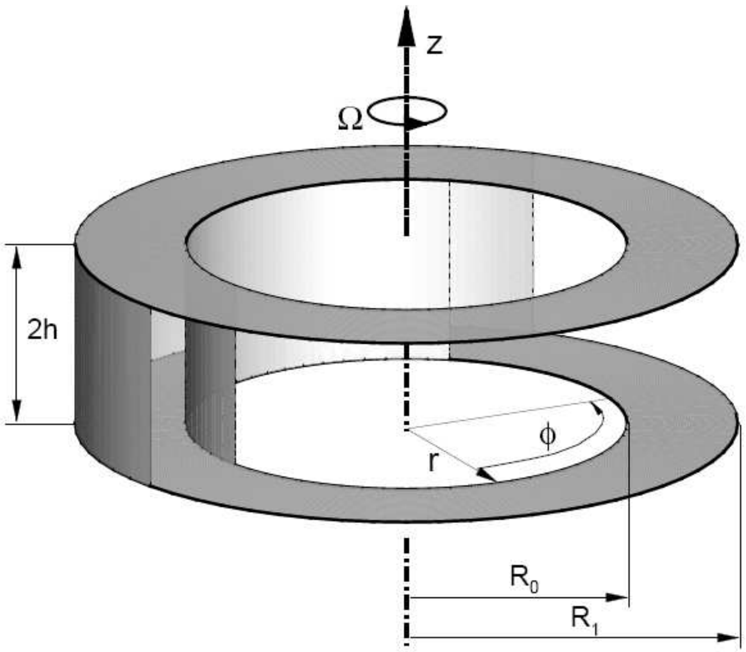






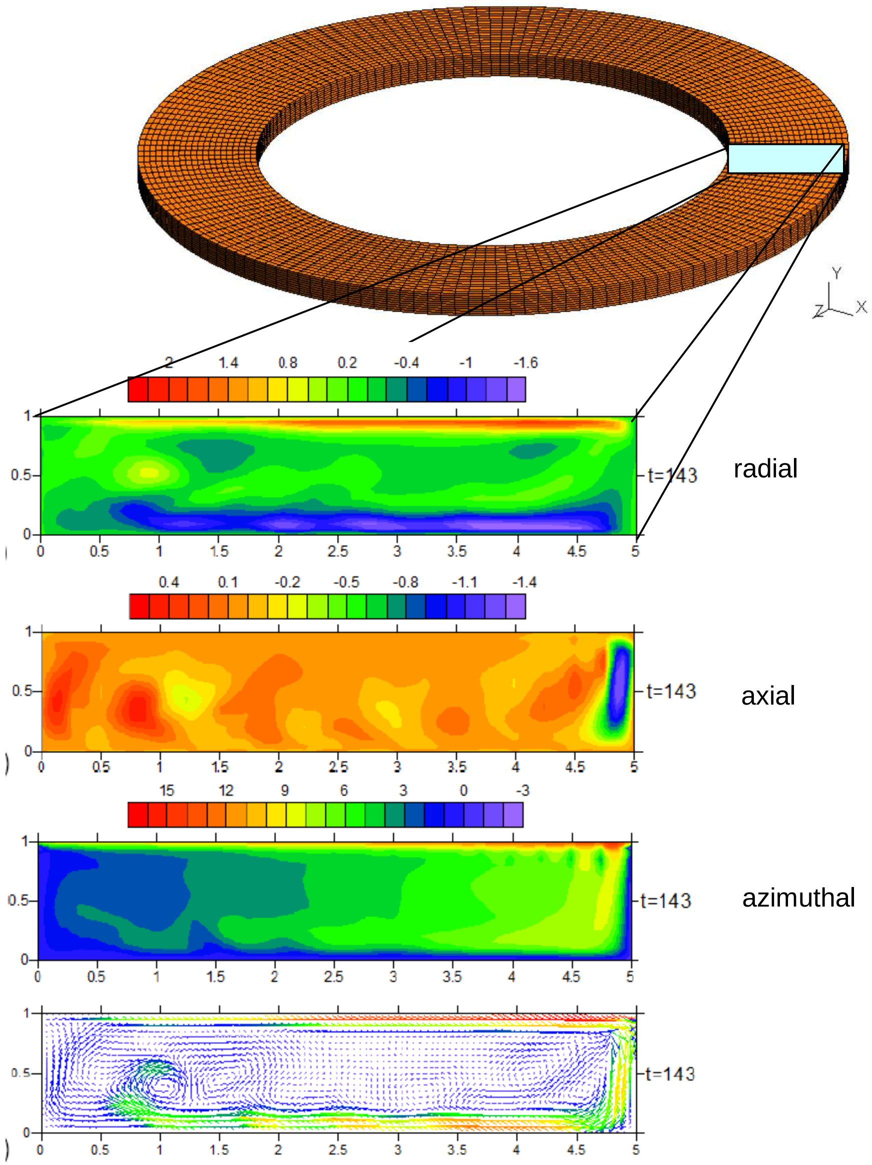
Flow between rotating disks (smooth)

$20{\times}160{\times}10$ elements, $P = 2$



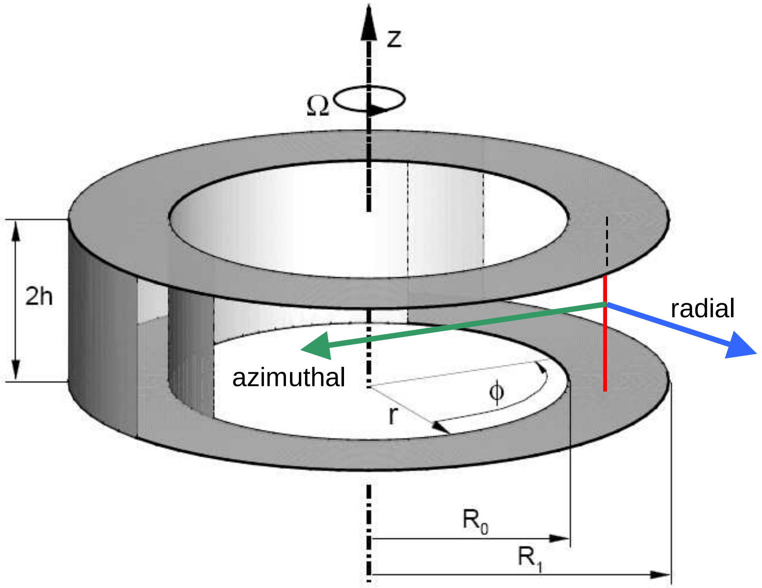
$$\mathrm{Re} = \frac{\Omega R_1^2}{\nu} = 70000$$

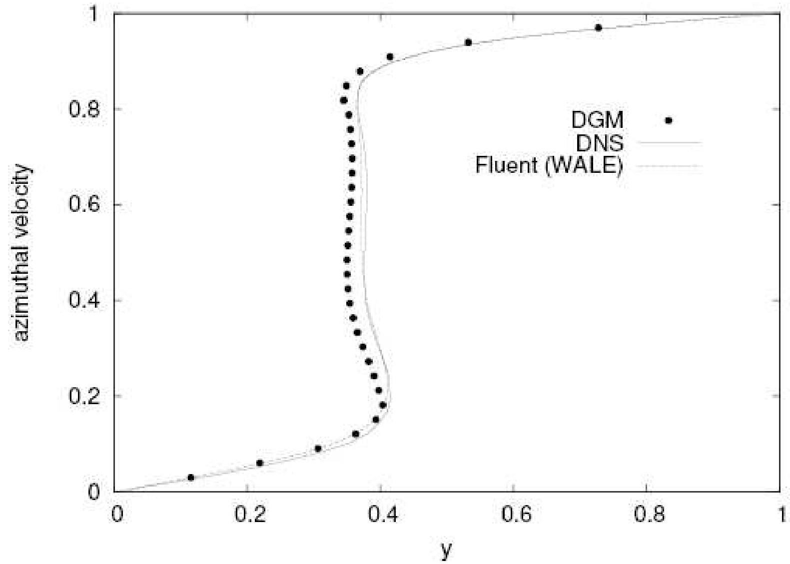
Instantenous flow

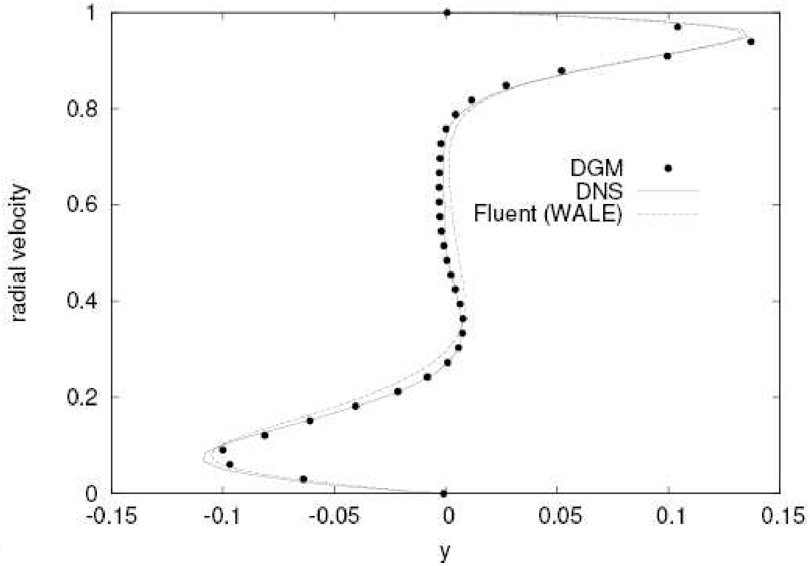


Flow between rotating disks (smooth)

- Comparison with DNS and LES data

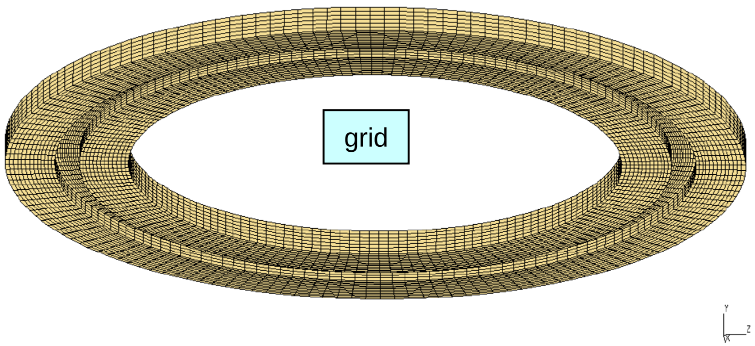




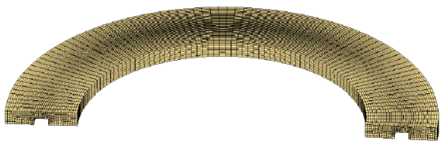


Disks with a step

290×160 el. (total)

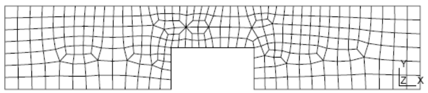


a)



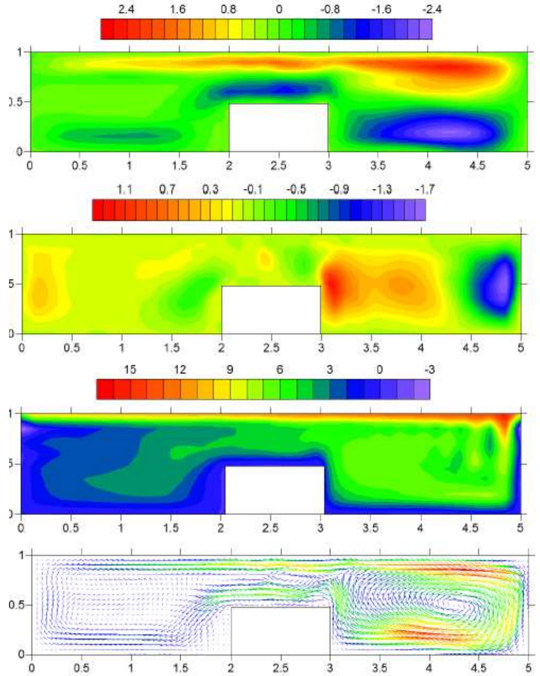
b)

290 elements



c)

Instantenous flow

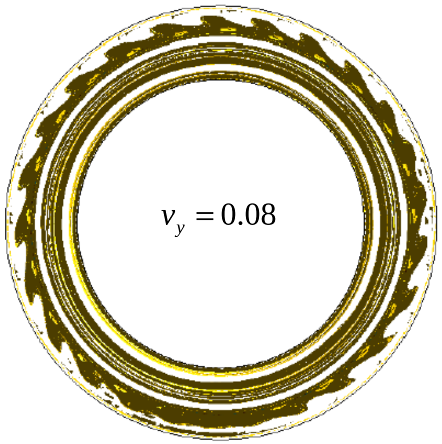


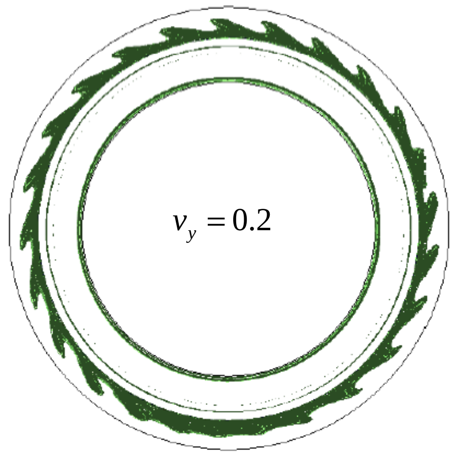
radial

axial

azimuthal

Instability patterns (axial velocity comp. isosurface)





Summary

- Implementation of DGM has been developed (DioGenes code)
  - Full DG treatment of incompressible Navier-Stokes equation (including diffusion terms and Poisson equation for the pressure)
  - Correcteness verified in a number of test cases
    - Cavity flow
    - Temporal shear layer
    - Free, round jet
    - Rotating cavity
- Advantages of the new code:
  - Can be applied to complex geometries (unstructured grids)
  - Due to locality of formulation, parallelisation is very efficient
  - The order of the approximation can be locally changed
  - Simple implementation of *p*-adaptation

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Thank you for your attention