Laminar – Turbulent By-Pass Transition Affected By the Intensity and Scale of Turbulence

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Introduction

Since the earliest investigations, the combined influence of intensity and scale of turbulence was suspected to cause earlier transition.

- enhanced turbulence level of an external flow -
  an earlier boundary layer laminar-turbulent transition
    which increases skin friction and heat transfer.

-the main features of turbulence
  its intensity and scale, related to a velocity along a streamline.

- Most often its dissipation length is used as the length scale.

Assumptions

- the process of by-pass transition in the boundary layer of a plate depends on the mutual interaction of the external flow and the boundary layer from the leading edge LE to point x_t of turbulence inception – and, of course, also further on to the end of transition.

- the history of the flow between these two points (leading edge to x_t) is considered important.

- The turbulence intensity and scale should be averaged between these two points. Here we assume that the integral average is more appropriate to describe the changes of turbulence intensity and scale than the arithmetical average or the Tu and Lu value in the LE

- Hitherto only the value of turbulence intensity (most often) or scale in the leading edge was taken into consideration.

- phenomena that cause Tollmien-Schlichting waves are deliberately neglected .
- So it is of course a simplification!

Goals

- To gain a new insight into some discrepancies in the data available in literature which concern the influence of turbulence scale

- The clou of the contradiction is whether the increasing turbulence scale of the external flow increases or decreases the Reynolds number Re_t^{**} of the inception of laminar-turbulent transition

1. Influence of turbulence intensity

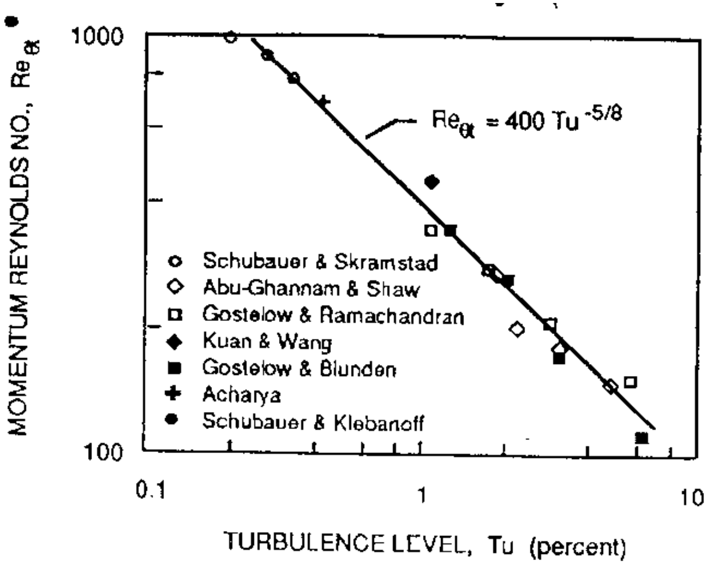
a) Abu-Ghanam and Shaw (1980),

$$Re_t^{**} = 163 + \exp(6.91 - Tu)$$

This correlation has two asymptotes: first, the transition can not occur below Re_t^{**}= 163; second, the transition occurs no later than when Re_t^{**}= 1165 for Tu =0.

b)

$$Re_t^{**} = A\,Tu^{-k}$$



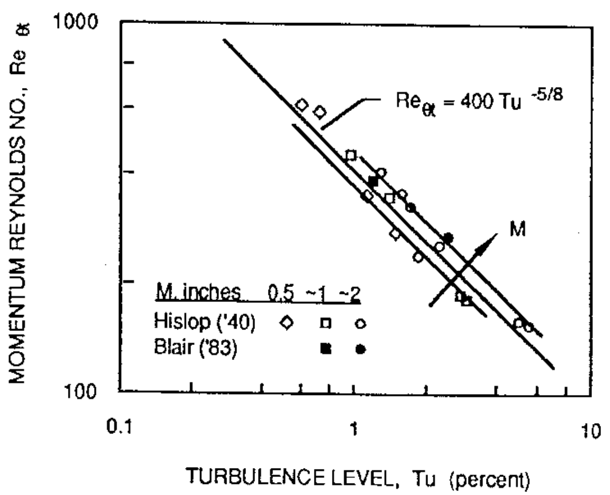
In literature two different pairs for A and k values can be found:
Hourmouziadis (1989) has proposed
A=460 and k=0.65,

while Mayle (1991) has proposed:
A= 400 and k=5/8.

According to Mayle (1991) correlations b) are more appropriate for flows with enhanced turbulence levels, for instance in turbomachinery

1. Influence of turbulence scale

a)



Mayle (1991) cited the results of Hislop, 1940, used also in Hall and Gibbings, (1972) and Blair (1983), where transition appears later when the mesh of the grid is greater.

Here we see that when the mesh size of grid M increases, then the transition inception Re_t^{**} also increases.

b) Jonas et al. (2000) also showed turbulence scale dependence on the inception of transition. They made their investigations for the constant turbulence level Tu=3% at the leading edge of a plate.

$$Re_t^{**} = (245 / Lu)^{0.535}$$

some shortcomings of this correlation:
- the influence of turbulence intensity not taken into account, only one value Tu=3% at the LE
- the length scale is not made dimensionless

There in contradiction to Mayle above, when the length scale M increases then Lu increases, then the Reynolds number of the transition inception Re_t^{**} also increases. - Later transition

And in Jonas correlation – Lu increases than Reynolds number decreases – earlier transition

Programme of investigation

I. Measurements of intensity of turbulence behind the grids

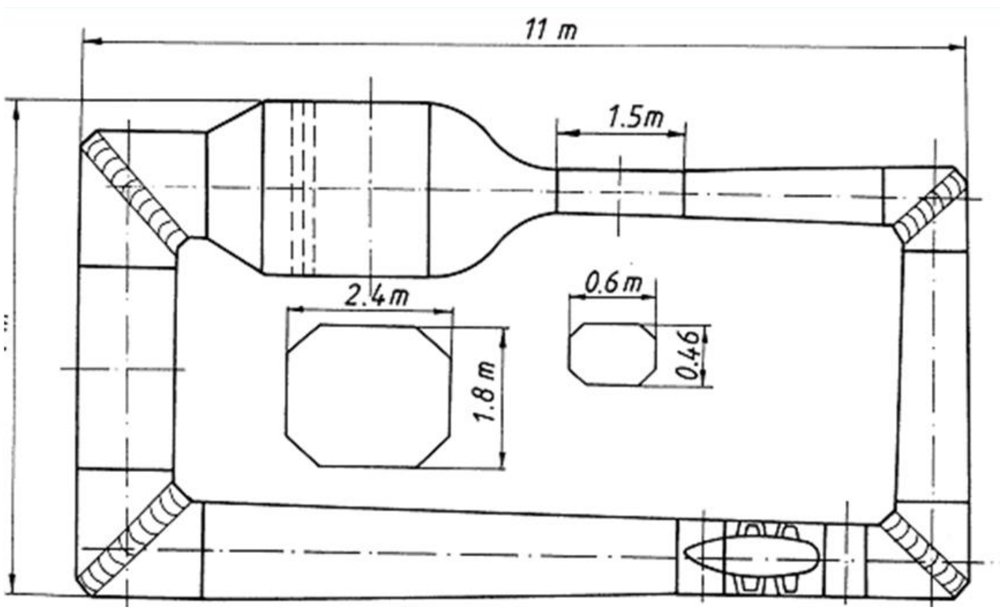
- determination of turbulence scale

II. Measurement of boundary layer parameters, δ, δ^*, δ^{**} and C_f

determination of transition beginning

III. Determination of experimental correlation

Experimental rig



Grid:

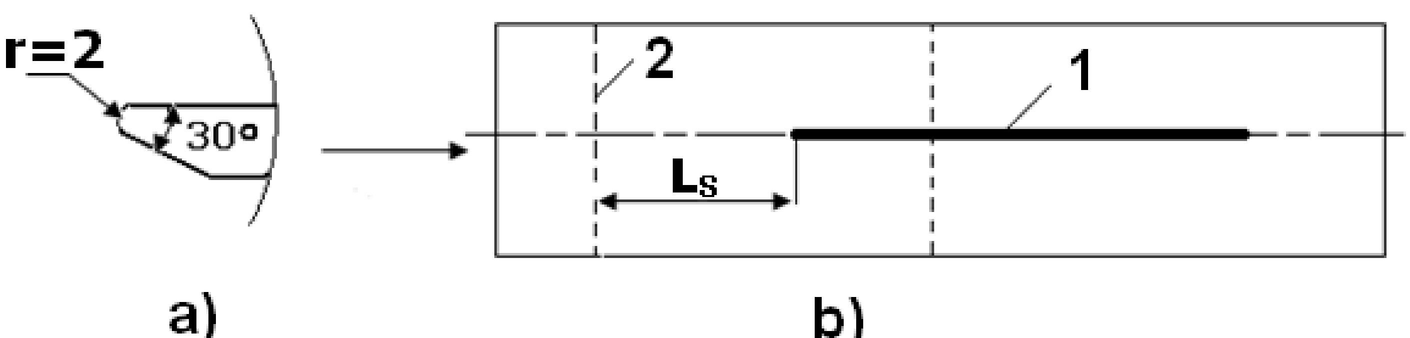
G1) d=0.3 *mm*, M=1 *mm*,
G2) d=0.6 *mm*, M=3 *mm*,
G3) d=1.6 *mm*, M=4 *mm*,
G4) d=3.0 *mm*, M=10 *mm*

Flow velocity:

6, 10, 15 and 20 m/s

Distance L_s between LE and grid

450, 410, 370 and 330 m



Non-dimensional integral parameters for Tu turbulence influence

$$Tu = c\left(\frac{d}{x}\right)^m \left(\frac{M}{d}\right)^p = C\left(\frac{d}{x}\right)^m$$

$$ITu_m = \frac{1}{x_t}\int_{L_S}^{L_S+x_t} Tu\,dx$$

1) Turbulence value in the LE

$$Tu(L_s = LE) = C\left(\frac{L_s}{d}\right)^m$$

2) Arithmetical average of Tu between LE and x_t

$$Tu_m = 0.5\,\mathrm{C}\left[\left(\frac{L_s}{d}\right)^m - \left(\frac{L_s + x_t}{d}\right)^m\right]$$

3) Integral average of Tu between LE and x_t (new)

$$Tu_m = \frac{\mathrm{C\,L_s}}{(1+\mathrm{m})\mathrm{x_t}}\left[\left(\frac{L_s}{d}\right)^m - \frac{L_s + x_t}{d}\left(\frac{L_s + x_t}{d}\right)^m\right]$$

Non-dimensional integral parameters for length scale Lu influence

Dissipative length scale
$$Lu = -\frac{\left[\left(\overline{u'^2}\right)\right]^{3/2}}{U\,\overline{\dfrac{\partial(u'^2)}{\partial x}}}$$
$$Lu = \frac{1}{2m}\, x\, Tu$$

1) Length scale at the leading edge
$$Lu(L_s) = C\, L_s \left(\frac{L_s}{d}\right)^m$$

2) Arithmetical average of Lu between LE and x_t

$$Lu_m = C\left[L_s\left(\frac{L_s}{d}\right)^m - (L_s + x_t)\left(\frac{L_s + x_t}{d}\right)^m\right]$$

3) Integral average of Lu between LE and x_t (new)

a) $$ILu_{\delta,m} = \frac{1}{x_t}\int_{L_S}^{L_S + x_t} \frac{Lu}{\delta^{**}}\, dx$$

b) $$ILu = \frac{1}{x_t}\int_{L_S}^{L_S + x_t} Lu\, dx$$

$$\delta^{**} = 0.664\, x\, \mathrm{Re}_x^{-0.5}$$

After completing of integration
$$Lu_{\delta,m} = \frac{2Cd}{K_\delta}\left(\frac{L_s}{d}\right)^{2/7}\sqrt{\frac{\nu}{x_t U}}\;{}_2F_1\left[\frac{1}{2},\frac{5}{7},\frac{3}{2};-\frac{x_t}{L_s}\right];\ m = -5/7$$

New correlations for transition inception

$$\mathrm{Re}_t^{**} = k \cdot ITu_m^{m1} \cdot ILu_{\delta,m}^{m2}$$

$$\mathrm{Re}_t^{**} = k \cdot ITu_m^{m1} \cdot (ILu / d)^{m2}$$

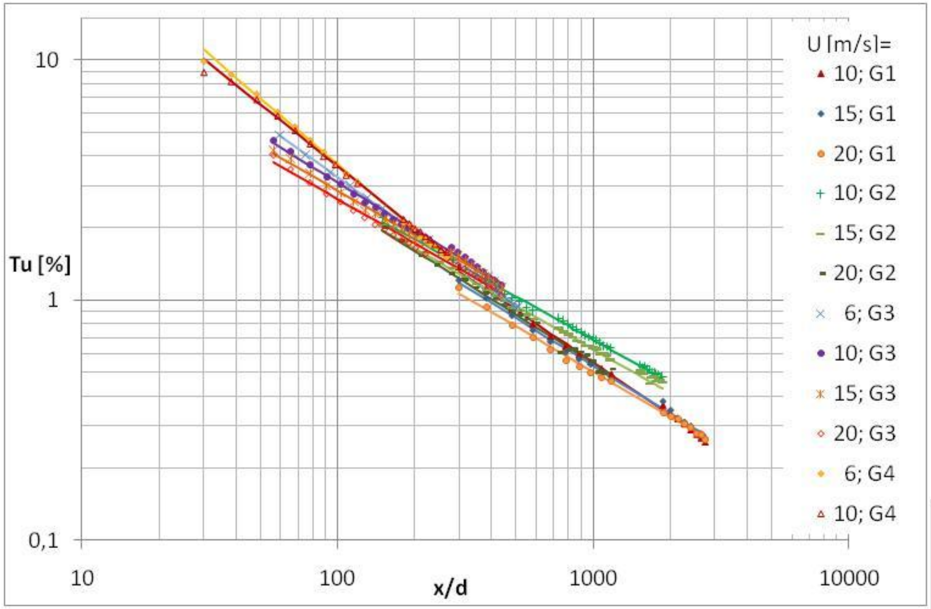
This correlation is based on an experimental investigation where the turbulence decay law behind the grid, the length scale variation, the momentum thickness of the boundary layer and the point of turbulence inception x_t are all measured.

-a generalization and extension of the correlation given by Hourmouziadis and Mayle, and also Jonas

- more complex - the integral averaged parameters ITu_m and $ILu_{\delta,m}$ or ILu/d have the value x_t in their integral upper limit and also on the left side of the correlation,

- better describes the complexity of the phenomenon of transition inception because the location of the point x_t depends on the flow history, especially the history of velocity fluctuations in external flow between the LE and the x_t

Investigation results, turbulence instensity Tu and scale Lu



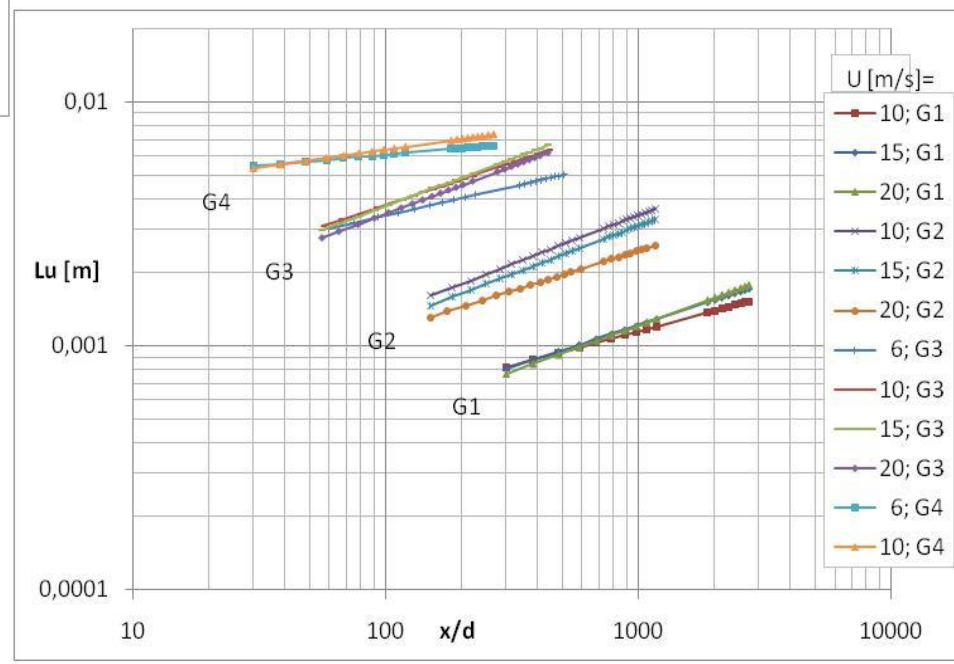
Turbulence decay behind the grid; L_s*=450mm*

Grid:
G1) d=0.3 *mm*, M=1 *mm*,
G2) d=0.6 *mm*, M=3 *mm*,
G3) d=1.6 *mm*, M=4 *mm*,
G4) d=3.0 *mm*, M=10 *mm*

In connection to Mayle's review
- the greater the M
the greater the length scale Lu

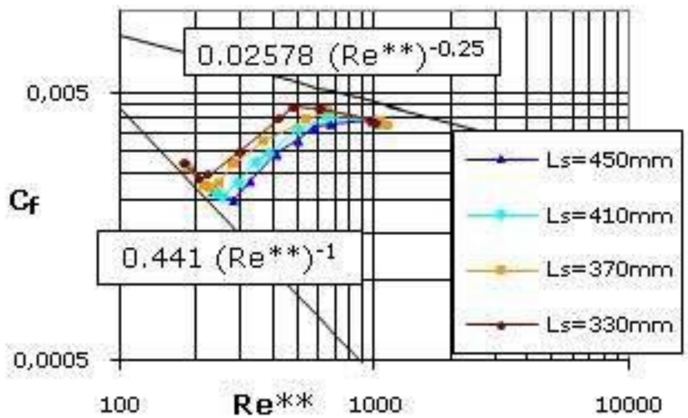
$$Tu = c\left(\frac{d}{x}\right)^m \left(\frac{M}{d}\right)^p = C\left(\frac{d}{x}\right)^m$$

c=0.193÷2.357,
m =0.562÷0.943,
p≈0.15



Scale of turbulence behind the grid

Determination of transition inception point x_t



The local shear stress coefficient C_f for Grid2, velocity 15 m/s, and four different distances L_s

$$\gamma(x) = 1 - \exp\left(- \frac{\mathrm{Re}^{**} - \mathrm{Re}_t^{**}}{\mathrm{Re}_\theta^{**} - \mathrm{Re}_t^{**}} \right)^{\alpha}$$

Weibull distribution function

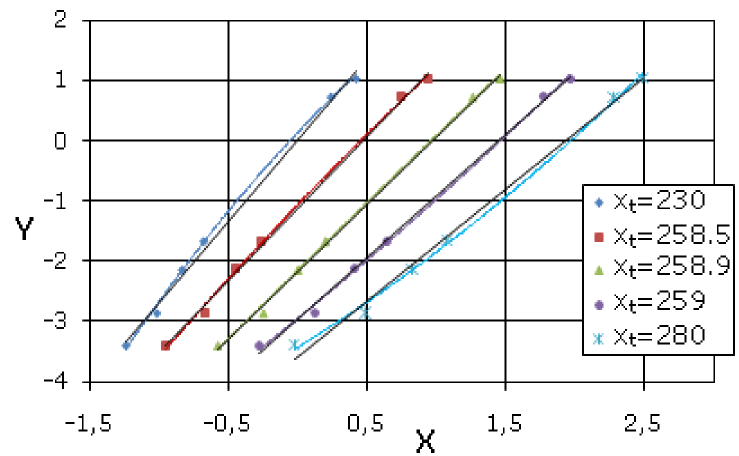
$$Y = \ln\ln\left(\frac{1}{1-\gamma}\right) \qquad X = \ln\left(\frac{\mathrm{Re}^{**} - \mathrm{Re}_t^{**}}{\mathrm{Re}_\theta^{**} - \mathrm{Re}_t^{**}}\right)$$

$\mathrm{Re}_\theta^{**} = \mathrm{Re}^{**}(\gamma = 0.632)$,

intermittency factor

$$\gamma = \frac{C_f - C_{fl}}{C_{ft} - C_{fl}}$$

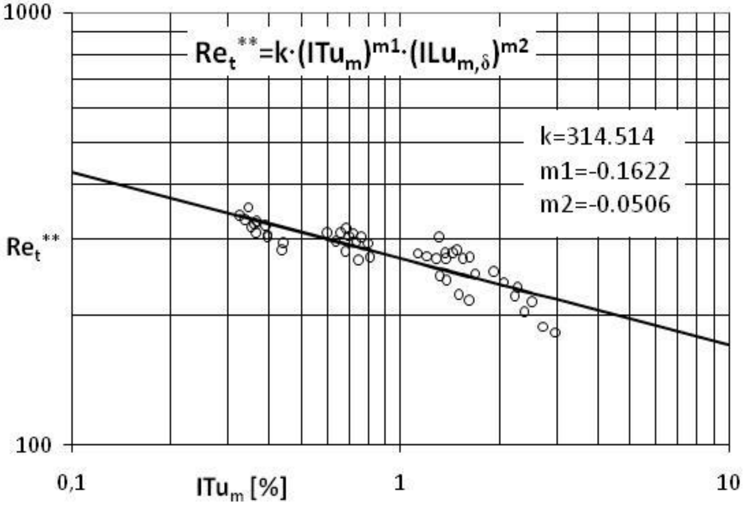
$Y = \alpha X$



$\mathrm{Re}_t^{**} = \mathrm{Re}^{**}(\gamma = 0.05)$, $Y = -2.95$

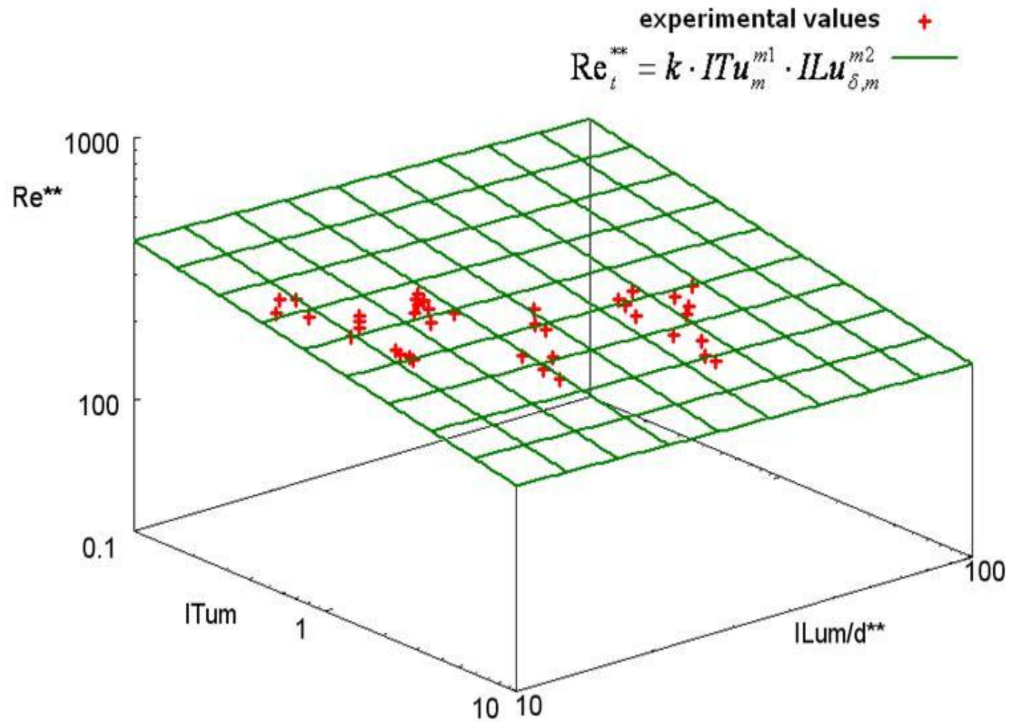
$x_t = 258{,}9$ mm for best $r = 0.9989$

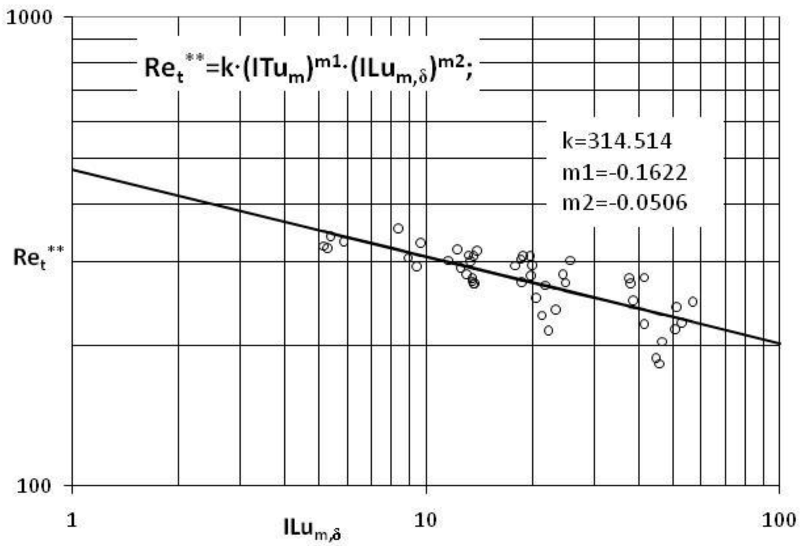
Transition inception correlation (first)



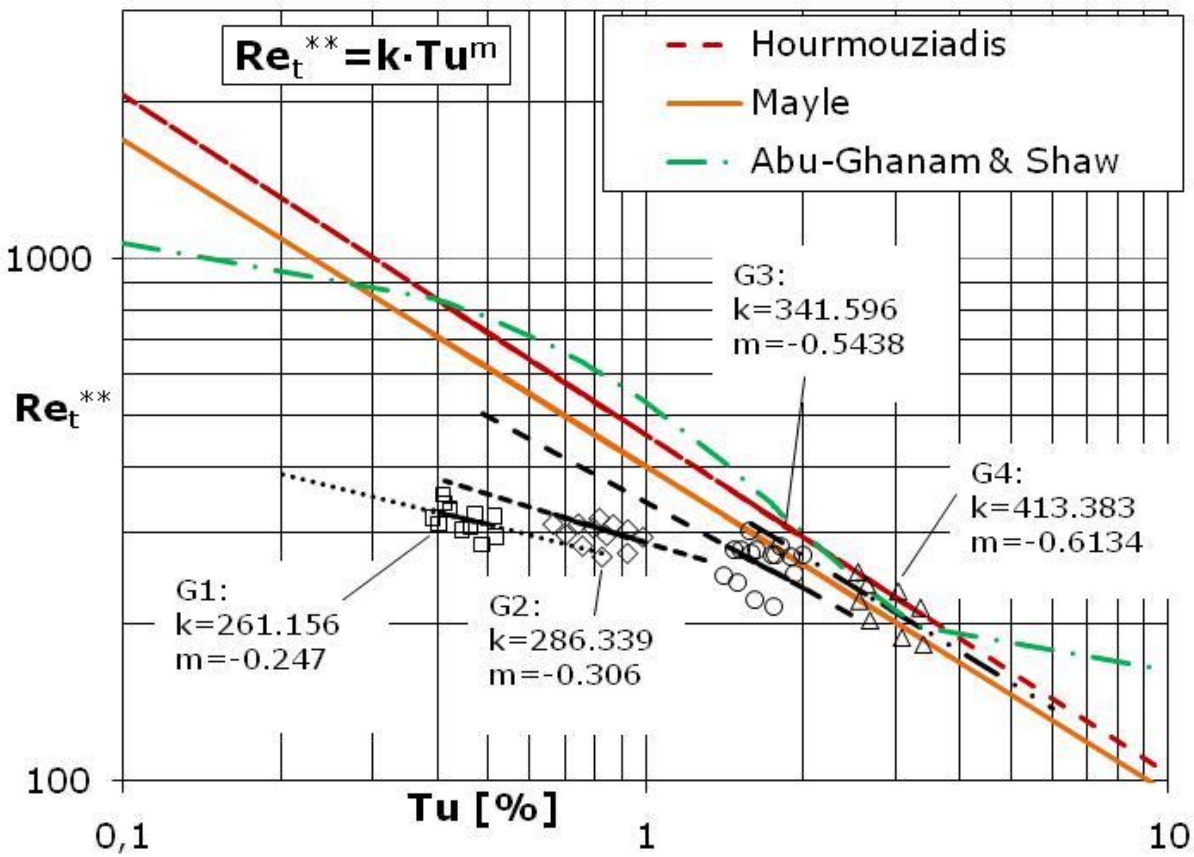
$$Re_t^{**} = k \cdot ITu_m^{m1} \cdot ILu_{\delta,m}^{m2}$$

k = 314.44
m1=-0.1618
m2=-0.050





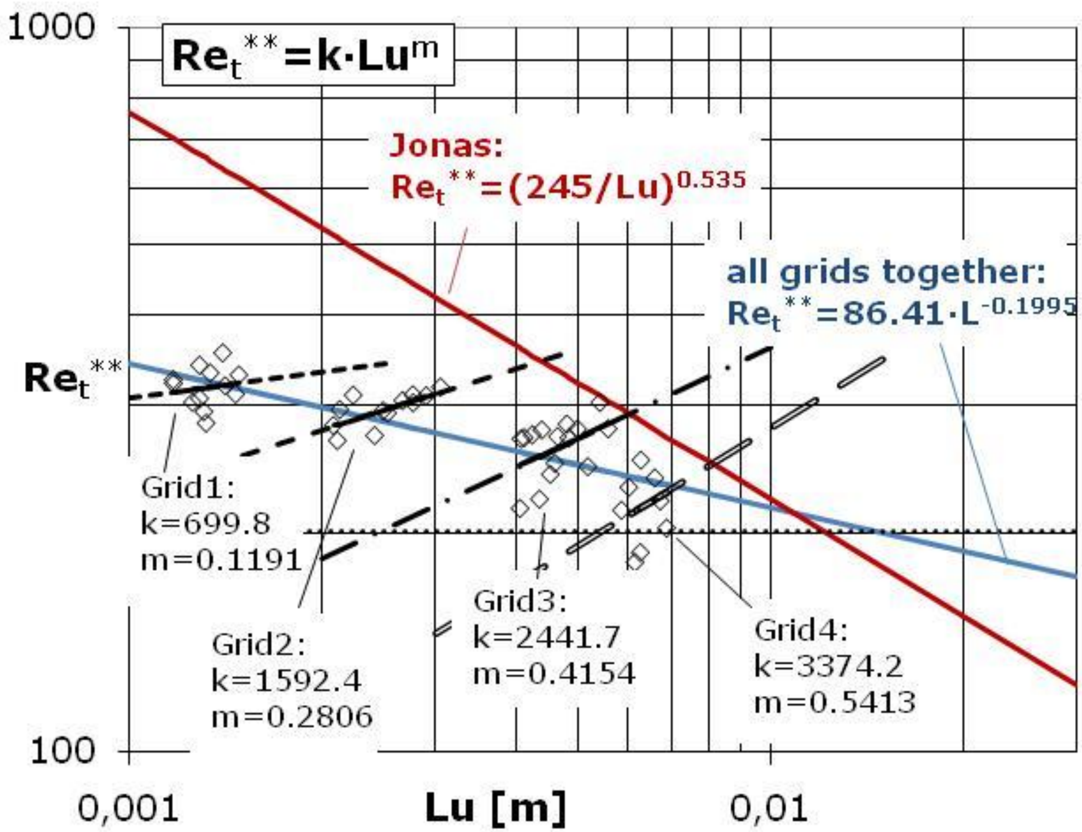
Comparison with literature



Results of G3 and G4 where the turbulence level is equal 1 to 4 percent, converge quite well with results known from the literature,

There are some discrepancies for the grid G1 and G2 where the turbulence level are below 1 %.

Comparison with literature



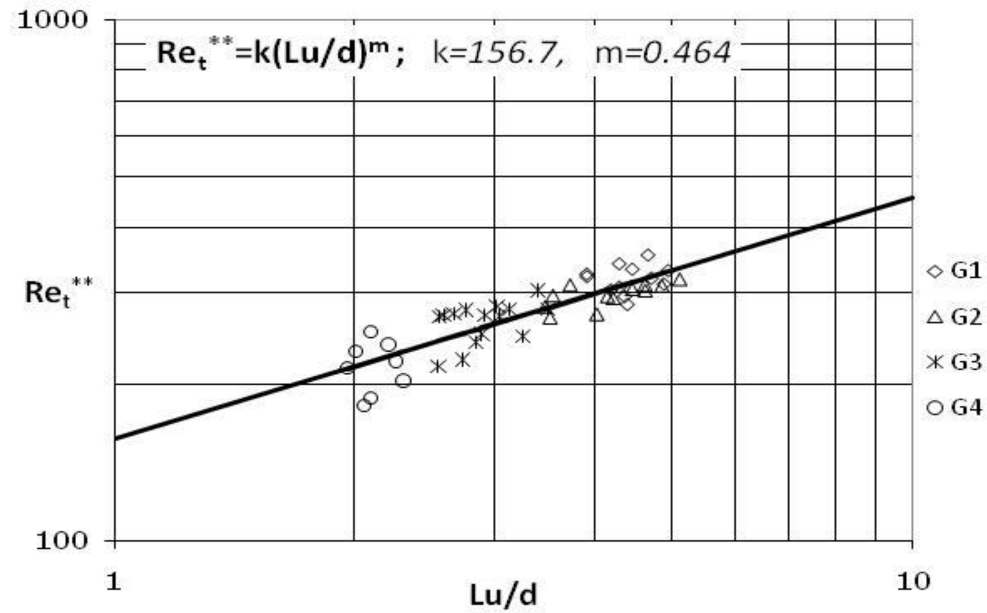
Correlation by Jonas- red line
.
Re_t^{**} decreases
as *Lu* increases,

also true in our investigation :
if we make a correlation for all grids together., blue line
There is also such an inclination i.e. Re_t^{**} decreases when the Lu increases

But when we make separate correlations for individual grid, the result appears to be quite the opposite.
Re_t^{**} for each individual grid increases as *Lu* increases.
Besides, the higher the values of *d* and *M*, the higher the value of coefficient *k*.

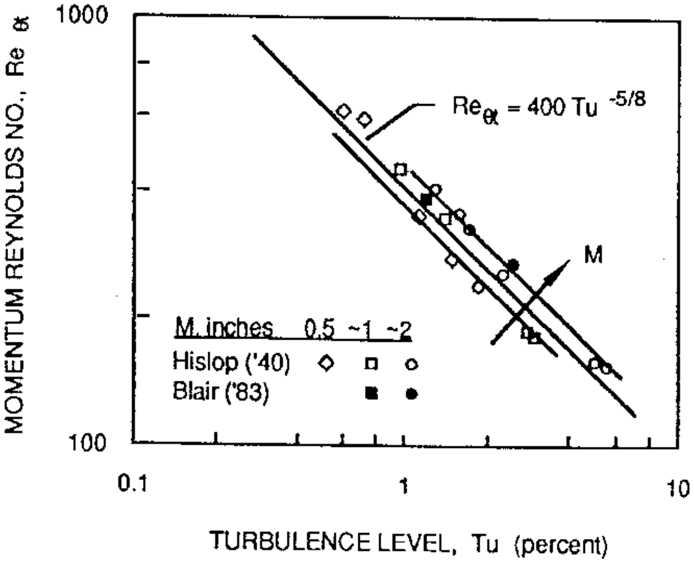
Our results

Re_t^{**} increases when Lu/d increases



Results reported by Mayle

Re_t^{**} increases when M increases



•Interesting result is shown where Re_t^{**} is demonstrated versus non-dimensional Lu /d
d- diameter of grid wire.

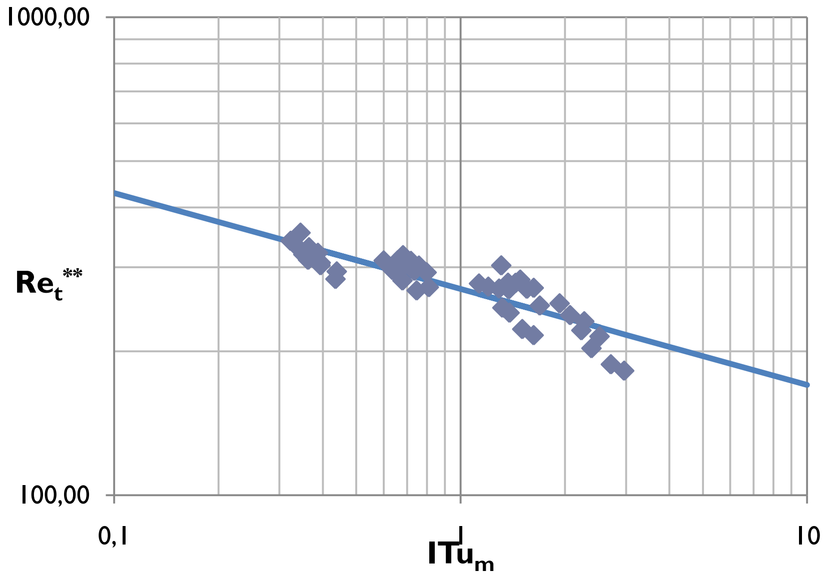
•quite good linear correlation is obtained with increasing Re_t^{**} with increasing Lu/d.

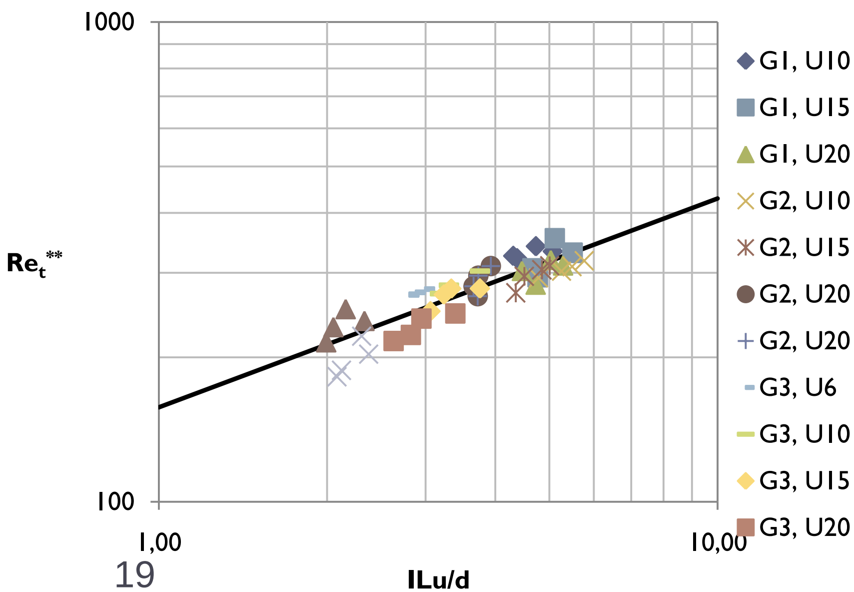
Transition inception correlation (second)

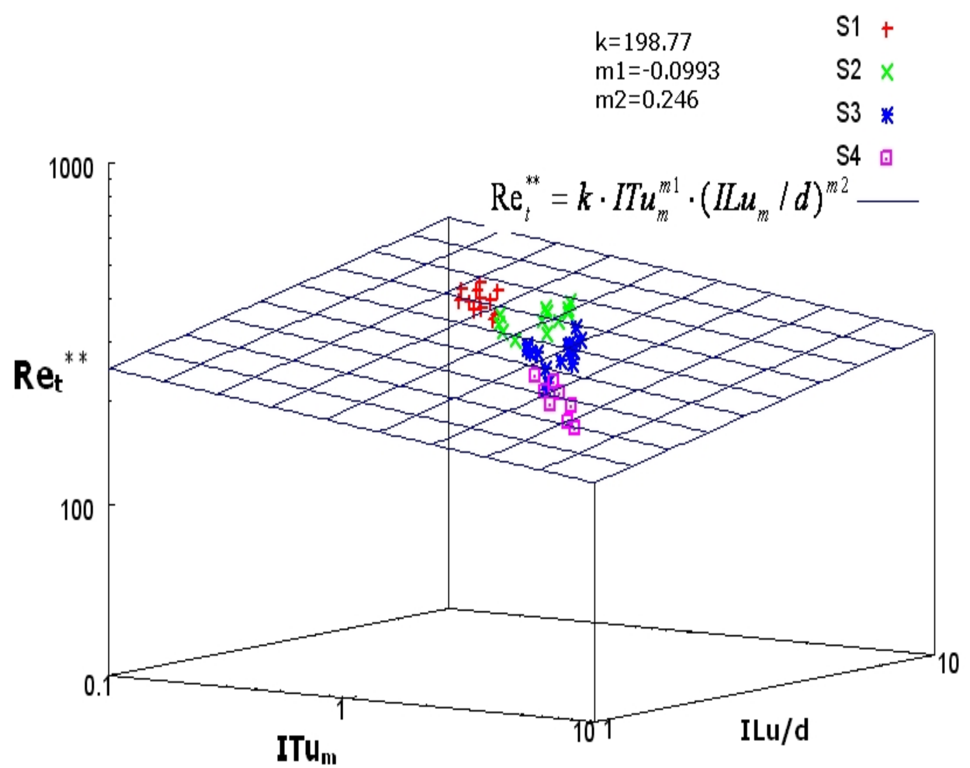
$$Re_t^{**} = k \cdot ITu_m^{m1} \cdot (ILu/d)^{m2}$$

k = 199
m1=-0.099
m2=0.246

Corr. coeff. 3D regression:
r12=-0.87, r13= 0.877,
r23= -0.887







Conclusion

- Two non-dimensional mean parameters for the turbulence intensity ITu_m and scale ILu as an averaged integral values from the leading edge to the point of transition inception are applied to get the correlation,
- with m1=-0.099, and m2=0.246 and k=199

$$\mathrm{Re}_t^{**} = k \cdot ITu_m^{m1} \cdot (ILu / d)^{m2}$$

- So Re_t^{**} increases, when Lu/d increases,
- thus it can be supposed that the results cited in Mayle are rather correct
- It is clear that initial conditions (wire diameter of grid) of turbulence generation have rather strong influence on the proces of transition
- To clarify some doubts more investigation is needed